A HOLISTIC APPROACH FOR DECIDING ON AND DESIGNING COLLISION AVOIDANCE MANEUVERS

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ABSTRACT

Space traffic congestion has significantly increased the number of conjunctions that could potentially lead to collisions. Consequently, the workload associated with conjunction analysis, the decision on the need to perform a collision avoidance maneuver, and, where appropriate, its design, has increased significantly. It is necessary to improve the tools available for decision support and for the detailed design of the maneuver to cope with the current situation. This paper presents a framework that supports satellite operators in the decision-making and design process for a collision avoidance maneuver. In a first step, a multi-objective problem is posed and solved to identify the most appropriate maneuver, given the desired risk minimization, compatible with other objectives such as the time required for return to the nominal orbit or the required propellant. Once the most appropriate maneuver has been identified, a constrained optimal control problem is proposed, the solution of which provides the optimal maneuver compatible with the platform and operations. This framework has been tested with real-life case studies using CDMs. This paper presents preliminary results of the tool, which verify the feasibility of the proposal and its ability to provide a holistic solution to the problem of deciding and designing collision avoidance maneuvers.

Keywords: collision avoidance; Genetic algorithm; Nonlinear programming.

1. INTRODUCTION

Collision avoidance is one of the main pillars of the strategy to mitigate the growth of the space debris population. The current trends in space traffic have significantly increased the number of objects in LEO and, accordingly, the number of conjunctions. Satellite operators must deal with this new situation with new tools to avoid high operating costs. To achieve this, among other improvements, it is necessary to have tools that help better decision-making when carrying out collision avoidance maneuvers and that allow these maneuvers to be designed in an efficient and safe manner. This translates into the requirement to properly calculate the metrics associated with the collision risk (e.g., the probability of collision or miss-distance), to carry out a screening of the possible future conjunctions considering the maneuvering plan, to take into account the uncertainty associated with the realization of the maneuver, and to carry out safe maneuvers in case of a sudden propulsion system failure in the middle of avoidance operation.

This field has been the subject of intense research in the last years. Solutions of different optimization formulations using different optimization methods have been studied in the literature. Bombardelli and Hernando-Ayuso [1] explored several analytical and semi-analytical methods to solve the minimum energy problem to get the lowest Δv to meet a desired distance in the encounter plane or to maximize the statistical or Euclidian distances for a given Δv. Gonzalo et al. [2] also implemented Gauss' Planetary equations to analytically solve for an impulsive avoidance maneuver when maximizing the statistical or Euclidian distances. Different heuristic algorithms have also been explored in literature: genetic algorithms have been implemented to solve for a single impulsive avoidance maneuver by Lee et al. [3] as well as to solve for avoidance of multiple threats by Kim et al. [4]. Gradient-based direct and indirect optimization has been used to solve several formulations which include energy-optimal, fuel-optimal, and time-optimal problems with some even considering uncertainty reduction along the way [5].

Although the works cited have addressed the problem from multiple perspectives, few of them have considered the problem as a multi-objective problem and were focused on only one objective function. To make informed decisions about whether and how to perform a collision avoidance maneuver, it is necessary to consider the impact of the maneuver on multiple aspects related to satellite operations. This is the rationale behind posing the problem as a multi-objective optimization problem, in which the user can evaluate and assess how a given CAM affects the payload time off, the propellant cost, the decrease in PoC and/or miss-distance, amongst others. This first stage consists of a global search based on surrogate models for supporting decision making. Based on the operator decision, an initial guess for the trajectory and maneuver is selected, as well as the weights of the

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different cost functions. In a second stage, the single-objective optimization problem is solved using a sequential programming approach in which all the constraints relevant for the maneuver are taken into account. From this perspective, the second stage is a multidisciplinary optimization problem, in which models for the satellite subsystems, such as propulsion, power, AOCS, etc., are incorporated to constraint the search space and to produce the optimal feasible maneuver compatible with collision risk mitigation. In this line, Dutta and Misra [6, 7] explored the effect of convex and non-convex optimization for avoidance trajectories along with return when considering linear and nonlinear evolution of uncertainties. Alternatively, De Vittori et al. [8] developed an analytical method concerning avoidance from short-term encounters with low thrust using indirect optimization. In addition, some recent works have already explored the capabilities of sequential quadratic programming [9] with second-order cone programming, and differential algebra in the context of multiple encounters.

The method is tested on realistic scenarios involving different orbit regimes – LEO and GEO. The performance of the short-term encounter model is tested in the GEO regime when it comes to providing an initial CAM hypothesis in the global optimization problem. The method is also tested to comply with different propulsion types and accommodate several uncertainties like control actuation errors, temporal and spatial uncertainties in CDM information, etc.

2. GENERAL APPROACH

The collision avoidance maneuver design is posed as a multi-objective optimization problem because the realization of the CAM has an impact on different aspects of the mission life, in addition to the reduction of the collision risk. The propellant consumption, the off time of the payload, the required related attitude maneuvers are to be considered jointly with the risk reduction in order to decide which CAM is the optimal. The solution to a multi-objective optimization problem is the set of Pareto optimal solutions, that is, solutions that cannot be improved in any of the objectives without degrading at least one of the other objectives. Although there are other philosophies to obtain the Pareto front (set of Pareto optimal solutions), when decision making is emphasized, the objective is to inform the decision maker about all the possible Pareto optimal options and the corresponding trade-off, supporting him or her in finding the most preferred solution according to their preferences. The ultimate goal is the selection of one solution that will be implemented in practice. In this way, the human decision maker, an expert in the problem domain, is at the core of the optimization process. There are different philosophies for solving multi-objective optimization problems, depending on how the information is provided. The classical classification of the methods is: i) no-preference methods, in which a neutral compromise solution is identified without preference information, ii) *a priori* methods, where preference information is first asked from the decision maker, and then a solution best satisfying these preferences is found, iii) a posteriori method, in which a representative set of Pareto optimal solutions is first found, and then the DM must choose one of them and iv) interactive methods, the decision maker is allowed to search for the most preferred solution iteratively. In the case at hand, we opt for a posteriori method, providing the decision maker with all the required information for the selection of the type of CAM to be implemented.

According to the strategy followed by the optimization methods to find the set of Pareto optimal solutions, they can be classified as mathematical programming-based, evolutionary algorithms and deep learning algorithms. The mathematical programming-based methods are based on the scalarization of the problem to find sequentially individual solutions of the Pareto-front. From that point of view, they can be computationally expensive and were discarded. Deep learning conditional approaches have recently arisen as methods to build Pareto fronts from a reduced data set of optimal points [6]. These methods were not considered suitable for the problem at hand because they do not provide any information about the characteristics of the CAM, only its performance. Finally, evolutionary algorithms are commonly used in solving multi-objective optimization problems. In general, they implement Pareto-based ranking schemes to directly provide the Pareto front without scalarization.

