MANOEUVRE IDENTIFICATION AND CHARACTERISATION FROM TWO LINE ELEMENTS

Yeerang Lim[(1)], Camilla Colombo[(1)]

[(1)] *Politecnico di Milano, Department of Aerospace Science and Technology, Via La Masa 34, 20156 Milan, Italy,*
Email: yeerang.lim@polimi.it, camilla.colombo@polimi.it

ABSTRACT

Detecting and reconstructing manoeuvres from given data is one of the important topics in the field of space situational awareness. A novel formulation to estimate unknown manoeuvres from historical orbit data is presented. The manoeuvre is identified from orbital element change, based on least square estimation, which is widely used for orbit determination problems. By applying the proposed approach, a manoeuvre and its time information can be estimated from inverting Gauss' planetary equations. The proposed method is validated against the TLE history from the Envisat spacecraft.

1 INTRODUCTION

Tracing unknown events in orbital environment from historical data is important, especially as the population in orbital regimes increases dramatically and as manoeuvre data are not publicly shared. Such data interpretation often provides useful information that is not necessarily shared, from which spacecraft is still active, to which behaviours can be expected and considered in the decision-making process. Manoeuvre detection and characterisation have been a key component in space situational awareness in that sense [1-5]. The estimation is possible from the orbit dynamics and observation history. If there are significant changes observed in the trajectory which cannot be explained by orbit propagation, an impulsive manoeuvre or a similar event can be assumed.

Past works focused on space event detection from existing data based on statistical technique [1][2][3]. Two Line Elements (TLE) data is often used as database because of its public accessibility and regular update. Patera proposed the moving window method, to filter orbital anomalies from smaller variations due to perturbations or noise [1]. The approach was applied to detect various events, i.e. collision, manoeuvre, even atmosphere fluctuation due to increased solar activity. Focusing on the manoeuvre detection, orbital boost manoeuvres were detected from an orbital energy plot. Kelecy et al. focused on out-of-plane manoeuvres, i.e. inclination change, to aid area-to-mass ratio estimation [2]. Similar to [1], data difference detection from adjacent filtered segments is proposed, which smooths the data in a sliding interval. Polynomial fit is calculated both for the trailing segment and the leading segment, and the difference in the middle between the two data is examined. Both orbit raising and inclination change were detected from orbital energy plot and inclination history. Lemmens and Krag proposed TLE Consistency Check (TCC) as well as TLE Time Series Analysis (TTSA) [3]. TCC approach detects events by comparing a published state with a propagated state, while TTSA detects them by extrapolating the behaviour of the series to measurement epoch outside of the extrapolation window.

There are relatively less literatures addressing manoeuvre reconstruction. While simply calculating delta-v magnitude from a jump in orbital energy or in inclination history is widely used, there are two studies estimating manoeuvre characteristics from historical data [4][5]. Both Pastor et al. [4] and Porcelli et al. [5] used raw measurements. Pastor et al. applied least square method to optical measurements [4]. Like the classical orbit determination, it provides manoeuvre estimation which minimises the residual between pre- and post-manoeuvre orbits. Two formulations were proposed for single burn and double burns cases, namely track-to-orbit and orbit-to-orbit. Then Porcelli et al. applied the similar approach to the track-to-orbit approach to radar measurements [5].

We introduce here a new formulation for identification and characterisation of a manoeuvre from historical data in forms of Keplerian orbit elements, based on the least-square optimisation. Compared to Cartesian coordinates, Keplerian elements are beneficial since they provide orbit's shape and orientation instantaneously and most of the elements vary slowly over time. While previous literatures mainly focus on the manoeuvre detection from trajectory change, we attempt to extract more information about the manoeuvre from the orbit elements history itself. The Gauss' planetary equations in the literature are used together with the Keplerian element formulation for manoeuvre identification, which lead to linear relationships with an impulse and the orbit elements difference, which then also can be propagated with the state transition matrix. This formulation can be transformed into a linear least square problem in terms of true anomaly of the manoeuvring point without loss of generality. A manoeuvre can be identified by searching the true anomaly i.e. the epoch of the manoeuvre, which minimises the difference between the orbit measurement and the propagation of the least-square solution delta-v to

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the epoch of measurement.

