

QUALITY CRITERION FOR ORBITS OF IN-SITU IMPACT DETECTION MISSIONS

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ABSTRACT

Space debris poses significant dangers to satellites, potentially shortening mission durations or causing orbital break-ups. Despite its importance, still high uncertainties prevail concerning the modelling of the small particle population. In the upcoming decade, new missions are expected to deliver data with unprecedented quality on this non-trackable debris population. This work proposes a quantitative metric for the assessment of orbits of such missions, thus aiding mission analysis in their design. This metric is derived by restructuring the χ^2 -test to formulate an equation that indicates the width of a 2σ -interval of a Poisson distribution relative to its mean. The statistical process of counting debris is assumed to be a (non-homogeneous) Poisson process, resulting in such a Poisson distribution. Thus, the equation provides a measure of the statistical uncertainty of detected impacts on an in-situ measurement mission.

Keywords: in-situ detection missions; impact detection; uncertainty quantification; probability interval; space debris statistics; Poisson statistics; orbit selection.

1. INTRODUCTION

In-situ space debris detection missions are expected to be a valuable resource for further improving statistical models of the space debris environment, such as ESA's Meteoroid And Space Debris Terrestrial Environment Reference (MASTER). This paper focusses on the selection of orbits for impact-based in-situ measurements. The mechanism of detecting impacts is assumed to be ideal, and thus only the uncertainty related to moving through a randomly distributed cloud of particles is taken into consideration.

1.1. Relevant Statistics

Impacts of space debris on a surface can be modelled as a Poisson process [2]. A Poisson process is

a statistical counting process in which events occur at random points in time independent of previously occurred events. It has only one parameter, which is the mean rate λ at which these events occur per time unit $\Delta t = 1$. The rate $\lambda(t)$ can be time dependent. The time dimension in statistics can also be seen as a one-dimensional position. This property is very useful for the case of space debris, as the satellite generally moves through areas of various amounts of particles per volume.

The resulting count at the end of a Poisson process is distributed as a Poisson distribution with the mean at $\lambda \cdot \Delta t$. With Δt being the number of time units that have passed since the start of the observation.

1.2. Exemplary Mission

An exemplary mission is used for demonstration in this paper. The detection surface shall be a 100 square metre sail. It is assumed that the mechanism for detecting hits on this sail is ideal for all objects larger than a specified minimum size and does not have false negatives or false positives. Different possible orbits of such a mission will be discussed in section 4.

2. METHOD AND INTERPRETATION

Furumoto and Sahara [2] presented a statistical method that indicates if the flux of debris changed in a measurement. Specifically, they calculated how large the relative difference between two counts of separate intervals in the measurement has to be to determine with a 95% certainty that they are based on different fluxes of debris.

For this paper, the basic approach of Furumoto and Sahara is used and modified. The resulting equation estimates the borders of the 2σ -interval of the distribution around the number of impacts indicated by a model for which MASTER is used in this case.

2.1. Approach

First, a χ^2 -test was constructed with the inputs of λ_{sat} , the hypothetical measured rate in a detection mission, and λ_{model} , the simulated rate for the same mission. Continuing, the time interval Δt is always reduced to $\Delta t = 1$. The structure is shown in table 1. The first line with $i = 1$ is the actual comparison between the measurement and the best guess (the model). After that, a theoretical measurement is done comparing the model with the best guess (again the model). This is done to follow the rules of the χ^2 -test. Using equation 1 for the χ^2 -test this concludes in equation 3.

Table 1. Inputs for χ^2 -test

| i | Measurement x_i | Expectation e_i |
|---|-------------------|-------------------|
| 1 | λ_{sat} | λ_{model} |
| 2 | λ_{model} | λ_{model} |

$$\chi^2 = \sum \frac{(x_i - e_i)^2}{e_i} \quad (1)$$

$$\Rightarrow \chi^2 = \frac{(\lambda_{sat} - \lambda_{model})^2 + (\lambda_{model} - \lambda_{model})^2}{\lambda_{model}} \quad (2)$$

$$= \frac{(\lambda_{sat} - \lambda_{model})^2}{\lambda_{model}} \quad (3)$$

Then, a factor of difference s is introduced with $\lambda_{model} = \lambda_0$ and $\lambda_{sat} = s \cdot \lambda_0$. This substitution concludes in Equation 5.