Among them, Genetic Algorithms (GAs) have been selected for its adaptability for different problems and types of optimization variables. The method evolves the initial population towards the global optimal solution by checking the performance of the chosen candidates based on the defined objectives. Since these methods involve direct comparison of each candidate solution against the objectives and does not involve any gradient method to move towards optimality, the objectives, constraints or dynamical functions do not have to be continuous or differentiable like in the direct or indirect optimization methods. So, these methods can handle complex dynamical systems along with more intricate constraints and solve for the global optimal solution. However, the performance of the method depends on the number of parameters to be tuned. The selection of the initial population plays a role in the final converged solution. If the initial population is not diverse enough, the entire feasible solution space will be more difficult to explore in order to find the optimal global solution. The algorithm also requires the crossover and mutation schemes and values to be chosen to determine how to combine the properties of the acceptable candidates from a generation and introduce novel characteristics to explore and create better candidates in the following generation to move the closer to the global optimal solution. Restricting genetic changes across the generations can lead converging to a locally minimum solution whereas a high frequency of genetic change can lead to failure of efficient transmission of useful information from one generation to the next. Thus, adjusting these parameters is a trial-and-error process and so the global optimal solution is not guaranteed. Unfortunately, GAs are computationally intensive. For that reason, it is advisable to use surrogate models to reduce the computational cost of the algorithm. It is also advisable to guide, to the extent possible, the search of the GA defining individuals with an a priori satisfactory performance. This is the justification for resorting to analytical or semi-analytical methods for the computation of the performance of the individuals (here individuals correspond to the minimum number of variables required to completely define a unique collision avoidance maneuver). An analytical formulation can provide quick solutions. However, deriving analytical results involves simplifying the problem, the constraints alongside the use of quite a few approximations. In section 3, we provide the survey of methods included in the performance computation within the GA framework.

A multi-objective formulation of the collision avoidance problem can provide a global view of the possible trade-offs between objectives, but the surrogate models using for exploring the search space do not provide the required accuracy or level of detail for the maneuver design. Therefore, in a second stage, it is mandatory to solve the full collision avoidance problem without any assumptions and approximations. In this form, it is a non-convex optimization which has multiple local minima, so the manner of formulating the problem and the optimization method chosen plays crucial role in determining the solution time and the converged optimum. Therefore, a gradient-based solver is used within this context to provide the single-objective solution of the problem. In section 4, we include a literature review on works using gradient-based methods for the design of collision avoidance maneuvers, and the formulation used in this work.

3. MULTI-OBJECTIVE OPTIMIZATION

The general mathematical formulation of a multi-objective optimization problem is as follows:

$$\max_{\mathbf{x}\in X} \ \mathbf{f}(\mathbf{x}) \ , \tag{1}$$

with $X \subset \mathbb{R}^n$ and $\mathbf{f} \in \mathbb{R}^m$. Genetic algorithms generate a population of random individuals $\mathbf{x}_i \in X$ and evaluate their performance $\mathbf{f}(\mathbf{x}_i)$. According to their performance, the individuals are sorted and combined for producing a new generation. For details on the operation of non-sorted we refer to [7]. Fast evaluation of the performance is therefore key to keep the computational cost of the GA as low as possible. Thus, we explored analytical and semi-analytical methods that can help in exploring the search space in an efficient manner.

In this line, Bombardelli performed several studies developing fast analytical and semi-analytical methods to solve for a single impulsive avoidance maneuver. The problem formulations are concerned with the avoidance of the oncoming space object and do not deal with the return of the satellite back to its nominal orbit. An analytical linear relationship is established between the impulsive Δv and the displacement at TCA in [8]. The relationship is derived under the assumption that the velocity vector at the time of closest approach is the same as the nominal velocity vector even after the application of the impulsive maneuver at time t_m (where $t_m < t_c$). The derivation also assumes a Keplerian orbit and the fact that the displacement due to the avoidance maneuver is much smaller when compared to the orbital radius of the satellite. The linear dependence of the displacement on the impulsive Δv aids in framing the objective function as a quadratic function which can later be solved as an eigenvalue problem when maximizing the miss distance or minimizing the collision probability (in the form of maximizing the Mahalanobis distance in the b-plane at TCA). In [1], the optimal single impulsive maneuver direction is solved using the eigenvalue problem for a given Δv magnitude to find the maximum Euclidean and Mahalanobis distance possible. The gravitational perturbations were also found to have negligible effects on the result obtained. Ayuso [9] also designed for the minimum Δv requirement for pre-determined Mahalanobis distance or Euclidean distance in the b-plane using a semi-analytical method. The problem needs to be solved at several maneuver times (t_m) to find the best maneuver location resulting in the lowest Δv requirement to equal a certain Mahalanobis distance at the TCA. Although the Bombardelli's method for computing PoC is limited to LEO and high-velocity conjunctions, it is proposed to extend the utilisation of the method in MEO and GEO. The rationale for that is the use of the Mahalanobis distance as proxy for PoC and risk metric. In LEO, Mahalanobis distance at TCA works perfectly well as a proxy for risk. In MEO and GEO, there exists the possibility that this proxy does not work as well as in LEO, because TCA is less deterministic (better described by a probability density function). Notwithstanding, Bombardelli's method is a perfect candidate as a surrogate model because one would not expect big changes in tendency / meaning in the Pareto front obtained with the surrogate versus the full optimal control problem.

Assumptions used in the method are listed below:

- A Keplerian orbit is assumed. It was also shown that perturbations like J2 does not have much effect on the result [1].
- The displacement of the satellite for the avoidance is much smaller than its orbital radius.
- The avoidance maneuver conducted at maneuver time (t_m) results in no changes in the nominal velocity vector of the satellite at the time of closest approach (TCA = t_c).

The formulation uses Pelaez' orbital elements (also known as DROMO elements) to derive the deviations in radial, out-of-plane directions and the accumulated time delay from the nominal at t_c resulting from the impulsive Δv at t_m. Using the assumption, the unchanged (nominal) velocity vector at t_c is then used to resolve these deviations into deviations in the b-plane at the TCA (t_c).

Thus, the method establishes a direct linear relationship between the Δv applied at a chosen maneuver time (t_m) and the resulting displacement in the b-plane at the time of closest approach (t_c).

$$\mathbf{r} = \mathbf{r_e} + \mathbf{M}\mathbf{\Delta v} \tag{2}$$

where $\mathbf{r}$ is the relative position between the primary and secondary at TCA, $\mathbf{r_e}$ is the nominal miss distance at TCA and $\mathbf{M}$ is the matrix establishing the linear relationship between the $\mathbf{\Delta v}$ components in the RTN frame and the displacement vector in the encounter plane.

The chosen procedure is a semi-analytical method when trying to solve for the minimum Δv requirement while the primary satellite is constrained to be at a fixed Mahalanobis distance (M_{d_0})from the secondary object at the time of closest approach.

Cost function: $J = \mathbf{\Delta v}^T \mathbf{\Delta v}$

Constraint: $\mathbf{r^T P^{-1} r} = M_{d_0}^2$

where is the $\mathbf{\Delta v}$ vector in the RTN frame, $\mathbf{P}$ is the covariance associated with the position coordinates in the encounter plane. Using $\mathbf{P}$ as identity matrix would convert the Mahalanobis distance into a desired Euclidean miss distance.