We also used TLE data for validation. Any outliers in the data are filtered first and then difference in orbit elements are calculated. Instead of six Keplerian elements, the argument of latitude replaces both the argument of perigee and the true anomaly, considering their accuracy in the measurements. The feasibility of the proposed approach is examined with the orbit history of the Envisat spacecraft, whose manoeuvre history is also available [6]. Simulations are performed for out-of-plane manoeuvres and in-plane manoeuvres separately, as the two manoeuvres are often not combined. Expected limitations from the TLE accuracy seen in the simulations are also addressed, together with a sensitivity analysis of the approach with synthetic orbit data.

2 MANOEUVRE IDENTIFICATION VIA GAUSS' PLANETARY EQUATION

When there is a discontinuous change in a spacecraft trajectory caused by an impulsive manoeuvre, the linear relationship between a manoeuvre vector $\delta\mathbf{v} = \begin{bmatrix}\delta v_t & \delta v_n & \delta v_h\end{bmatrix}^T$ and consequential changes in orbit elements $\delta\boldsymbol{\alpha} = \begin{bmatrix}\delta a & \delta e & \delta i & \delta\Omega & \delta\omega & \delta M\end{bmatrix}^T$ is given as Gauss' Planetary Equation as a function of true anomaly f [7].

$$\mathbf{G}_v(f) = \begin{bmatrix} \frac{2a^2 v}{\mu} & 0 & 0 \\ \frac{2(e+\cos f)}{v} & -\frac{r}{av}\sin f & 0 \\ 0 & 0 & \frac{r\cos\theta}{h} \\ 0 & 0 & \frac{r\sin\theta}{h\sin i} \\ \frac{2\sin f}{ev} & \frac{2e+(r/a)\cos f}{ev} & -\frac{r\sin\theta\cos i}{h\sin i} \\ -\frac{b}{eav}2\left(1+\frac{e^{2r}}{p}\right)\sin f & -\frac{b}{eav}\frac{r}{a}\cos f & 0 \end{bmatrix} \quad (1)$$

where $\theta = f + \omega$ is the argument of latitude, b is the semi-minor axis of the orbit, p is the semilatus rectum, h is the norm of the angular momentum, v is the magnitude of the velocity, and r is the radial distance.

Assume that two states in Keplerian elements $\boldsymbol{\alpha}_0$ and $\boldsymbol{\alpha}_1$ are available at the epoch t_0 and t_1 respectively, which makes $\boldsymbol{\alpha}_0 = \boldsymbol{\alpha}(t_0)$ and $\boldsymbol{\alpha}_1 = \boldsymbol{\alpha}(t_1)$, and the difference between two states cannot be explained by propagation. An impulsive manoeuvre within $[t_0, t_1]$ can be expected, with its epoch $t = t_1 - \Delta t$ to be solved. As shown in Figure 1, it is equivalent to a Two-Point Boundary Value Problem (TPBVP) with unknown Δt,

$$\delta\boldsymbol{\alpha}(t_1) = \mathbf{G}_M(\Delta t)\mathbf{G}_v(t)\delta\mathbf{v} = \mathbf{G}(\Delta t)\delta\mathbf{v} \quad (2)$$

where $\mathbf{G}_M$ is the State Transition Matrix (STM) to describe the changes in the error states $\delta\boldsymbol{\alpha}$ along the coasting arc. Assuming small error states, it is possible to simplify $\mathbf{G}_M$ as below by considering Keplerian motion only.

