Formulating the Space Footprint: A Model for Quantifying Space Sustainability

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ABSTRACT

As the use of Earth's orbital space intensifies with the rapid expansion of satellites, mega-constellations, and space debris, an advanced impact indicator is essential to quantify and support space sustainability. This paper presents the Space Footprint model, a comprehensive impact indicator grounded in mathematical modelling, developed to assess the cumulative strain human activities impose on orbital environments. By offering a unified measure of space utilization and sustainability, the model addresses the long-term safety, functionality, and resilience of space as a shared resource.
Space sustainability is defined here as the capacity of orbital regions to support ongoing and future operations without compromising safety or usability due to escalating risks from collision, debris proliferation, orbital crowding, and environmental degradation. Unlike traditional capacity-based models with static thresholds, the Space Footprint model introduces a dynamic, adaptive mathematical framework that captures a broader range of impacts from space operations. Beyond accounting for space debris, the model quantifies factors such as satellite density, technological advancements, operational efficiencies, and environmental effects, each represented as a unique variable within its core equation.

This paper derives the mathematical framework of the Space Footprint model, detailing the formulation of its core equation and demonstrating how each component reflects space sustainability factors through weighted, time-dependent variables. The model incorporates both linear and non-linear functions to capture complex interactions, including dynamic feedback loops from debris generation and non-linear risk escalations in high-density orbits. Key components, such as satellite population, debris density, and environmental drag, are represented as time-dependent variables, enabling the model to simulate and predict evolving orbital strain. To validate the model's applicability, the Space Footprint is applied to real-world scenarios, including assessments of mega-constellations in Low Earth Orbit (LEO) and projected impacts of space debris mitigation policies like the Zero Debris Policy. Using continuous real-time data from sources like the ESA MASTER/DISCOS databases, the model recalibrates in response to changing orbital conditions, offering stakeholders a reliable and continuously updated metric.

The paper demonstrates how the Space Footprint model can guide sustainable decision-making in space traffic management, satellite constellation planning, and debris mitigation strategies. By quantifying the long-term cumulative impact of human activities on space resources, the model provides a practical tool that aligns space operations with sustainability goals, promoting a safer and more resilient orbital environment for future generations.

1. INTRODUCTION: THE NEED FOR A SPACE SUSTAINABILITY INDICATOR

1.1 The Growing Challenge of Orbital Sustainability

Earth's orbital environment has become increasingly congested due to the rapid expansion of satellite deployments, space debris accumulation, and the rise of large constellations. As of March 2025, there are over 30,000 tracked objects in orbit, including active satellites, defunct spacecraft, and debris fragments exceeding about 10 cm in size [1]. The problem is most pronounced in Low Earth Orbit (LEO), where

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satellite density is highest due to the presence of Earth observation, communication, and military assets [2].

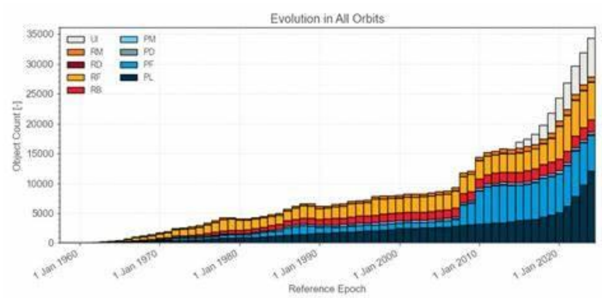

Figure 1: Evolution of the number of objects in all orbits. Retrieved from ESA Space Debris report 2024

A key concern is collisional cascading, also known as Kessler Syndrome, where each collision generates further debris, leading to an exponential growth in fragmentation events and the risk of eventual loss of certain orbital regions for safe operations [3]. This process depicts how an initial satellite collision produces debris fragments that propagate further collisions, amplifying the hazard. Past fragmentation events, such as the Fengyun-1C anti-satellite test (ASAT, in 2007) [4] and the Iridium-Cosmos collision (2009) [5], have demonstrated how individual events can significantly increase collision risks for other operators.

With the deployment of large constellations, including SpaceX Starlink, OneWeb, and Amazon Kuiper, LEO is set to host over 100,000 satellites by 2030 [6]. However, as illustrated in Figure 3, the expansion is not limited to established operators—China's planned mega-constellations are expected to grow significantly, further intensifying competition for orbital slots and spectrum resources.