Thus, the Lagrangian function (L) can then be written as:

$$L(\Delta V, \lambda) = \Delta \boldsymbol{V}^T \Delta \boldsymbol{V} + \lambda \left(\left(\boldsymbol{r}_e + \boldsymbol{M}\Delta \boldsymbol{V}\right)^T \boldsymbol{P}^{-1} \left(\boldsymbol{r}_e + \boldsymbol{M}\Delta \boldsymbol{V}\right) - M_{d_o}^2 \right)$$

Now letting $\boldsymbol{A} = \boldsymbol{M}^T \boldsymbol{P}^{-1} M$, $\boldsymbol{b}^T = \boldsymbol{r_e}^T \boldsymbol{P}^{-1} M$, $c = \boldsymbol{r_e}^T \boldsymbol{P}^{-1} \boldsymbol{r_e} - M_{d_o}^2$

$$L(\Delta V, \lambda) = \Delta \boldsymbol{V}^T (\boldsymbol{I} + \lambda \boldsymbol{A}) \Delta \boldsymbol{V} + 2\lambda \boldsymbol{b}^T \Delta \boldsymbol{V} + \lambda c \tag{3}$$

To find the $\mathbf{\Delta v}$ for which L is minimum:

$$\frac{\partial L}{\partial \Delta \boldsymbol{V}} = 0 \Rightarrow \Delta \boldsymbol{V} = -\lambda (\boldsymbol{I} + \lambda \boldsymbol{A})^{-1*} \boldsymbol{b} \tag{4}$$

where -1*: pseudo-inverse.

Using the eigen values (λ_1^*, λ_2^*), eigenvectors ($\mathbf{s_1^*}, \mathbf{s_2^*}$) of $\mathbf{A}$ and the dual formulation of the Lagrangian, root of the following function F(λ) would provide the required λ to be substituted in the previous equation to obtain the optimal $\mathbf{\Delta v}$.

$$F(\lambda) = \frac{\left(s_1^{*T} b\right)^2 \lambda \left(2 + \lambda_1^* \lambda\right)}{c \left(1 + \lambda_1^* \lambda\right)^2} + \frac{\left(s_2^{*T} b\right)^2 \lambda \left(2 + \lambda_2^* \lambda\right)}{c \left(1 + \lambda_2^* \lambda\right)^2} - 1 = 0 \tag{5}$$

The above equation can be solved using Newton's method starting from the initial guess($\lambda^{(0)}$):

$$\lambda^{(0)} = \max \left(\frac{1}{\lambda_1^*} \left(-1 + \frac{1}{\sqrt{1 - \frac{\lambda_1^* c}{\left(s_1^{*T} b\right)^2}}} \right), \frac{1}{\lambda_2^*} \left(-1 + \frac{1}{\sqrt{1 - \frac{\lambda_2^* c}{\left(s_2^{*T} \boldsymbol{b}\right)^2}}} \right) \right) \tag{6}$$

Since this method solves for the minimum Δv that would be required at a certain time (t_m) for the satellite to meet the constraints at t_c, therefore the problem must be solved at different time steps leading up to the close approach at t_c to find the time that would require the lowest possible Δv to meet the constraints. This can be implemented by the genetic algorithm which can solve this method with different t_m, where t_m can be a gene (optimization variable) of an individual candidate (chromosome).

Now considering the return to nominal true anomaly using impulsive maneuvers, it can be defined in two ways:

- Return to the nominal orbit at the nominal true anomaly: This is especially useful for constellation satellites, sun-synchronous satellites, GEO satellites where respecting the assigned true anomaly at a given time is of utmost importance for their operations.
- Return to the nominal orbit at an arbitrary true anomaly in the orbit: This can be chosen for the satellites where only traversing the orbit path matters. This return approach can be employed if it delivers a more fuel-efficient return maneuver compared to that when returning to the nominal true anomaly.

When the nominal true anomaly is to be matched, Lambert's solution can be used to position the satellite from the avoidance trajectory after the TCA to the desired position in the nominal orbit in the required time to match the nominal true anomaly. The primer vector can be used to adjust the maneuver times to find the fuel-optimal orbital transfer from the avoidance trajectory to the nominal trajectory [28]. Finding the fuel-optimal orbit can involve changing the initial and final maneuver times for return or using additional mid-course impulses depending on the profile of the magnitude of the primer vector. Depending on the other mission and avoidance requirements meeting those optimal conditions may or may not be possible, such as, increasing the final time for return or adding operational burden by adding more impulses for a small gain in Δv. The primer vector ($\mathbf{p}$(t)) follows the same dynamical system as the orbit states for this optimal transfer problem as follows [28]:

$$\begin{bmatrix} \boldsymbol{p}(\mathrm{t}) \\ \dot{\boldsymbol{p}}(\mathrm{t}) \end{bmatrix} = \boldsymbol{\Phi}(t, t_0) \begin{bmatrix} \boldsymbol{p}(\mathrm{t}_0) \\ \dot{\boldsymbol{p}}(\mathrm{t}_0) \end{bmatrix} \tag{7}$$

where $\mathbf{\Phi}(t, t_0)$ is the state transition matrix partitioned using 3 X 3 matrices:

$$\mathbf{\Phi}(t, t_0) = \begin{bmatrix} \boldsymbol{M}(t, t_0) & \boldsymbol{N}(t, t_0) \\ \boldsymbol{S}(t, t_0) & \boldsymbol{T}(t, t_0) \end{bmatrix} \tag{8}$$

Also, for the first guess to obtain the profile of primer vector, $\mathbf{p_0}, \mathbf{p_f}$ are assigned using the Δv required to move from the avoidance trajectory to Lambert's transfer trajectory at time t_0 and Δv required to move from Lambert's transfer trajectory to the nominal state at t_f respectively.

$$\boldsymbol{p}(t_0) = \boldsymbol{p_0} = \Delta \boldsymbol{V_0}/\Delta V_0 \text{ and } \boldsymbol{p}(t_f) = \boldsymbol{p_f} = \Delta \boldsymbol{V_f}/\Delta V_f \tag{9}$$

Thereafter, $\dot{\mathbf{p}}(t)$ and $\mathbf{p}(t)$ calculated in the following equations help indicate the adjustments required for optimality. As quoted from [28],"If $\dot{p}_0 > 0$, an initial coast will lower the cost. Similarly, if $\dot{p}_f < 0$, a final coast will lower the cost" and " if the value of $p(t)$ exceeds unity along the trajectory, the addition of a midcourse impulse at a time for which > 1 will lower the cost".

$$\dot{\boldsymbol{p}}_0 = \boldsymbol{N_{f0}^{-1}} \left[\boldsymbol{p}_f - \boldsymbol{M_{f0}}\boldsymbol{p_0}\right] \text{ and } \dot{\boldsymbol{p}}_{\boldsymbol{f}} = \boldsymbol{S}_{f0}\boldsymbol{p_0} + \boldsymbol{T}_{f0}\dot{\boldsymbol{p}}_0$$
$$p(t) = \frac{\boldsymbol{N_{t0}}\boldsymbol{N_{f0}^{-1}}\Delta \boldsymbol{V_f}}{\Delta V_f} + \left[\boldsymbol{M_{t0}} - \boldsymbol{N_{t0}}\boldsymbol{N_{f0}^{-1}}\boldsymbol{M_{f0}}\right]\frac{\Delta \boldsymbol{V_0}}{\Delta V_0} \tag{10}$$

When nominal anomaly is not to be matched, the return maneuver can be chosen to be executed when the satellite returns to its initial location after one orbital period, to complete the avoidance and return in just two maneuvers.