$$\mathbf{G}_M(\Delta t) = \begin{bmatrix} 1 & \mathbf{0}_{1\times4} & 0 \\ \mathbf{0}_{4\times1} & \mathbf{I}_{4\times4} & 0 \\ -\frac{3}{2}\frac{\sqrt{\mu}}{a^{5/2}}\Delta t & \mathbf{0}_{1\times4} & 1 \end{bmatrix} \quad (3)$$

Since the matrix $\mathbf{G}(\Delta t)$ is 6x3, solving $\delta\mathbf{v}$ is an over-determined problem, meaning that there is no solution in general and therefore the problem should be dealt in a statistical way, i.e. regression analysis. In this regard, the problem in Eq. 2 can be converted to an optimal Δt searching in $[t_0, t_1]$, which minimises

$$\left\| \delta\boldsymbol{\alpha}(t_1) - \mathbf{G}(\Delta t)\delta\mathbf{v}_{LSQ} \right\| \quad (4)$$

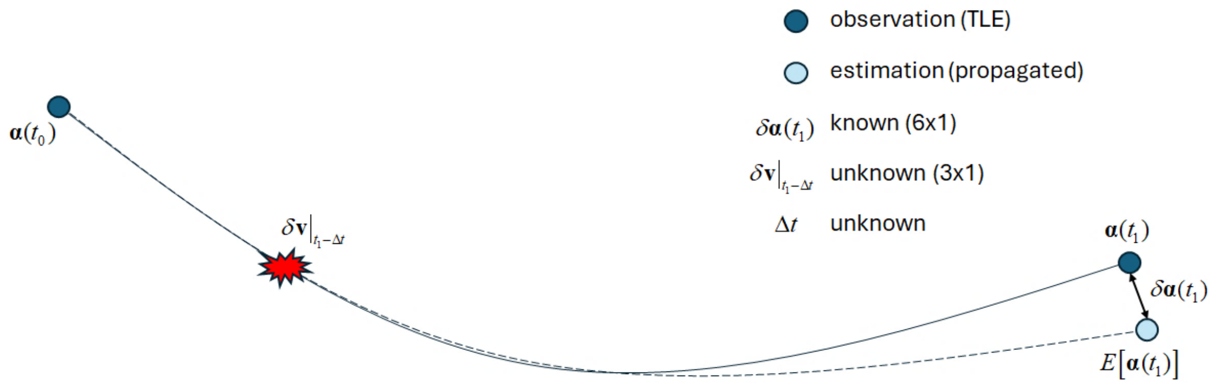


Figure 1 Schematic problem definition of maneuver reconstruction

By assuming a fixed Δt , a least-square solution is available as Eq. 5.

$$\delta \mathbf{v}_{LSQ} = \left(\mathbf{G}^T \mathbf{G}\right)^{-1} \mathbf{G}^T \delta \boldsymbol{\alpha} \tag{5}$$

when there is no perturbation or measurement noise present, Eq. 4 becomes zero with the correct Δt .

The formulation in Eq. 4 and Eq. 5 is validated with numerical simulations. At first, an initial orbit in Table 1 is propagated with GMAT [8] from 21545 MJD, with an impulsive manoeuvre described in Table 2. EGM-96 gravity model with 0 degree and 0 order is used, i.e. no Earth oblation is considered. Atmospheric drag is also not included in the propagation.

Table 1 Test case (1) – initial and final orbits

Initial orbit @ modified Julian date 21545					
X (km)	Y (km)	Z (km)	V_X (km/sec)	V_Y (km/sec)	V_Z (km/sec)
7100	0	1300	0	7.35	1
Final orbit @ modified Julian date 21545.486					
X (km)	Y (km)	Z (km)	V_X (km/sec)	V_Y (km/sec)	V_Z (km/sec)
6163.539	-3520.865	648.782	3.674	6.369	1.541

Table 2 Test case (1) – impulsive manoeuvre to be solved

t_0 (MJD)	21545
t_f (MJD)	21545.486
Δt	30000 sec
$t_{\delta \mathbf{v}}$	t_0 + 12000 sec
$\delta \mathbf{v}$ (TNH)	$\left[1.0 \quad 0 \quad 2.0\right]^T$ (m/sec)

Figure 2 is a plot of the error magnitude with the least square solution in Eq. 4 for each Δt candidates. As shown in Figure 2 and also as summarised in Table 3, Δt with the minimum error matches with the true value in Table 2, and also the manoeuvre vector is estimated quite well.