Figure 2 provides a detailed projection of major LEO satellite constellations between 2025 and 2030, showing a sharp increase in satellite deployments, particularly from SpaceX, Amazon, and China. SpaceX's Starlink remains the dominant operator, with deployments expected to surpass 12,000 satellites by the end of the decade, while Amazon's Project Kuiper will complete its 3,236-satellite constellation by 2029. Meanwhile, China is on track to deploy up to 15,000 satellites, with ambitions exceeding 43,000 in the long term.

In this study, space sustainability is defined as the long-term stability, quality, and utility of the orbital environment—beyond just human use. While orbital space is essential for economic, scientific, and security applications, it also has intrinsic value, as it is part of the natural night sky and plays a role in ecosystem functions such as animal navigation [7]. The Space Footprint (SF) model aligns with this broader sustainability perspective by quantifying the cumulative impact of human activities, accounting for both operational strain (e.g., congestion, fragmentation) and natural regulatory forces (e.g., atmospheric drag, solar radiation pressure). Unlike compliance-based models that focus on maintaining usability, SF assesses whether orbital conditions are stabilizing, degrading, or reaching critical thresholds, ensuring that sustainability is framed as a system-wide environmental metric rather than just an operational guideline.

This exponential growth in satellite populations poses significant challenges for space sustainability, including collision risk, space debris proliferation, and radio-frequency interference. However, current governance frameworks remain fragmented, reactive, and inconsistently enforced, failing to ensure long-term orbital sustainability [8]. Without enhanced space traffic coordination, regulatory oversight, and active debris mitigation, the rapid expansion of LEO constellations could lead to operational instability and further increase the probability of catastrophic collisional events.

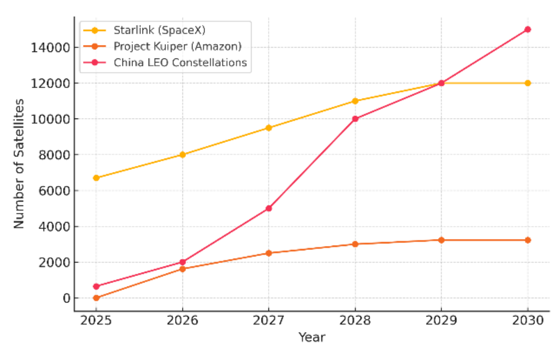

Figure 2: Projected growth of major LEO satellite constellations (2025-2030).

Additionally, significant concerns arise regarding the increasing difficulty of maintaining a stable and sustainable orbital environment due to gaps in technology, environmental factors, and operational protocols. The rapid pace of technological innovation in satellite design and deployment often outstrips the development of standardized operational practices, leading to inconsistent collision avoidance measures and debris mitigation strategies across different operators [9].

The long-term implications of current orbital practices underscore an intergenerational challenge analogous to climate change. Just as greenhouse gas emissions have enduring effects on Earth's climate [10], [11], today's space activities will influence the orbital environment for decades to come. The timeline of orbital degradation and debris accumulation implies that actions taken now will have profound and lasting impacts on future generations, thereby necessitating proactive, sustainable space governance policies that consider both immediate risks and long-term outcomes [12], [13]. This perspective reinforces the urgency for early intervention and coordinated international efforts, much like the global response required to mitigate the impacts of climate change.

Drawing further parallels with terrestrial sustainability issues, lessons learned from climate change mitigation—such as the importance of early intervention and the high economic costs of inaction—can provide valuable insights for space policy. By adopting similar strategies, policymakers can better manage the risks of orbital debris and degradation, ensuring that space remains a viable resource for scientific, commercial, and cultural purposes in the long term.

Given the complex and evolving nature of orbital sustainability, further empirical research is essential. Multidisciplinary approaches that integrate engineering, environmental science, policy studies, and economics are needed to refine current models and develop robust mitigation strategies. International collaboration will be key to addressing these challenges, ensuring that future research and policy initiatives are comprehensive and globally coordinated.