Linear Quadratic Regulator (LQR) for finite thrust maneuvers: Finite horizon LQR can be used to ensure that final states approach the desired values while reducing the total control cost over the considered optimization time. LQR results in a continuous control requirement. The weighting matrix (R) for the control in the cost function of LQR can be adjusted accordingly over time to shift the control requirement from continuous to finite arcs. The LQR optimization problem is formulated as follows [29]:

- State dynamics: $\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)$
- End constraint: $\mathbf{x}(t_f) = \mathbf{x}_r(t_f)$, where $\mathbf{x}_r(t)$ is the desired nominal state vector.
- Cost function:

$$J = (\mathbf{x}(t_f) - \mathbf{x}_r(t_f))^T \mathbf{Q}_f (\mathbf{x}(t_f) - \mathbf{x}_r(t_f)) + \int_{t_0}^{t_f} \Big[(\mathbf{x}(\tau) - \mathbf{x}_r(\tau))^T \mathbf{Q} (\mathbf{x}(\tau) - \mathbf{x}_r(\tau)) + (\mathbf{u}(\tau) - \mathbf{u}_r(\tau))^T \mathbf{R} (\mathbf{u}(\tau) - \mathbf{u}_r(\tau))\Big]$$

where $\mathbf{Q}_f^T = \mathbf{Q}_f \geq 0$, $\mathbf{Q}^T = \mathbf{Q} \geq 0$, $\mathbf{R}^T = \mathbf{R} \geq 0$, and $\mathbf{x}_r, \mathbf{u}_r$ are desired state and control references respectively. The matrices $\mathbf{A}(t), \mathbf{B}(t), \mathbf{Q}_f(t), \mathbf{Q}(t)$ and $\mathbf{R}(t)$ can vary with time but for easier representation these would be attributed as $\mathbf{A}, \mathbf{B}, \mathbf{Q}_f, \mathbf{Q}$ and $\mathbf{R}$ respectively from here on in this document.

The optimal control is calculated as:

$$\mathbf{u}(t) = \mathbf{u}_r(t) - \mathbf{R}^{-1}\mathbf{B}^T(\mathbf{S}_{xx}(t)\mathbf{x} + \mathbf{s}_x(t)) \tag{11}$$

$\mathbf{S}_{xx}(t)$ and $\mathbf{s}_x(t)$ can be solved by integrating the following equations backward starting from the accompanying final conditions at t_f

$$\dot{\mathbf{S}}_{xx}(t) = -\mathbf{Q} + \mathbf{S}_{xx}(t)\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{S}_{xx}(t) - \mathbf{S}_{xx}(t)\mathbf{A} - \mathbf{A}^T\mathbf{S}_{xx}(t)$$
$$\dot{\mathbf{s}}_x(t) = \mathbf{Q}\mathbf{x}_r(t) - \left(\mathbf{A}^T - \mathbf{S}_{xx}(t)\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\right)\mathbf{s}_x(t) - \mathbf{S}_{xx}(t)\mathbf{B}\mathbf{u}_r(t)$$

with terminal values $\mathbf{S}_{xx}(t_f) = \mathbf{Q}_f$ and $\mathbf{s}_x(t_f) = -\mathbf{Q}_f\mathbf{x}_r(t_f)$.

Genetic algorithm to find the Pareto-optimal front of multi-objective. Genetic algorithm is a kind of heuristic evolutionary algorithm that evolves a randomly generated initial population to produce fitter solutions based on a given set of objectives. In case of the presence of multiple objectives, each of the non-dominated candidates of the final population form the Pareto-optimal front. A candidate is termed as "dominated" if there exist other candidates in the population which are better than that considered candidate in terms of every objective.

Genetic algorithm starts with a diverse population of candidates to be evaluated against the given objectives or a fitness function which determines the probability of a candidate getting selected for crossover to generate candidates for the next population. There are different selection and crossover schemes available and different schemes can be adapted depending on the logic and requirement of the optimization problem.

Some of the different selection criteria involve 'Roulette Wheel Selection', 'Stochastic Universal Sampling' both of which allow the fitter candidates from a generation a greater chance of being selected by offering a greater fraction of the wheel to them. 'Tournament selection' is a ranking based selection where the fittest candidate from a random group of candidates is selected as a parent. 'Elitism' is a procedure which ensures that the top solutions from a generation are not lost when moving onto the next generation and getting passed on. The 'elite group' is defined as a certain percentage of the population which performed the best as per the fitness function. A desired percentage of the top elite candidates can be chosen to produce the candidates for the next generation while the rest of the elite population can be chosen to perform cross-over with randomly selected 'non-elite' members [10]. Thus, there are multiple heuristic ways of selecting parents for the next generation to preserve the best solutions and generate even better ones with trial-and-error process.

There are different methods of performing the cross-over as well. The cross-over candidate generated depends on the number and location of cross-over points for information exchange between the two parents to create the next generation candidate. Mutation is a process of introducing new information to the population by introducing features to the candidates absent in either of the parents. This helps in better exploration of the solution space. Finally, when a desired number of generations/iterations are reached or the result has converged within a desired tolerance, the genetic algorithm can be terminated resulting in the best fit candidate as the global optimal answer.

The main advantage of this method is that it does not the objectives, constraints or the system dynamics to be continuous or differentiable mathematical functions but can just explore the solution space for the global optimal candidate by measuring the performance of probable candidate solutions over several generations. However, as can be observed there are too many decision parameters to be tweaked by trial-and-error to determine the selection, cross-over and mutation schemes.

For solving the optimal avoidance trajectory, the genetic algorithm is used to find the optimal locations of the maneuver times and magnitudes for the most fuel-efficient transfer alongside causing the least hindrance to the satellite mission in terms of time spent off the nominal orbit. The purpose of using genetic algorithm in this problem formulation is to generate the Pareto-optimal front candidates providing solutions with different trade-offs between the objectives. For each candidate, which comprises a unique set of start and end times for the avoidance trajectory, the genetic algorithm runs the previously mentioned algorithms to find the lowest delta-v requirement for that situation. Thereafter, the Pareto-front is generated with different combinations of start, end times, number of maneuvers and Δv requirements. The user is then required to choose the desired trade-off which would then be used as the initial guess for gradient - based optimization which would be customized with more operational and system constraints.