Table 3 Test case (1) – manoeuvre estimation result

$\delta \mathbf{v}_{LSQ}$ (m/sec)	$\left[1.000 \quad -0.015 \quad 1.998\right]^T$
$\left\|\delta \mathbf{v}_{LSQ} - \delta \mathbf{v}\right\| / \left\|\delta \mathbf{v}\right\|$	0.68 %
Δt	30000 sec

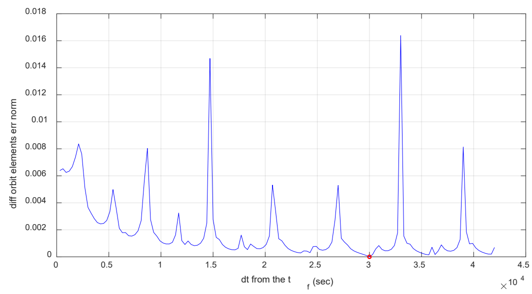


Figure 2 Δt searching via least square estimation

The same approach is extended to include J_2. The final orbit changes under the Earth oblateness. As summarised in Table 5, the manoeuvre estimation is significantly degraded by assuming a Keplerian motion. To overcome this issue, (1) we switched from osculating orbit elements to Brouwer Long mean elements [9], (2) modified the STM in Eq. 3 to include secular drift, and (3) included the secular change in absolute orbit elements when calculating $\mathbf{G}(\Delta t)$. The modified STM is given in Eq. 6,

$$\mathbf{G}_M^{J_2}(\Delta t) = \mathbf{G}_M(\Delta t) + \mathbf{\Phi}^{J_2}(\Delta t) \tag{6}$$

while the details of the additional matrix $\mathbf{\Phi}^{J_2}(\Delta t)$ is available in [10]. The estimation gets much more accurate with the modified equation as in Table 6.

Table 4 Test case (2) – final orbit

Final orbit @ modified Julian date 21545.486					
X (km)	Y (km)	Z (km)	V_X (km/sec)	V_Y (km/sec)	V_Z (km/sec)
6466.935	-2887.862	866.385	3.010	6.726	1.422

Table 5 Test case (2) – manoeuvre estimation result

$\delta\mathbf{v}_{LSQ}$ (m/sec)	$[0.9806 \quad -0.6445 \quad 2.2604]^T$
$\left\|\|\delta\mathbf{v}_{LSQ} - \delta\mathbf{v}\right\|\| / \|\|\delta\mathbf{v}\|\|$	31.10 %
Δt	30000 sec

Table 6 Test case (2) – manoeuvre estimation result

$\delta\mathbf{v}_{LSQ}$ (m/sec)	$[0.9946 \quad -0.0273 \quad 2.0293]^T$
$\left\|\|\delta\mathbf{v}_{LSQ} - \delta\mathbf{v}\right\|\| / \|\|\delta\mathbf{v}\|\|$	1.81 %
Δt	30000 sec

A couple of notes should be added for the proposed manoeuvre reconstruction. First and foremost, there is an ambiguity in orbital revolution as shown in Figure 3 due to periodic $\mathbf{G}(\Delta t)$, unless there is a secular drift in along-track direction. For an orbit-normal case the ambiguity exists in every half orbit with a manoeuvre in the opposite direction. Second, the derivation includes the first order approximation for linearization. Therefore, it is expected that the estimation error gets larger when the propagation time is longer.