1.2 Limitations of Existing Sustainability Approaches

Several previous frameworks aim to define aspects of orbital sustainability. The 25-Year Rule: Requires satellites in LEO to deorbit within 25 years of mission end. However, studies show that compliance is low, and even adherence to this rule would not prevent long-term debris proliferation [14]. In recognition of these limitations, recent policy discussions have signaled a transition toward a stricter deorbit requirement. The U.S. Federal Communications Commission (FCC) has already introduced a 5-year deorbit rule for non-geostationary satellites, and other international regulatory bodies are considering similar reductions to deorbit timelines to curb the escalating debris problem [15]. However, long-term debris models indicate that reducing deorbit timelines alone has limited effectiveness compared to achieving high post-mission disposal (PMD) adherence (>90%) [16], [17]. The effectiveness of PMD strategies far outweighs the difference between a 25-year and a 5-year deorbit rule when it comes to controlling long-term sustainability risks [18].

The European Space Agency's (ESA) MASTER Model: A widely used tool for characterizing the space debris population and associated risk assessments. While MASTER provides critical insights into debris behavior, it does not quantify sustainability as a single metric and was not originally designed for impact-based sustainability analysis. As a result, it remains a valuable predictive tool rather than a holistic sustainability framework [19].

Carrying Capacity Models: Carrying capacity models attempt to define a maximum sustainable population of satellites per orbit, but they fail to account for the evolving nature of orbital dynamics. Traditional models assume a static threshold beyond which sustainability collapses, neglecting real-world influences such as debris mitigation, satellite decommissioning, and new launches. Unlike static models, which impose an arbitrary population limit, an adaptive model adjusts based on fragmentation rates, reentry dynamics, and evolving mitigation strategies. This perspective underscores the need for sustainability metrics that incorporate real-time feedback mechanisms, rather than relying on fixed, outdated thresholds.

The Space Sustainability Rating (SSR): The SSR, developed by the World Economic Forum in collaboration with ESA and the Massachusetts Institute of Technology (MIT), is a framework designed to rank space missions based on sustainability criteria [9]. The rating assesses a mission's adherence to best practices, such as collision avoidance strategies, debris mitigation efforts, and post-mission disposal compliance. By assigning a score, the SSR aims to encourage satellite operators to adopt more sustainable practices through market-driven incentives rather than strict regulatory enforcement. While the SSR provides an important benchmarking tool, it lacks dynamic adaptability and does not assess cumulative, system-wide sustainability impacts. The rating evaluates individual missions rather than the aggregate impact of all activities within an orbital regime, meaning that a high SSR score for a single mission does not necessarily translate to overall sustainability improvements. Additionally, SSR does not account for real-time orbital evolution, such as the interplay between increasing satellite density and mitigation advancements, which are critical for long-term sustainability assessments. This limitation highlights the need for impact-based metrics like the Space Footprint (SF), which quantify net orbital sustainability beyond mission-level evaluations.

Unlike per-mission sustainability metrics, which assess the environmental and operational performance of individual satellite missions, the Space Footprint (SF) is a system-wide measure of orbital sustainability. While per-mission assessments evaluate compliance with best practices—such as collision avoidance maneuvers, controlled deorbiting, and fuel reserves for end-of-life disposal—they do not measure the overall health, stability, or long-term usability of an orbital region itself.

SF, in contrast, does not rate individual missions but quantifies the cumulative effects of all space activities within an orbital regime. It accounts for the interplay between satellite populations, debris accumulation, mitigation efforts, and environmental influences, providing a dynamic measure of how an orbital region is evolving over time. This distinction is crucial: even if individual missions follow sustainability best practices, the overall orbital environment may still degrade due to external factors such as fragmentation events, historical debris, or non-compliant actors.

That said, per-mission sustainability frameworks are still relevant to broader sustainability efforts. While they do not directly measure orbital sustainability, high compliance with sustainable mission practices (e.g., post-mission disposal, collision avoidance) can contribute to maintaining orbital stability. However, because orbital sustainability is an emergent property of system-wide interactions rather than just the sum of individual missions, a holistic metric like SF is necessary to assess whether an orbit is stabilizing, degrading, or reaching critical thresholds.