Direct optimization. With the initial guess for the avoidance trajectory available from the previously mentioned algorithms and the trade-off between the multiple objectives determined, a direct optimization problem can be formulated. Direct optimization involves discretizing the state dynamics and constraints over the timeline and thereafter using a non-linear programming problem (NLP) to solve for the optimal states and controls at each of the discretized time steps. Different kinds of discretization scheme can be used for setting up the NLP. Uniform trapezoidal discretization implements linear functions between two consecutive time steps whereas Hermite-Simpson collocation discretizes the timeline uniformly assuming a cubic spline between two consecutive time steps. Non-uniform discretization can be particularly useful if the functions are not very smooth (which is required for such a gradient-based method). Since the sharp edges of the functions can be discretized in a finer way compared to the rest of the timeline. This also helps in reducing the number of optimization variables by reducing the number of nodes compared to uniform fine grid on the timeline and distributing the nodes where its more needed. Unlike the indirect optimization method, the optimization variables are not doubled in number by the introduction of Lagrange multipliers. The introduction of Lagrange multipliers also requires furnishing initial guesses for them which can be of different orders of magnitude as compared to the sate variables and may also lack physical significance. Path constraints are also easier to enforce in direct optimization. The basic structure of a direct optimization using Hermite-Simpson collocation is shown below [11]:

$$\min_{\mathbf{u}_k,\mathbf{x}_k} J(\mathbf{u}_k) = \sum_{k=0}^{N-1} \frac{h_k}{6}\left(u_k + 4u_{k+\frac{1}{2}} + u_{k+1}\right) \tag{12}$$

$\forall k = 0, 1/2, 1, ..., N$, subject to

$$\mathbf{x}_{k+\frac{1}{2}} = \frac{1}{2}\left(\mathbf{x}_k + \mathbf{x}_{k+1}\right) + \frac{h_k}{8}\left(\mathbf{f}_k - \mathbf{f}_{k+1}\right)$$

$$k = [0, ..., N-1]\ , \text{ interpolation constraints} \tag{13}$$

$$\mathbf{x}_{k+1} - \mathbf{x}_k = \frac{h_k}{6}\left(\mathbf{f}_k + 4\mathbf{f}_{k+1/2} + \mathbf{f}_{k+1}\right)$$

$$k = [0, ..., N-1]\ , \text{ collocation constraints} \tag{14}$$

$$-d_{\max} \leq q_1 \leq d_{\max}\ , \text{path constraints} \tag{15}$$

$$-u_{\max} \leq u \leq u_{\max}\ , \text{path constraints} \tag{16}$$

$$\mathbf{x}_0, \mathbf{x}_N \text{ given}, \qquad \text{boundary conditions} \tag{17}$$

4. NON LINEAR PROGRAMMING PROBLEM

The use of optimal control theory to solve the collision avoidance problem has a long tradition in the literature. In particular, the use of collocation methods and the translation into a non-linear programming problem has been used in previous works. The following review of the literature is not extensive but covers similar approaches to the one presented in this section. Some of them were already mentioned in the section 1.

Scharf et al. [12] presented a Reactive Collison Avoidance (RCA) scheme to be implemented in real time to handle planar scenarios and avoid oncoming space objects by considering a pre-specified avoidance region to be maintained around the space objects. Martinson [13] solved for impulsive thrusts with the assumption of Clohessy-Wiltshire equations to solve for the avoidance of multiple encounters using feedback mechanism. Avoidance was ensured by maintaining pre-determined ellipsoidal regions around space objects and the solution was based on a weighted objective function. Sales [14] studied the effect of continuous as well as impulsive in-plane and out of-plane thrusts for collision avoidance maneuvers using Radau Pseudospectral method in GPOPS-II software. Starting from Bombardelli's solution [8],

G. Salemme [15] extended it to finite thrust maneuver to avoid the collision. Results were obtained using numerical solutions of multiple two-point boundary value problem to get to the bang-bang thrust profile using homotopy method. Zimmer [5] explored reducing uncertainty and making the trajectory more observable in addition to minimizing the fuel cost using indirect optimization. It was found that while the result was dependent on the initial condition of the data but reducing uncertainty of a relatively good estimate can come at a relatively high price. Also, while different paths had different uncertainties and fuel cost associated with them minimizing the covariance at a certain instant in the cost function could also very well lead to maximizing it at some other instant. To avoid the problem of initial guess, convex optimization has been used in different domains like collision avoidance, rendezvous or formation flying problems. A successive second-order cone programming problem was used to solve the convex rendezvous problem without the consideration of uncertainty [16], while in [17] a formation flying problem was solved using linear programming problem. Robustness was implemented by requiring the same avoidance constraint to be satisfied for several uncertain initial relative states. Li et al. [18] used a combination of multiresolution technique using convex optimization and mesh-refinement on Clohessy-Wiltshire equations. The avoidance region was handled using a tangent method similar to the one used in [17] for planning close-proximity operations. Armellin [19] used differential algebra and successive linearizations to solve multi-impulse convex optimization problem concerning only the avoidance maneuvers without return to the original trajectory. Dutta and Misra [20, 21, 22] explored the effect of convex and non-convex optimization for avoidance trajectories along with return when considering linear and nonlinear evolution of uncertainties. De Vittori et al. developed an analytical method concerning avoidance from short-term encounters with low thrust in [23] using indirect optimization and in [24] solved for the avoidance trajectory while incorporating Gaussian nature of uncertainty and the return as well for space objects in LEO.

4.1. Problem Statement

The collision avoidance maneuver is designed as the solution of an optimal control problem with a single objective function.

$$
\begin{aligned}
\min_{\mathbf{u}(t),t_f} J(\mathbf{u}(t),t_f) \;&=\; w_1 \int_{t_0}^{t_f} ||\mathbf{u}(t)||_2 \mathrm{d}\tau \;+ \\
&+\; w_2(t_f - t_0) \\
&-\; w_3 \min_t \left[(\mathbf{x}_{\text{ref},2}(t) - \mathbf{x}_1(t))^T \mathbf{P}(t)^{-1} (\mathbf{x}_{\text{ref},2}(t) - \mathbf{x}_1(t))\right] + \\
&-\; w_4 \min_t \left[(\mathbf{x}_{\text{ref},2}(t) - \mathbf{x}_1(t))^T (\mathbf{x}_{\text{ref},2}(t) - \mathbf{x}_1(t))\right] + ...
\end{aligned} \tag{18}
$$

subject to

$$
\begin{aligned}
\delta\mathbf{x}_1(t_0) &= \mathbf{0} && \text{Initial conditions} && (19)\\
\delta\mathbf{x}_1(t_f) &\in \chi_{\text{goal}} && \text{Terminal conditions} && (20)\\
\delta\dot{\mathbf{x}}_1(t) &= \mathbf{f}(\delta\mathbf{x}_1(t), \mathbf{u}(t), t) && \text{System dynamics (4.2)} && (21)\\
\delta\mathbf{x}_1(t) &\in \chi_{\text{free}}, \forall t \in [t_0, t_f] && \text{Collision avoidance (4.3)} && (22)\\
\mathbf{u}(t) &\in \mathcal{U}(\mathbf{x}(t)) && \text{Control feasibility} && (23)
\end{aligned}
$$