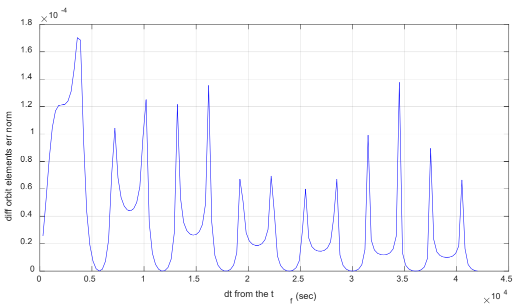


Figure 3 Periodic ambiguity in least-square solution search

3 MANOEUVRE RECONSTRUCTION FROM TLE DATA

The proposed approach is now tested with the actual data, TLEs in this study, mainly because of its public availability. To apply the approach to the actual TLE history, TLE outlier removal in [8] is applied. We do not address the outlier filtering in details here, it can be summarised in five steps as follow.

(1) Remove the TLEs corresponding to a correction of the immediately previous elements, according to a minimum threshold of the update time between two subsequent TLEs

(2) Identify large gaps between TLEs to define time windows in which outliers will be searched

(3) Remove the TLEs with values of inclination that are not coherent

(4) Remove the TLEs with values of eccentricity that are not coherent

(5) Remove the TLEs with negative values of B* drag term

Once the filtered TLEs are obtained, the discontinuous changes in the values are selected as the initial and the final point of the problem. Please note that, we do not include the autonomous manoeuvre detection in this study, so once the outliers are filtered a peak in the TLE history is manually picked up for now. Figure 3 shows the simulation process.

Actual manoeuvre history in orbit is available for specific satellites by the International DORIS Service (IDS) [6]. When applying the approach first, we witness the large $\delta\omega$ and δM values with the opposite signs. It is suspected that the perigee direction in TLEs varies a lot due to the small eccentricity, so we reduced the dimension of the problem from (6x3) to (5x3), by using argument of latitude $\lambda = \omega + M$ instead of argument of perigee and mean anomaly separately.