$$\chi^2 = \frac{(s\lambda_0 - \lambda_0)^2}{\lambda_0} \quad (4)$$

$$s = 1 \pm \sqrt{\frac{\chi^2}{\lambda_0}} \quad (5)$$

In this case, χ^2 is a chosen value and is set to the value equivalent to 2σ (about 95.45%). Using the inverse χ^2 distribution, this amounts to $\chi^2 = 3.9999 \dots \approx 4$.

With this equation, s indicates the relative distance from the mean to both the start and the end of the 2σ -interval and therefore the interval p in figure 1. Figure 1 illustrates an exemplary Poisson probability mass function (PMF) and cumulative distribution function (CDF) with visual representations of the discussed intervals and values. However, the Poisson distribution is obviously discrete, as it describes a counting process. Thus, the interval p has to be corrected to an interval u . This actual interval u is set to be equal to or greater than the desired interval p , corresponding to the common implementation of a Poisson inverse CDF. With $F(X)$ being the Poisson CDF and $\frac{1-p}{2} = \hat{p}$, the following formulations can be constructed:

$$F(n_{end} - 1) < 1 - \hat{p} \leq F(n_{end}) \quad (6)$$

$$F(n_{start}) \leq \hat{p} < F(n_{start} + 1) \quad (7)$$

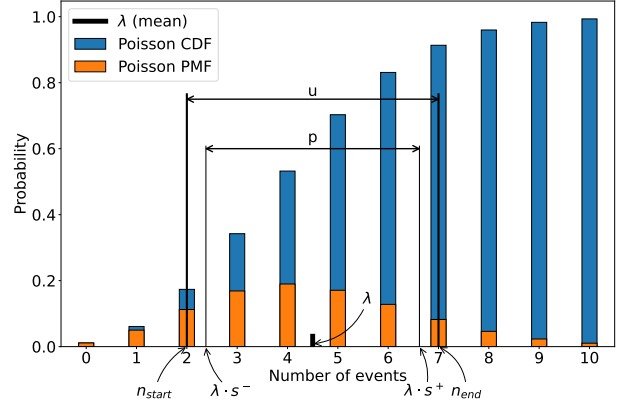


Figure 1. Exemplary Poisson distribution with a calculated interval p and an actual interval u around a mean λ .

2.2. Method Algorithm

The following algorithm is then created to calculate the quality criterion for given orbits with the previously described mathematical approach.

1. Define the mission environment with mission epoch and duration.
2. Define the lower threshold for detectable debris.
3. Define the impact surface in MASTER.
4. Choose orbits of interest.
5. Run MASTER for the mission duration with the orbit in the environment.
6. Multiply the resulting total flux by the total detection surface.
7. Calculate the associated s -values with a χ^2 -value of 3.999999999998461 (Equation 5).

Note: Alternatively, an implementation of the inverse Poisson CDF can be used. The Python module SciPy allows to get the according limits of an interval as the absolute values. The s -values are the limits normalised by the mean.

The s^+ -value is the associated quality criterion for the condensed population. It denotes the width of the 2σ -interval (95.45%) relative to the expected number of impacts. For example, an s^+ -value of 1.5 indicates that with a probability of about 95.45% the count of particles on the satellite will lie within $\pm 50\%$ of the mean.

3. INSIGHTS INTO THE METHOD

3.1. Test of the method

In discussions about the method, the question arose, how it can be viable for the inhomogeneous environment of space debris. The question itself is based on the satellite flying through volumes of different densities of particles. This is replicated in MASTER, making it natural to test the theory with this software. One of the outputs of the MASTER simulation is the Cell Passage Event (CPE) data [3]. The CPEs represent the unique passages through discrete volumes of the orbit environment. For each volume, the mean probability of a collision with a particle can be calculated [1]. Therefore, realising each single homogeneous Poisson process will give a reasonable count of the number of particles that impacted the satellite in its environment. This was run as a Monte Carlo simulation (without adapting the base parameters). The output of each single run is the total number of particles that hit the satellite. For the full Monte Carlo simulation, the output has a distribution that is actually just a Poisson distribution.