where $\mathbf{x}_1 = \mathbf{x}_{\text{ref},1} + \delta\mathbf{x}_1$ is the state of the primary, $\mathbf{x}_{\text{ref},1}$ is the nominal trajectory of the primary without performing the collision avoidance maneuver, $\delta\mathbf{x}_1$ is the relative state of the primary with respect to the nominal and $\mathbf{x}_{\text{ref},2}$ is the nominal state vector of the secondary; $\mathbf{P}(t)$ is the combined covariance of both primary and secondary at time t, $\mathbf{R}(t)$ and the covariance resulting from the maneuver uncertainty $\mathbf{P}(t) = \mathbf{C}_1(t) + \mathbf{C}_2(t) + \mathbf{R}(t)$. The weights are computed from the solution of the global optimization in the previous step. Let $\mathbf{m}$ be the objective function of the global optimization, with $m_1 = \int_{t_0}^{t_f} ||\mathbf{u}(t)||_2 \mathrm{d}\tau$, $m_2 = (t_f - t_0)$, and so on. If a specific solution is selected from the Pareto front, $\mathbf{m}_{GO}$, then the weights can be defined as $w_i = m_{GO,i}^2 / ||\mathbf{m}_{GO}||_2^2$. Equation (18) establishes the mathematical description of the objectives. The collision probability is minimized using the Mahalanobis distance as a proxy (third element in Eq. (18)).

4.2. System Dynamics

The propagation of the nominal trajectories of both primary and secondary is performed using a high-fidelity dynamical model.

$$
\dot{\mathbf{x}}_{1,2}(t) = \mathbf{f}_{HF}(\mathbf{x}_{1,2}, t)\,, \qquad \mathbf{x}_{1,2}(t_0) = \mathbf{x}_{10,20}\,, \tag{24}
$$

where the subscripts 1 and 2 have been used to denote primary and the secondary. The dynamics considered in the optimal control problem are the analytical relative motion approximation with respect to the nominal trajectory of the primary, $\mathbf{x}_{\text{ref},1} = [\mathbf{r}_{\text{ref},1}^T, \mathbf{v}_{\text{ref},1}^T]^T$,

$$\mathbf{f} = \begin{bmatrix} \delta\dot{x} \\ \delta\dot{y} \\ \delta\dot{z} \\ \left(\frac{2\mu}{r_{\text{ref},1}^3} + \frac{h_{\text{ref},1}^2}{r_{\text{ref},1}^4}\right)\delta x - \frac{2(\mathbf{v}_{\text{ref},1}\cdot\mathbf{r}_{\text{ref},1})h_{\text{ref},1}}{r_{\text{ref},1}^4}\delta y + 2\frac{h_{\text{ref},1}}{r_{\text{ref},1}^2}\delta\dot{y} \\ \left(\frac{h_{\text{ref},1}^2}{r_{\text{ref},1}^4} - \frac{\mu}{r_{\text{ref},1}^3}\right)\delta y + \frac{2(\mathbf{v}_{\text{ref},1}\cdot\mathbf{r}_{\text{ref},1})h_{\text{ref},1}}{r_{\text{ref},1}^4}\delta x - 2\frac{h_{\text{ref},1}}{r_{\text{ref},1}^2}\delta\dot{x} \\ -\frac{\mu}{r_{\text{ref},1}^3}\delta z \end{bmatrix},$$

where $\delta\mathbf{x}_1 = [\delta x, \delta y, \delta z, \delta\dot{x}, \delta\dot{y}, \delta\dot{z}]^T$, μ is the Earth's gravitational constant, and $h_{\text{ref},1}$ is the modulus of the specific angular momentum of the primary refrence trajectory. The state transition matrix can be computed as

$$\dot{\mathbf{\Phi}}(t, t_0) = \mathbf{A}(t)\mathbf{\Phi}(t, t_0), \qquad \mathbf{\Phi}(t_0, t_0) = \mathbf{I}, \tag{25}$$

with the Jacobian matrix $\mathbf{A} = \partial\mathbf{f}/\partial(\delta\mathbf{x}_1)$ and can be used for the propagation of the covariances, if required.

A non-dimensional version of the dynamics is used in the implementation of the prototype to avoid scalability issues in the non-linear programming problem. Thus, the non-linear version of the dynamics reads: $\delta\dot{\mathbf{x}}_\mathbf{1} = \hat{\mathbf{f}}(\delta\mathbf{x}_\mathbf{1}) + [\mathbf{0}_{\mathbf{3}\times\mathbf{1}}, \mathbf{u}_\mathbf{1}^\mathbf{T}]^\mathbf{T}$, where states involved are scaled correspondingly by the characteristic length and time.

4.2.1. Transformations from ECI to RTN reference frames

The state of the secondary must be transformed to the reference frame relative to the primary reference trajectory. The reference frame is a local RTN, defined by $\mathbf{x}_{\text{ref},1}$. Hereafter, for clarity, vectors with components in the RTN or ECI reference frames will be denoted with a superscript (RTN or ECI, respectively). The rotation matrix from RTN to ECI is called $\mathcal{T}$, and thus, for any general vector $\mathbf{s}$:

$$\mathbf{s}^{ECI} = \mathcal{T}\mathbf{s}^{RTN}$$

The matrix $\mathcal{T}$ has as columns the unit vectors $\mathcal{T} = [\mathbf{u}_r; \mathbf{u}_\theta; \mathbf{u}_h]$, defined as:

$$\mathbf{u}_r = \frac{\mathbf{r}}{r} \qquad \mathbf{u}_h = \frac{\mathbf{r}\times\mathbf{v}}{|\mathbf{r}\times\mathbf{v}|} \qquad \mathbf{u}_\theta = \mathbf{u}_h \times \mathbf{u}_r$$

4.3. Collision avoidance constraint

Collision avoidance constraints are dealt with as a Keep Out Zone (KOZ) constraint. KOZ is defined as the complementary of the free space, in terms of the covariances of the primary and secondary.

$$\chi_{\text{KOZ}} = \chi_{\text{free}}^C \tag{26}$$

$$\chi_{\text{KOZ}} = \left\{\mathbf{x} | (\mathbf{x}-\mathbf{x}_2)^T\mathbf{E}(\mathbf{x}-\mathbf{x}_2) \geq M_{d_0}^2\right\} \tag{27}$$

where $\mathbf{E} = C_1 + C_2$, with C_i, $i = 1, 2$, the covariance matrices of primary and secondary, respectively and $M_{d_0}^2$ is the desired Mahalanobis threshold to be maintained.