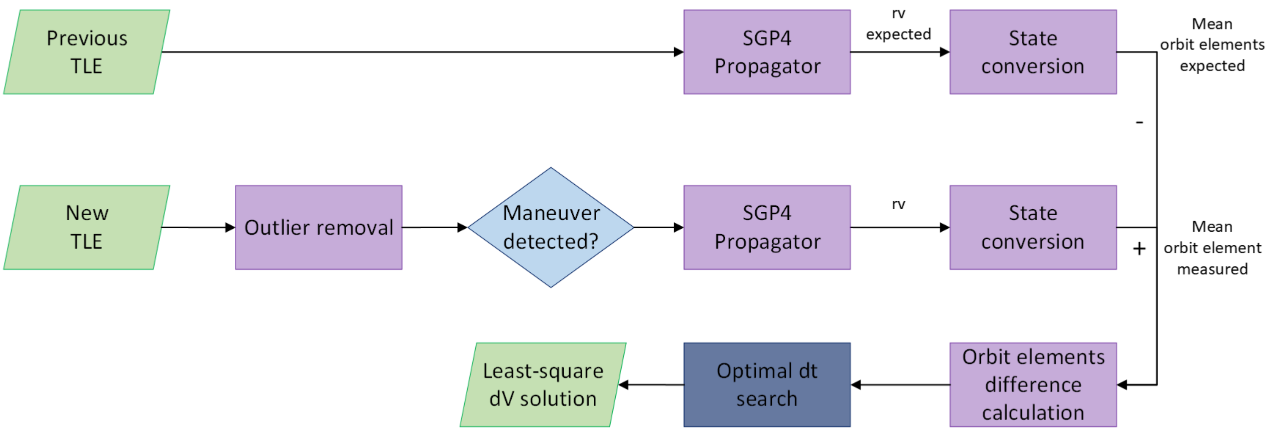


Figure 4 Manoeuvre identification process from TLE data

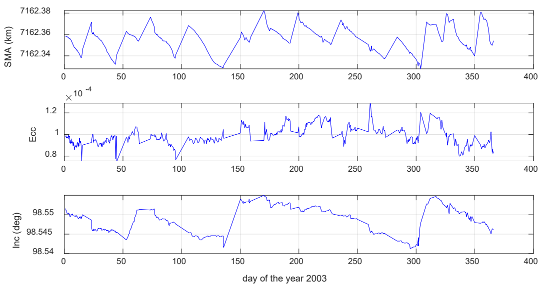


Figure 5 Envisat TLE history, year 2003

The estimation result is summarised in Table 7. It is already mentioned in [2], that the post-manoeuvre TLE typically suffer a time lag on the order of days before showing the full effect of a known manoeuvre, which also can be observed in Figure 5. Therefore we do not include Δt estimation study in the table, as a peak in the TLE plot does not match with the actual epoch of the orbit change.

Table 7 Envisat out-of-plane manoeuvre estimation, happened between 2002 to 2005

Epoch	02252	02352	03135	03300	04034	04105	05006	05076
$\delta\mathbf{v}$ happened (m/sec, TNH)	0 0 1.6960	0.0117 0.0214 1.4566	0.0113 0.0186 1.8717	0.0051 0.0469 2.0042	0.0028 0.0327 1.7468	0.0085 0.0160 1.7320	0.0033 0.0478 1.9151	0.0101 0.0148 2.0157
$\delta\mathbf{v}$ estimated (m/sec, TNH)	0.0251 0.0491 -1.715	0.0384 -0.1369 -1.5168	0.0230 -0.3065 1.2879	0.0247 -0.6596 1.5225	0.0043 0.1413 1.5523	0.0195 0.0168 2.0196	0.0194 -0.0065 2.0204	0.0132 0.0027 -1.9571
$\left\| \|\delta\hat{v}_h\| - \|\delta v_h\| \right\| / \|\delta v_h\|$	1.12 %	3.97 %	31.19 %	24.03 %	11.13 %	3.90 %	16.61 %	2.91 %

Table 8 Envisat in-plane manoeuvre estimation, happened in 2003

Epoch	03043	03063	03094	03135	03157	03227	03272	03304	03349
$\delta\mathbf{v}$ happened (m/sec, TNH)	0.0216 -0.0001 -0.0010	0.0211 0.0000 0.0011	0.0252 -0.0001 -0.0010	0.0164 0.0000 0.0011	0.0175 0.0000 -0.0002	0.0154 0.0000 -0.0007	0.0163 0.0000 -0.0009	0.0348 0.0001 -0.0013	0.0212 0.0001 -0.0011
$\delta\mathbf{v}$ estimated (m/sec, TNH)	0.0236 0.6704 0.3868	0.0175 1.4251 0.6503	0.0269 -0.2248 -0.8235	0.0246 0.2036 -1.2781	0.0250 -1.9872 0.6235	0.0155 0.4637 0.2880	0.0159 0.3227 0.4560	0.0302 0.1657 -0.6776	0.0288 0.1310 -0.0271
$\left\| \|\delta\hat{v}_T\| - \|\delta v_T\| \right\| / \|\delta v_T\|$	9.26 %	17.06 %	6.75 %	50.00 %	42.86 %	0.65 %	2.45 %	13.22 %	35.85 %

4 SENSITIVITY ANALYSIS

A sensitivity analysis is conducted under two scenarios, (1) orbit plane change manoeuvre and (2) orbit raising manoeuvre. It is a tricky to perform a single, uniform analysis, as the estimation performance also depends on the magnitude and direction of the reference manoeuvre. Therefore, the case scenarios were divided into two cases, (1) when there is 1.0 m/sec out-of-plane manoeuvre happened and (2) 1.0 m/sec along-track manoeuvre happened. Then a position error up to 100 m in transverse/along-track/orbit-normal direction added for each case.

