METHODS FOR ROBUST PROPAGATION AND IMPROVED COVARIANCE REALISM IN LEO

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ABSTRACT

Accurate trajectory modeling and covariance realism in Low Earth Orbit (LEO) are critical for safe space operations, but challenges in ensuring numerical stability over long-term propagation with reliability are formidable. Supported by the Intelligence Advanced Research Projects Activity (IARPA) Space Debris Identification and Tracking (SINTRA) program and the Office of Space Commerce (OSC), we are developing a next-generation drag modeling framework that accurately characterizes atmospheric density uncertainty due to space weather through a physics- and data-driven approach. During the development process, it was observed that the 2-norm of the position standard deviation exhibited an unexpected dip when a LEO orbit was numerically propagated beyond three days. This prompted a deeper investigation into the numerical stability and Gaussian assumptions underlying the propagation framework. In this paper, the findings of this study are compared with Monte Carlo results and the corresponding conclusions are shared to improve the accuracy of orbit prediction methods.

Keywords: Orbit Propagation; Atmospheric Drag Modeling; Covariance Realism; Mathematical Modeling; Monte Carlo Simulation.

1. INTRODUCTION

This study highlights observations made during the development of a next-generation drag modeling framework. In our previous research, development of the Stochastic Unscented Transform (SUT) framework was presented. This is a mathematical formulation designed to capture the joint statistics of probabilistic atmospheric density models and their probabilistic drivers or inputs. During the development process, an unexpected dip in the 2-norm of the position standard deviation was observed, when the LEO orbit was numerically propagated beyond three days. This nonphysical trend emerges when the state covariance is orthogonalized and sigma points are recomputed at a fixed cadence, indicating the introduction of nonlinearities. This setup has been employed such that the half-life, i.e., the temporal correlation, in atmospheric density can be modeled. The presence of nonphysical trends in position and velocity uncertainties during propagation provides a basis to investigate deviations from Gaussian assumptions, specifically the divergence of sigma points from the underlying Gaussian distribution.

Firstly, a quick comparison with Monte Carlo simulations showed that Gaussian assumptions are maintained throughout the 7-day propagation time period. This indicated some numerical instability is emerging due to orthogonalization implementation and hence, normalization techniques were applied to improve numerical conditioning. The position and time units are scaled with respect to the Earth's radius and the orbital period of the given LEO orbit, respectively. Although this adjustment leads to measurable improvements in the conditioning of the covariance matrix, it does not fully eliminate the instability. Consequently, the standard Cholesky decomposition, used in the covariance orthogonalization process, was replaced by the singular value decomposition (SVD). The use of SVD significantly enhances numerical stability, allowing for an increase in the covariance-orthogonalization update cadence from a 5 to 20 minutes, without introducing excessive instability.

However, the analysis revealed that for update cadences exceeding 20 minutes, the non-physical trends persist. To thoroughly understand these trends, a qualitative analysis is performed using a 10,000 sample Monte Carlo (MC) simulation. The K-test and the normal probability tests are applied to the MC statistics to replicate the propagation scenarios and evaluate the impact of nonlinear deviations in the covariance. The Monte Carlo results confirm

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that the Gaussian assumptions are strongly maintained for most of the position and velocity components and the non-physical trend is not observed. This means that repeated covariance orthogonalization with repeated sigma point calculation is the reason for the observed anomalies in the position and velocity standard deviations.

This comprehensive investigation underscores the challenges of maintaining numerical stability in the complex dynamics of LEO modeling. In the following sections, motivation, introduction to the SUT framework, and investigation methodology are presented. Lastly, results are presented along with detailed commentary on the interpretation of the observations, and possible solutions are highlighted, respectively.

2. MOTIVATION

Accurate modeling of atmospheric density and, therefore, drag perturbation is critical for LEO operations due to their dominant role in orbital decay and long-term trajectory prediction [2, 15]. Incorporating temporal correlations and spatial effects due to diurnal changes, solar rotational variations, solar cycle variations, and associated uncertainty improves real-time estimation [12].

Several research articles have contributed to the development of the current suite of physics-based and data-driven models [5, 7, 8, 9, 10, 18]. Although these models are critical for simulating the dynamics of atmospheric density, inherent limitations make accurate characterization difficult, especially for real-time operations [4]. The work presented here is a continuation of our efforts to develop a next-generation drag modeling framework [3, 4, 13, 14].