Additionally, the Functional requirement COO3-CAM-FUN-13 [1] enforces the consideration of more than one CDM when available. In such a case (when two CDMs A and B are available), the KOZ is defined as

$$\begin{aligned}\chi_{\text{KOZ}} \;=\; & \{\mathbf{x}|(\mathbf{x}-\mathbf{x}_2^A)^T\mathbf{E}^A(\mathbf{x}-\mathbf{x}_2^A) \geq M_{d_0}^2 \wedge \\ & \wedge(\mathbf{x}-\mathbf{x}_2^B)^T\mathbf{E}^B(\mathbf{x}-\mathbf{x}_2^B) \geq M_{d_0}^2\}\end{aligned}$$

4.4. Fail-safe maneuver constraint

An additional constraint is included to ensure that the CAM is a fail-safe maneuver, i.e. the collision risk is not increased at any point of the maneuver with respect to the no maneuver case. Thus it ensures that the probability of collision is lowered even in the case of a propulsion failure. The condition is checked at or in a time-window around the nominal TCA (t_c). The constraint is defined as follows for a time step t_{c_i} in the time window around t_c.

$$\begin{gathered}(\Phi(t_{c_i}, t_k)\mathbf{x}(t_k) - \mathbf{x}_2(t_{c_i}))^T\mathbf{E}(\Phi(t_{c_i}, t_k)\mathbf{x}(t_k) - \mathbf{x}_2(t_{c_i}))^T \geq \\ (\Phi(t_{c_i}, t_{k+1})\mathbf{x}(t_{k+1}) - \mathbf{x}_2(t_{c_i}))^T\mathbf{E}(\Phi(t_{c_i}, t_{k+1})\mathbf{x}(t_{k+1}) - \mathbf{x}_2(t_{c_i}))^T \\ \forall k \in [0, c_i].\end{gathered}$$

5. RESULTS

The prototype is being progressively developed to handle all the required constraints and objectives, with the goal to de-risk and accelerate the subsequent production implementation. The prototype has implemented the multi-objective optimization to generate the Pareto front and provide a quantitative analysis of the trade-offs among different objectives. The primary constraints like avoidance, return, fail-safe constraints have been implemented so far in the prototype while some of the more enhanced features are yet to be progressively implemented.

In this section the prototype has been tested for a fuel-optimal avoidance trajectory. The fail-safe criterion is implemented whereby the control at any time step would ensure that the two satellites never get closer than the distance they would have had without the implementation of the control at that time. The satellite is also constrained to return to the same orbit and match its nominal true anomaly at the end of the avoidance trajectory. The scenario for the example has been described as follows:

[1] CAM shall measure the collision risk considering different sources of CDMs (if available).

5.1. Example scenario

A close approach scenario between two LEO satellites with TCA at midnight UTC on January 1, 2007.

The nominal orbit of the primary has the following Keplerian parameters:

- $a = 7004.7$ km;
- $e = 0.00137$;
- $i = 97.4869$ deg;
- RAAN= -49.2587 deg;
- ω= 58.071 deg.

The orbit of the secondary has the following Keplerian parameters:

- $a = 7004.656$ km;
- $e = 0.001346$;
- $i = 89.51$ deg;
- RAAN = -74.589 deg;
- ω = 60.07deg

The mass of the primary is 156 kg, and the relative distance at TCA is 357.4 m. In turn, the relative position vector at TCA:

- In the ECI frame: [-163.71649735, 290.44110637, 128.83620463] m
- In the b-plane: [-21.75, 356.77] m

Combined covariance at TCA in the b-plane :

$$\mathbf{P} = \begin{bmatrix} 164.03 & -85.11 \\ -85.11 & 224874.08 \end{bmatrix} \text{m}^2.$$

5.2. Result from the multi-objective optimization:

The Non-dominated Sorting Genetic Algorithm - II (NSGA-II) has been implemented to find the solutions representing the Pareto-optimal front. The objectives considered in this step are the following.

- Minimization of the total Δv used for both avoidance and return
- Minimization of the total time spent off the nominal orbit
- Maximization of the Mahalanobis distance between the two concerned RSOs at TCA

For this example, the genetic algorithm evaluates the Δv requirement for avoidance as per the semi-analytical method proposed in [9] which solves the minimum energy problem for the Δv required to meet a certain Mahalanobis distance at TCA. Following which the Lambert's problem is implemented to find the return maneuvers to match the nominal states of the primary. The genetic algorithm shuffles through different relative positions at TCA and different maneuver times marking different departure and return locations from the nominal orbit. The two-maneuver solution is also explored by which the satellite returns to the nominal orbit after an integral number of complete orbits.

Fig. 1 shows the Pareto-front for the three-burn maneuver solution for above mentioned multi-objective problem. It can be seen in the following figures that the maximization of Mahalanobis distance and minimization of time off the nominal orbit comes at the cost of a higher overall Δv requirement, which is as would be expected. But this analysis brings forth the quantitative trade-offs among the different objectives. Other than the obvious avoidance and return constraints, there are also several other constraints with configurable thresholds in place to refine the search space of the genetic algorithm, such as the following. Respecting these ensure that unnecessary high cost plans are not being included in the solution because it brings down the time or maximizes the Mahalanobis distance beyond the practically required thresholds.

- Minimum time before TCA for the avoidance maneuver
- Minimum time to be spent for the entire avoidance trajectory
- Minimum time to be allowed between second and third maneuvers for a 3-maneuver avoidance trajectory
- Maximum Mahalanobis distance at TCA
- Any additional thresholds from the user to restrict the search space

With the inclusion of the two-impulse solution in the search space, the Pareto-front solution is seen to favour the two-maneuver solutions more, so the candidates are seen to cluster more at the integer number of orbital periods spent off the nominal orbit. However, this step does employ only Keplerian dynamics with no orbital perturbations among several other approximations listed in Section 3.

5.3. Result from direct optimization

In the production implementation, the operator can select the desired combination of the objectives to use as

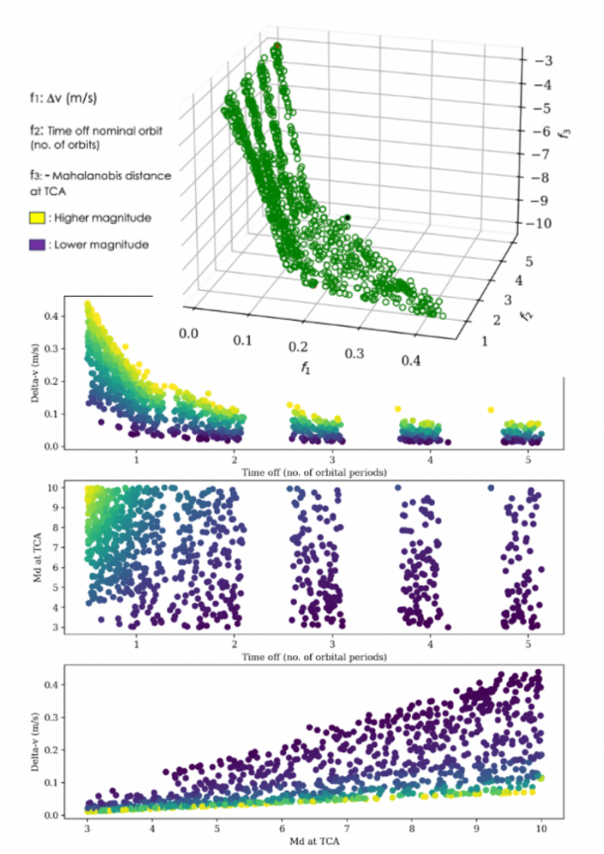

Figure 1. The Pareto front solution from the multi-objective optimization for the example scenario.

the baseline for step 2 of the CAM. In this case, a fuel-optimal solution over one orbit is chosen as the guess for the next step, which would return to the nominal orbit within one orbital period while maintaining a desired Mahalanobis distance of 3 between the concerned RSOs. The corresponding solution from the genetic algorithm serves as the initial guess for this step. The following results show how introducing more constraints over the basic avoidance and return impact the required control.