This is given by the superposition of multiple Poisson processes still being a Poisson process with all mean rates summed (simplified). The CPE data is already normalised to impacts per year. Therefore, the added up number of impacts per year can be used as the mean for a Poisson distribution. This is also true for a normalisation by area. Thus, CPE data, which is normalised by year and square metre, can be used by adding up 'flux contribution' for each CPE and multiplying the result by the area of the satellite. The resulting number is the mean of the Poisson distribution.

The next step in testing the approach is to check if the results of equation 5 coincide with the interval created by the inverse Poisson CDF. As the inverse Poisson CDF is discrete, the ceiling function has to be applied to both s -values after multiplying them by the corresponding mean. Then, both values can be compared to the values that give the correct interval according to the inverse Poisson CDF for different means. Setting the mean to any integer number up to 1 000 000, no deviation can be found.

3.2. Visual representation

The s -value can be plotted over the Poisson mean rate as visualised in figure 2. The 2σ -interval for a Poisson rate can be read from the vertical axis. Then, the s -value can be multiplied by the chosen Poisson rate to produce the absolute interval. The horizontal axis can also be recalculated to represent time (e.g. with a daily rate of impacts). Such a figure might aid in deciding on the duration of an observation period.

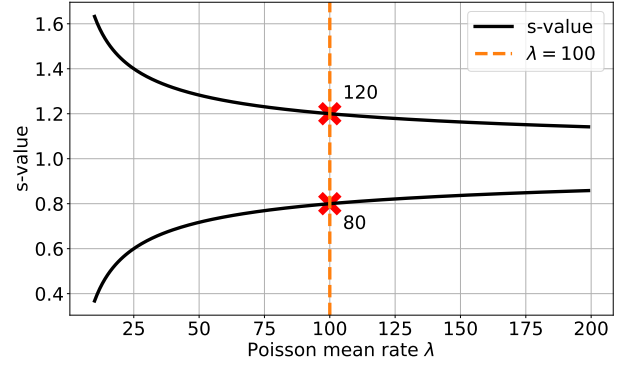


Figure 2. s -values over different Poisson rates determined with equation 5 ($\chi^2 \approx 4$).

4. APPLICATION OF THE METHOD TO AN EXEMPLARY MISSION

An exemplary in-situ detection mission with multiple hypothetical orbits is used to demonstrate the developed algorithm. The approaches used to find exemplary orbits for this mission are highlighted in the list below. The resulting orbits are then summarised in table 2. These do not represent an actual orbit analysis but give a rough idea of how the mission might perform in these environments. Sun-synchronous orbits (SSO) and heavily populated orbits are chosen for their relevance to in-situ detection missions.

- Choose a circular orbit via the MASTER bin with the highest spatial density.
- Choose a SSO similar to the point above.
- Choose a highly populated SSO (satellites launched after 01/2014).
- Choose a highly populated non-SSO orbit (satellites launched after 01/2014).
- Choose an orbit similar to the orbit of the International Space Station (ISS).

The MASTER environment was set to use particles of 0.1 mm to 100 mm size at the current reference epoch (01.08.2024). The bin with the highest spatial density was found using the output file `master_t._.1`. This file is created in the spatial density mode when demanding a figure of the spatial density over altitude and declination.

Highly populated orbits were identified using the dataset of tracked low-Earth orbit (LEO) objects from `space-track.org` [4]. The 30 day dataset from October 2024 was used. The data was filtered for the tag payload (no debris), for a periapsis altitude greater than 200 km and for launch dates after 01.01.2014. Optionally, the data was filtered for Starlink satellites. Any satellite that passed a defined orbit height ± 0.5 km was counted

towards the frequency of the population of that orbit. The orbits with the highest frequency are chosen.