Previous research by Qian et al. has attempted to identify these characteristics and quantify their effects during orbit propagation [12]. In addition, research by Wright et al. and Gao et al. has presented novel approaches to calibrate atmospheric density during real-time operations [6, 17]. However, none of these works discuss the integration of these models with orbit propagation for real-time operations. Articles by Vallado et al., Paul et al., and Bhatia et al. discuss different techniques to integrate high-fidelity dynamical modeling for atmospheric density within the orbit propagation architecture [4, 11, 16]. Although Vallado et al. propagate upto 4 days from epoch, the other two publications limit propagation to 3 days only. Reduced fidelity and the emergence of nonlinearities have been observed and reported as one of the reasons for this limitation.

Generally, for day-to-day operations, 3 to 4 days of propagation are reasonable. However, for conjunction assessment and collision avoidance, up to 5 days of propagation is preferred for the two-line elements (TLEs) issued by the 18th Space Defense Squadron (18th SDS) [1]. This is because the sooner a potential conjunction is identified, the easier it is to mitigate it. During scenarios such as geomagnetic storm, when uncertainty on model parameters can increase significantly, longer propagation is preferred.

This paper presents a comprehensive analysis of the nonlinearities encountered in LEO orbit propagation, with a particular focus on the half-life modeling of atmospheric density. By examining the interplay between numerical stability, covariance realism, and the preservation of Gaussian assumptions, we aim to contribute to the development of more accurate and robust orbit prediction models. Such advancements are crucial for a wide range of applications, including long-term propagation, collision avoidance, and high-precision orbit determination, all of which are essential for the safe and sustainable use of the increasingly congested LEO environment.

3. STOCHASTIC UNSCENTED TRANSFORM

The Stochastic Unscented Transform (SUT) is a novel mathematical formulation that captures the joint statistics of probabilistic atmospheric density models and their inputs [14]. This next-generation drag modeling framework leverages models that provide reliable physics- and data-driven uncertainty estimates; refer to Figure 1.

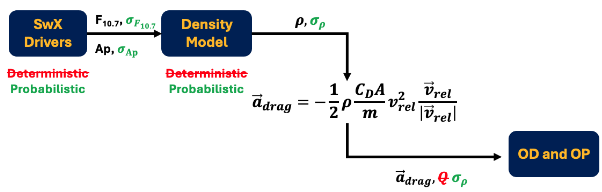

Figure 1. A schematic of the next-gen drag modeling framework. The elements of the new framework are highlighted in green while those being replaced are strikeout in red. SwX: Space Weather, OD: Orbit Determination, and OP: Orbit Propagation.

The SUT framework separates the system into input, model, and output components that are all probabilistic. Under Gaussian assumption, SUT uses smartly sampled sigma points to represent input uncertainty that are transformed through the probabilistic model to produce an overall output distribution, accounting for both input and model-induced uncertainties. This is represented in the SUT density block in Figure 2.

Although the current framework incorporates the uncertainty on atmospheric density, characterization of temporal correlation requires repeated covariance matrix orthogonalization to recompute the sigma points at each timestep. This additional characterization of temporal correlation using covariance orthogonalization results in an anomalous dip in the 2-norm of the position standard deviation when numerically propagated for more than 3 days.

In the next section, the simulation setup and methodol-

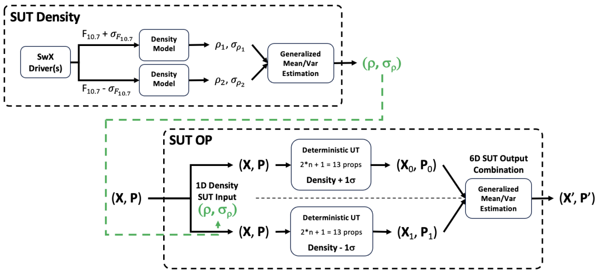

Figure 2. SUT Architecture

ogy for the investigation of this anomalous trend are presented.

4. METHODOLOGY

This section highlights the orbital specification, normalization techniques used, numerical integrators employed, and covariance matrix decomposition methods used to investigate observations made during long-term propagation using SUT and covariance orthogonalization.

4.1. Simulation Setup

To thoroughly investigate the nonlinearities in LEO orbit propagation, a comprehensive 7-day simulation is conducted. The initial conditions are selected to represent a typical LEO scenario, refer to Table 1.

The simulation duration of 7 days was chosen to capture both short-term and medium-term orbital evolution effects. To improve the numerical conditioning of the covariance matrix, position and time units are normalized. The Earth's radius is used as the reference length for position normalization, while the orbital period of the simulated LEO orbit serves as the time normalization factor. The mass is normalized by the given mass of the spacecraft, respectively.