Avoidance is enforced by using both the Euclidean and Mahalanobis distances between the two concerned RSOs over the entire time window. The primary satellite is again constrained to match the nominal states by the end of the avoidance trajectory. The maximum thrust constraint is also in place from the thruster model of the primary.

The convergence to a locally optimal solution is depicted by the resulting 'bang-bang' solution for the optimal control acceleration magnitude, whereby the magnitude of the control acceleration switches between the maximum and the minimum values (2). A 'bang-bang' solution is desirable because, at least theoretically, they produce the maximum available thrust acceleration at the locations of the orbit where it is effective and do not thrust at all when not effective. Fig.3 shows the deviation of the primary satellite position from its nominal along the avoidance trajectory and its return at the end. The total Δv requirement for this equals 0.041 m/s.

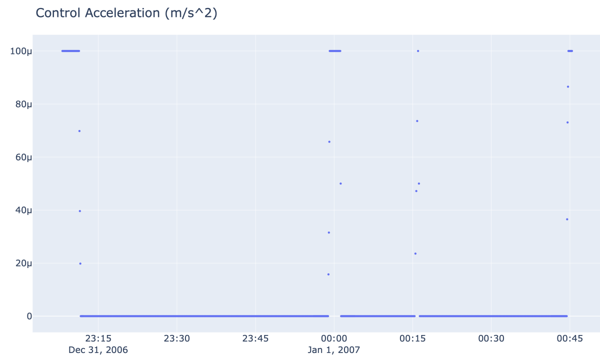

Figure 2. The control thrust profile for the necessary constraints.

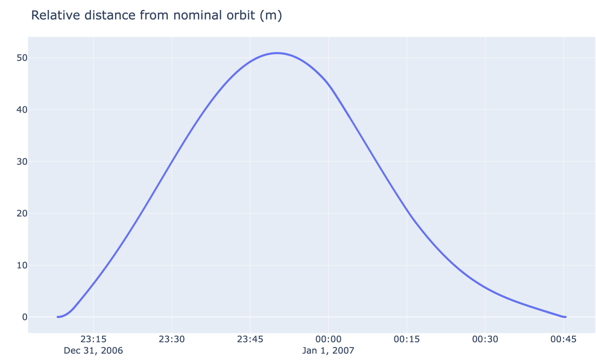

Figure 3. The deviation of the primary satellite position from its nominal.

Now the effect of the fail-safe constraint on the optimal control profile is explored. The fail-safe constraint ensures that the optimal avoidance trajectory is so designed that the control input at no point in time would make the satellite reach a riskier position with the secondary than without the control, thus ensuring safety in case of a sudden propulsion failure at any stage without completion of the entire planned avoidance trajectory. The satellite state (position and velocity) from every time step is propagated with and without the control at that time step upto a certain time window around the original TCA (where the orbits or primary and secondary are in proximity) and it is ensured that the Euclidean and Mahalanobis distance between the primary and the secondary with the control is not lesser than that without the control input.

With the fail-safe constraint switched on, the Δv requirement is observed to increase from 0.041 m/s to 0.072 m/s and the scenario is seen to require more contribution from normal component of thrust than when the fail-safe constraint is switched off.

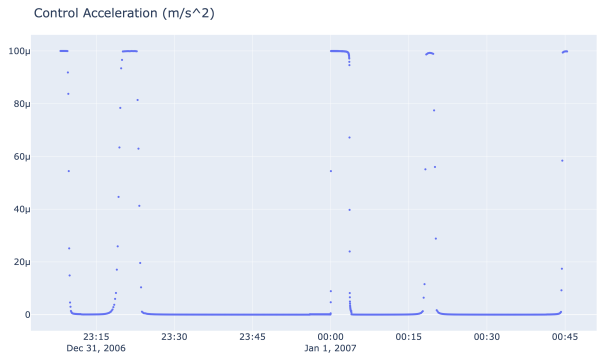

Figure 4. The control thrust profile with the fail-safe constraint alongside the necessary constraints.

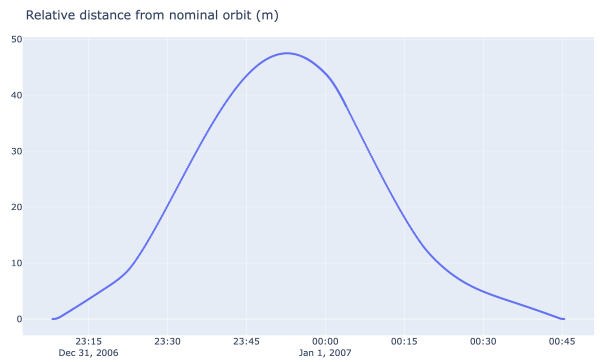

Figure 5. The deviation of the primary satellite position from its nominal when the fail-safe constraint is switched on.

6. CONCLUSIONS

In this work, a methodology for supporting the decision making in a conjunction assessment scenario and the design of an optimal collision avoidance maneuver compatible with all the ground and space segment constraints is presented in this paper. The framework consists of two steps: a first one devoted to explore the full search space; the second step provides with a detailed solution of the collision avoidance maneuver. The first step is based on the use of a heuristic algorithm, a genetic algorithm, that provides a trade-off among the different conflicting objectives, e.g. fuel consumption, time to come back to the nominal orbit, or risk reduction. In the scenario presented in this paper, the Pareto front in Fig. 1 shows the preliminary cost of the different optimal solutions and allows operators to select the type of maneuver that best fits the needs of the satellite mission. Once the decision of making a CAM is taken, a second step, based on the definition of an optimal control problem, is solved using non-linear programming. At this step, all the constraints related to the platform and the satellite operation can be included. The result of the second step is the history of the required control inputs (thrust and steering laws) to reduce the collision risk and satisfy all the constraints. In the preliminary results, we have shown an example of the design of a CAM and how the inclusion of constraints modify the control profile.

In this way, the tool presented here helps in the entire process related to CAM, from decision making to the detailed design of the maneuver.

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