Table 2. Hypothetical exemplary orbit parameters for demonstration of the quality criterion.

| ID | SMA [km] | INC [°] | Orbit Selection Short Description |
|----|-------------|------------|--------------------------------------|
| 1 | 7262.25 | 97.5 | By MASTER-bin |
| 2 | 7262.25 | 99 | SSO like '1' |
| 3 | 6910.00 | 82.5 | Most satellites |
| 4 | 6869.00 | 82.5 | '3' w/o Starlink |
| 5 | 6934.00 | 97.7 | Most sats (SSO) |
| 6 | 6869.00 | 97.4 | '5' w/o Starlink |
| 7 | 6794.60 | 51.6 | ISS eps=0.0007 |

Next, the method was used on the chosen orbits. The results of this method are highlighted in table 3.

Table 3. Results of the method for exemplary orbits with particles larger than 0.1 mm.

| ID | Flux [m ⁻² yr ⁻²] | Flux*Area [yr ⁻¹] | s ⁺ -value (1 year) |
|----|---|----------------------------------|-----------------------------------|
| 1 | 556.1 | 55610.0 | 1.0085 |
| 2 | 478.8 | 47880.0 | 1.0091 |
| 3 | 15.44 | 1544.0 | 1.0509 |
| 4 | 5.074 | 507.4 | 1.0640 |
| 5 | 26.29 | 2629.0 | 1.0390 |
| 6 | 13.12 | 1312.0 | 1.0552 |
| 7 | 2.784 | 278.4 | 1.1199 |

This table shows that generally the s⁺-value is very low for the debris size regime starting at 0.1 mm. The exemplary mission detects this size of debris even in the low-flux orbit of the ISS with a deviation of just ±11.99% around the mean. This can be sufficient for a mission. However, an in-situ measurement mission might not be able to detect such small debris. Thus, two other runs are presented with a simulation environment starting at 0.5 mm and 1 mm particle sizes. They are shown in the tables 4 and 5, respectively. The orbits 1 and 2 were not optimised for these two size regimes.

Table 4. Results of the method for exemplary orbits with particles larger than 0.5 mm.

| ID | Flux [m ⁻² yr ⁻²] | Flux*Area [yr ⁻¹] | s ⁺ -value (1 year) |
|----|---|----------------------------------|-----------------------------------|
| 1 | 0.1525 | 15.25 | 2.7741 |
| 2 | 0.1409 | 14.09 | 2.8457 |
| 3 | 0.03896 | 3.896 | 4.51 |
| 4 | 0.007232 | 0.7232 | 9.1469 |
| 5 | 0.0473 | 4.73 | 4.1856 |
| 6 | 0.007088 | 0.7088 | 9.2292 |
| 7 | 0.00413 | 0.413 | 11.7807 |

With these two size regimes, the s⁺-value has increased significantly to multiplicative factors of the mean. Although an uncertainty of a factor of 2 might still be

Table 5. Results of the method for exemplary orbits with particles larger than 1 mm.

| ID | Flux [m ⁻² yr ⁻²] | Flux*Area [yr ⁻¹] | s ⁺ -value (1 year) |
|----|---|----------------------------------|-----------------------------------|
| 1 | 0.005187 | 0.5187 | 10.6197 |
| 2 | 0.004961 | 0.4961 | 10.8364 |
| 3 | 0.001785 | 0.1785 | 17.3984 |
| 4 | 0.001081 | 0.1080 | 22.0819 |
| 5 | 0.001494 | 0.1494 | 18.9244 |
| 6 | 0.001221 | 0.1221 | 20.8273 |
| 7 | 0.000436 | 0.0436 | 34.1801 |

feasible for space debris in general, factors significantly higher than that seem to be out of scope for the exemplary mission, which would still have a significant cost tied to it. Thus, demanding seemingly accurate results.

5. CONCLUSIONS

A method for calculating a quality criterion for orbits of in-situ detection missions was presented. A hypothetical exemplary mission with a 100 m² detection surface was then used to demonstrate that single in-situ detection missions might only be feasible for the measurement of sub-millimetre particle fluxes. Aiming for particles above such a threshold might require the involvement of larger sails, longer observation periods or multiple missions with the same scope.

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