Although this normalization technique significantly improves the conditioning of the covariance matrix, it is important to note that it does not fully eliminate all instabilities. The persistence of some instabilities highlights that the underlying cause of the nonlinearities is something else.

Additionally, two different implementations of numerical integrators are tested: a fixed-step RK45 and a variable-step ode45. The dual approach was intended to evaluate potential trade-offs between computational efficiency and numerical stability in atmospheric density modeling. The implementation of variable-step ode45, based on the adaptive Dormand-Prince 4(5) method, employs adaptive step size control, adjusting time steps between 10^{-3} and 10^{2} seconds based on local truncation error estimates. This method achieves fifth-order accuracy using

Table 1. Simulation set-up for the orbit uncertainty propagation using SUT and MC approach.

Parameter	Value/Details
Initial position (ECI)	[3782.9, -5441.6, -1420.075] km
Initial velocity (ECI)	[-0.606, 1.539, -7.488] km/s
State	Position and Velocity
Dynamic Model	2-body, J2 and drag
Propagation period	7 days
Object shape/type	Spherical & symmetric
Cross-sectional AMR	0.002 m^2/kg
Drag coefficient	2.2
Initial epoch	00:00:00 UTC, October 29, 2003
Orbit propagation	SUT Modified MC (Paul et. al.)
Number of MC	10,000

six derivative evaluations per step, resulting in a $O(h^5)$ global error. In contrast, the fixed step RK45 configuration enforces constant time steps through a modified Runge-Kutta 4(5) implementation, using step locking to force $\Delta t = 0.1 - 10$ second increments.

Notably, despite the differing approaches of these integrators, our analysis revealed no significant differences in their ability to resolve the nonlinear trends observed in the LEO orbit propagation. Thus, all the results presented are using a fixed-step RK45 integrator as it has shorter computational time, respectively.

4.2. Decomposition Method

To further address the instability issues observed in covariance propagation, this study compares the results of the standard Cholesky decomposition with those of the singular value decomposition (SVD) for covariance orthogonalization.

The Cholesky decomposition, while computationally efficient, can sometimes lead to numerical instabilities when dealing with ill-conditioned or near-singular matrices. In contrast, SVD offers superior numerical stability, especially for matrices with high condition numbers.

5. RESULTS

In this section, the results are presented for different scenarios that were implemented during this investigation. Initially, a 5th day dip in the 2-norm of the position standard deviation reveals numerical instability; see Figures 3 and 4. These results are generated with the SUT framework for a half-life of 60 minutes, respectively. Comparison shown here is against the resulting position 2-norm

1σ trend obtained using the RK45 propagation method of the mean state only, i.e., not with sigma points and hence half-life is infinite (in other words, atmospheric density has a constant value). Clearly, the reason for the nonphysical trend can be linked to covariance orthogonalization.

To confirm this hypothesis, different scenarios are implemented, and the stability of the 2-norm of the standard deviation of the position is monitored, with covariance orthogonalization update cadences varied.

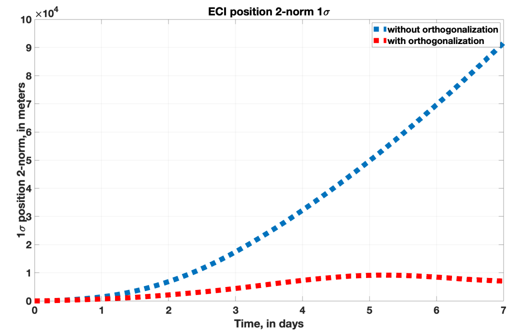

Figure 3. Comparison of the denormalized Earth-Cenetered Inertial (ECI) position 2-norm 1σ for SUT using orthogonalization and for RK45 propagation method of the mean state only without orthogonalization. Half-life is 60 minutes, SVD is used

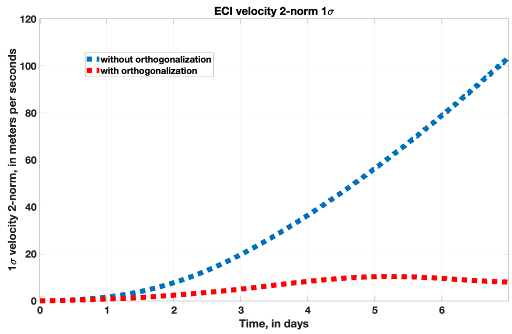

Figure 4. Comparison of the denormalized Earth-Cenetered Inertial (ECI) velocity 2-norm 1σ for SUT using orthogonalization and for RK45 propagation method of the mean state only without orthogonalization. Half-life is 60 minutes, SVD is used

5.1. Normalization Effects

The impact of normalization on conditioning is summarized in this section. Both constrained normalization and adaptive normalization were applied and it was noted that although numerical conditioning results in delayed nonlinearity, it does not completely remove it. Comparison of position magnitude with and without normalization is shown in Figure 5. Condition numbers are noted to improve by a magnitude of 4 for the normalization case.