First, it is obvious that the estimation is less affected by position error for orbit-normal manoeuvres. Second, transverse direction estimation often contains the largest error, compared to the other two axes. From these two observations, it is explained that why in-plane manoeuvre reconstruction contains large errors in the transverse direction. This behaviour is already expected from the Gauss' planetary equation in Eq. 1. As relative semi-major axis can be only changed by along-track manoeuvre, and so is relative inclination by orbit-normal manoeuvre only, the algorithm tends to compensate the residuals by estimating (or adding) transverse manoeuvre. Unless both the measurement and the propagation is highly accurate, it is recommended to discard along-track estimation.

One possible way to overcome the high sensitivity is to reduce the dimension of the problem, especially as it is expected that the routine orbit maintenances are done separately for orbit raising and orbital plane change. If one kind of maintenance is observed in the TLE history already, it is possible to extract out-of-plane manoeuvre part in the GVE in Eq. 1 and estimate an orbit-normal impulse only. Similar approach can be applied to in-plane cases. If it is expected that the unknown manoeuvre is a routine one, it is reasonable to assume that there was only an along-track manoeuvre, and a transverse delta-v is negligible. Of course, this approach is not valid for an unexpected event such as collision or explosion, it would not be possible to assume the direction of an impact. Then it is necessary to have both a propagator with high fidelity, as well as precise measurement.

5 CONCLUSION

A new approach to characterise an unknown impulsive manoeuvre is introduced, in a form of least-square estimation with reverse Gaussian planetary equation. The proposed method is applied both on simulated orbit and the actual TLE history. A couple of remarks can be made, focusing on the TLE data and the sensitivity analysis. First, despite the TLE accuracy and large search window up to more than 10 days, it was possible to reconstruct a manoeuvre with reasonable accuracy, especially for inclination changes. Second, due to the nature of TLE from batch orbit determination, the full effect of an impulse appears days after from the actual epoch, which adds delays in the estimation. Therefore, it is not meaningful to estimate time of manoeuvre with TLE data. Last but not least, considering its sensitivity, it is not recommended to use the in-track manoeuvre estimation when level of accuracy is low, and the propagation time is long. The second and the third points can be further investigated by using different measurements if applicable, which is more precise than TLE and reflect manoeuvres instantly.

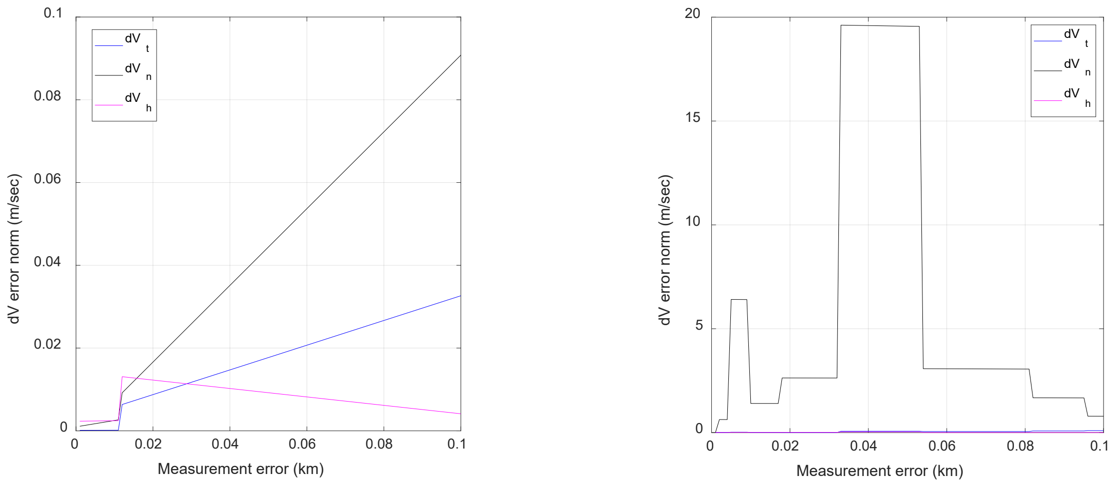


Figure 6 Error sensitivity analysis, transverse direction, out-of-plane (left) and in-plane (right) case