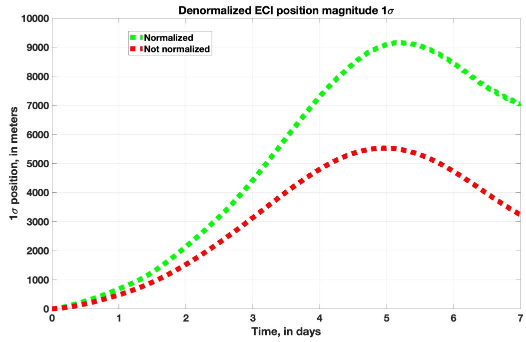

Figure 5. Comparison of denormalized ECI position magnitude standard deviation with and without normalization, 60 minutes half-life and SVD method

5.2. SVD vs. Cholesky Decomposition

Comparative performance between SVD and Cholesky decomposition is analyzed. The implementation of SVD significantly enhances the overall stability of the covariance propagation. This improvement allows for an increase in the covariance-orthogonalization update cadence from 5 to 20 minutes without introducing excessive instability.

The extended update cadence not only improves computational efficiency, but also provides a more realistic representation of how often the covariance matrix would be updated in practical orbital determination scenarios. However, it is worth noting that for update cadences exceeding 20 minutes, some residual nonphysical trends persist, indicating the complex nature of the nonlinearities involved in LEO orbit propagation; refer to Figure 6.

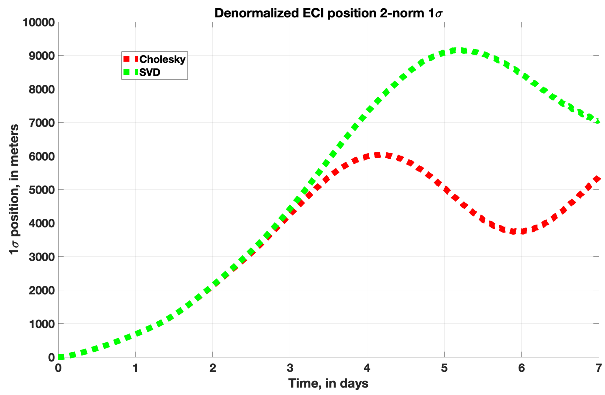

Figure 6. Results showing denormalized ECI position 2-norm standard deviation for 60 minute half-life using Cholesky and SVD decomposition

5.3. Monte Carlo Validation

Monte Carlo simulation is set up using the orbit propagation method highlighted in scenario 2 of the study by Paul et al. [11]. A simulation, with 10,000 sample size, is run for 180-minute half-life and infinite half-life, respectively. The initial orbit is according to the specifications given in Table 1. In Figures 7 and 8 it is clear that Gaussian assumptions are maintained throughout the 7-day propagation time period. This proves the hypothesis that the nonphysical trend observed with Unscented Transform (UT) propagation is due to nonlinearities emerging because of repeated covariance orthogonalization.

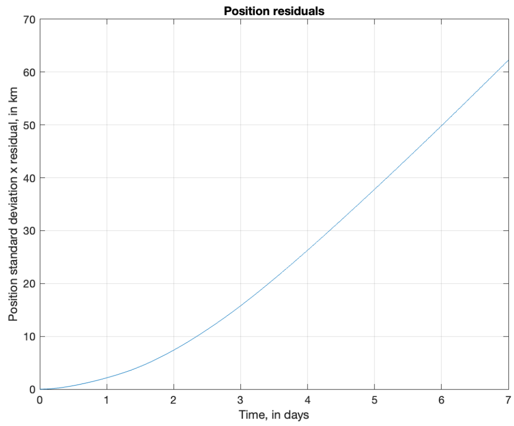

Figure 7. Monte Carlo results showing denormalized ECI position magnitude standard deviation for 180 minute half-life

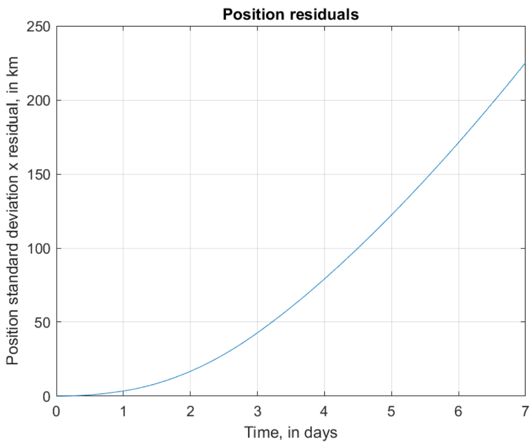

Figure 8. Monte Carlo results showing denormalized ECI position magnitude standard deviation for infinite half-life

Additionally, K-test and normal probability plots are generated for the day-wise (with data sourced at the end of each day) segmentation of the position components in the local vertical and local horizontal (LVLH) frame. The Normal probability plots matches the quantiles of sample data to the quantiles of a normal distribution. Whereas the Kolmogorov-Smirnov test returns a decision for the null hypothesis that the given data comes from a Gaussian distribution, against the alternative that it does not come from such a distribution. The result is 1 if the test rejects the null hypothesis at the 5 percent significance level, or 0 otherwise. The objective is to qualitatively test the Gaussianity of the data over the propagation time period. The results show that the radial position components become weakly non-Gaussian towards day 7, whereas for the cross-track and along-track position components, Gaussianity is maintained; see Figures 9, 10, and 11. Similar results are noted for the velocity components, respectively.

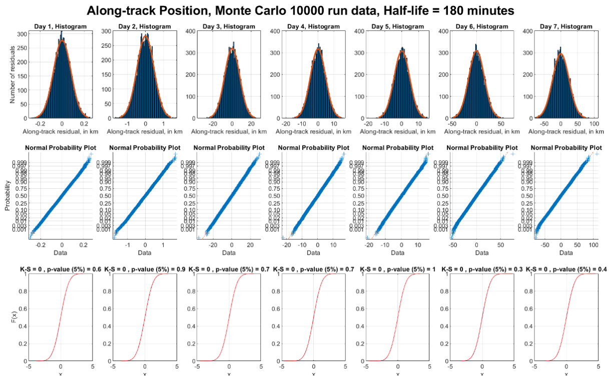

Figure 9. Monte Carlo results showing denormalized ECI position along track standard deviation for 180 minutes half-life

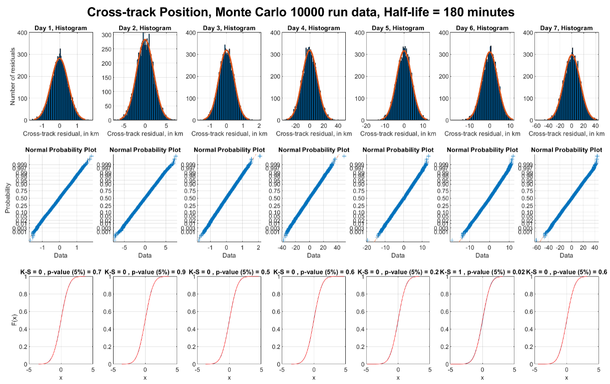

Figure 10. Monte Carlo results showing denormalized ECI position cross track standard deviation for 180 minutes half-life

6. CONCLUSION AND FUTURE WORK

This paper presented methods to improve numerical stability in LEO orbit propagation through SVD decomposition, normalization, and adaptive integration techniques. Implications for LEO orbit modeling and broader applications in space situational awareness and collision avoidance are highlighted, with ongoing research aimed at further improving stability and realism in uncertainty modeling.

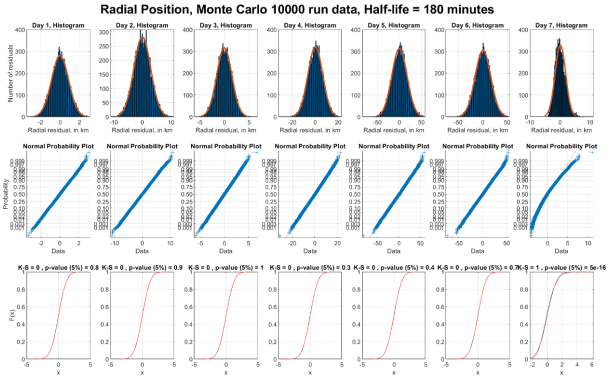

Figure 11. Monte Carlo results showing denormalized ECI position radial standard deviation for 180 minutes half-life

Non-linearity in covariance caused due to repeated covariance orthogonalization is concluded as a contributing factor to stability issues. Monte Carlo simulation with 10,000 sample was used to validate this conclusion.

Further work will introduce a new architecture that will forgo the need for repeated covariance orthogonalization to characterize the temporal correlation of atmospheric density while accurately incorporating the uncertainty on the same during real-time estimation.

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