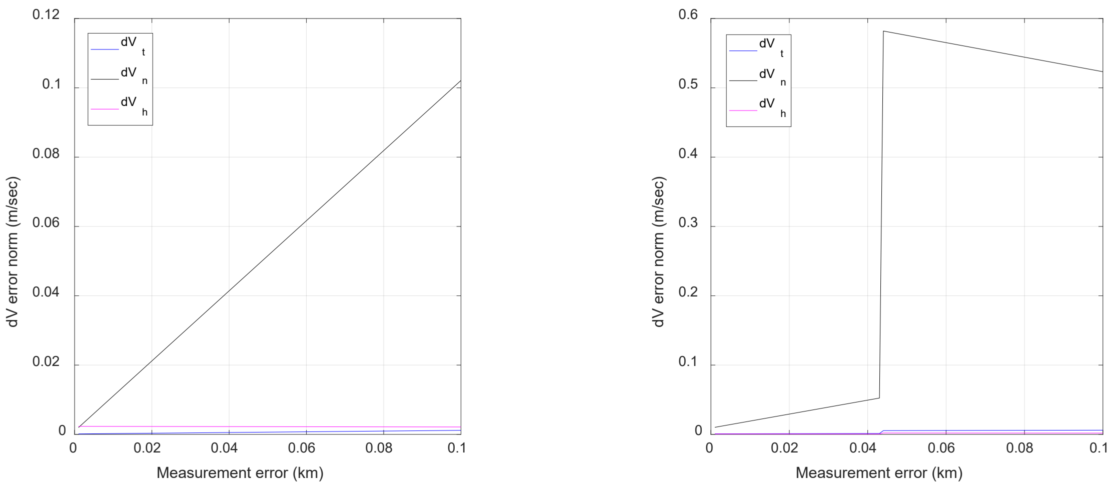
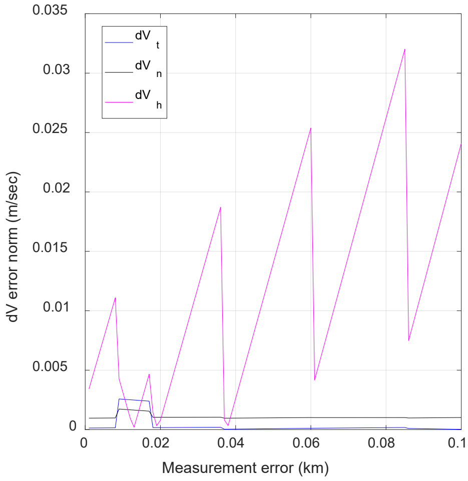


Figure 7 Error sensitivity analysis, along-track direction, out-of-plane (left) and in-plane (right) case



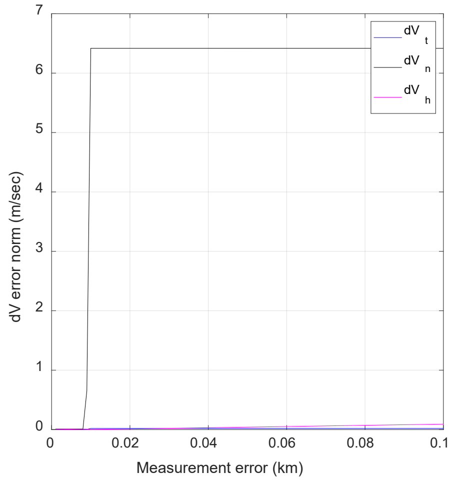


Figure 8 Error sensitivity analysis, orbit-normal direction, out-of-plane (left) and in-plane (right) case

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