Thus, SF is not positioned as a replacement for per-mission metrics but as a complementary measure that provides insight into the long-term trajectory of orbital sustainability beyond compliance-based assessments of individual actors.

2. RESEARCH APPROACH: ITERATIVE REFINEMENT OF SF

This study follows an iterative methodology to ensure the Space Footprint (SF) model is both robust and adaptable. The process begins with the development of a preliminary SF formulation based on representative conditions in Low Earth Orbit (LEO), using current data and established parameters to generate a baseline metric.

The model is then subjected to stress-testing to identify breaking points—conditions under which the formula fails to align with expected sustainability trends. These include cases of exaggerated mitigation effects or underestimation of debris growth. Based on these findings, refinements are introduced, incorporating empirical insights and simulation feedback to improve responsiveness to dynamic orbital conditions.

The refined SF model is then applied across a range of scenario analyses, including baseline evolution, large-constellation growth, enhanced mitigation, regulatory interventions, and worst-case collision cascades.

These simulations assess how SF behaves under both realistic and extreme conditions, offering insights into its policy relevance and predictive utility.

Following this, the model will be extended to Medium Earth Orbit (MEO) and Geostationary Orbit (GEO) to evaluate its generalizability across distinct orbital regimes with different environmental dynamics and population characteristics.

Finally, the SF formulation will be validated against historical case studies and real-world orbital data. This step ensures that the model captures known sustainability outcomes—such as post-event debris growth or mitigation-driven stabilization—ultimately producing the first fully operational version of the SF indicator (v1), ready for future refinement and real-world application.

3. THE STARTING POINT: DERIVING THE BASELINE SPACE FOOTPRINT FORMULA

3.1. In-Depth Analysis of SF Model Parameters

The Population Size (N) parameter represents the total number of space objects—encompassing active satellites, defunct spacecraft, and debris—present in a given orbital regime. A higher value of N directly implies greater orbital congestion and, consequently, a heightened likelihood of collision-induced debris generation [3].

Mathematically, the Population Size (N) parameter can be expressed as:

$$N = \sum_{i=1}^{n} w_i \qquad (1)$$

Where w_i is the weighting factor defined as the fraction of the orbital period that object spends within the considered altitude range. Thus, the total population (N) accounts for elliptical orbits, weighting each object's contribution by the proportion of its orbit spent in the altitude range under consideration.

Additionally, N can be categorized into:

$$N = N_{active} + N_{inactive} + N_{debris(\geq d)} \qquad (2)$$

Where N_{active} is the number of active operational satellites, $N_{inactive}$ is the number of defunct or inactive spacecraft, and $N_{debris(\geq d)}$ is the number of debris fragments equal to or larger than a defined size threshold, typically d=10 cm as per standard debris tracking limits.

A special note on smaller debris: Particles smaller than the tracking threshold 10 cm (e.g., micro-meteoroids or sub-centimeter debris fragments) are not explicitly counted within the N parameter due to current tracking limitations, but their collective impact is implicitly captured within the Environmental Influence parameter (A), as discussed later in this section.

The Environmental Influence (A) parameter quantifies the effect of natural forces—such as atmospheric drag, solar radiation pressure, and gravitational perturbations—on the orbital lifetimes of space objects. Unlike the other parameters, A is not influenced by direct human interventions but instead represents the natural processes that either mitigate or prolong the persistence of spacecraft and debris. The incorporation of A into the Space Footprint (SF) model acknowledges that orbital conditions vary significantly by altitude and region, profoundly affecting sustainability assessments.

Mathematically, the Environmental Influence (A) parameter can be expressed generally as a function dependent on altitude (h), solar activity index (S), and gravitational perturbations (P):

$$A = f(h, S, P, g) \qquad (3)$$

Where, H is Orbital altitude, S is Solar radiation pressure (dependent on solar activity cycle), D is Atmospheric density (dependent on solar and geomagnetic activity), and G is Gravitational perturbations (primarily from Earth, Moon, and Sun) Specifically, the parameter can be represented as:

$$A = f(D, S, G) \qquad (4)$$

In Low Earth Orbit (LEO)**,** atmospheric drag is the dominant natural influence, positively affecting sustainability by accelerating the orbital decay and removal of objects [2]. In mathematical terms, higher atmospheric density (D) increases the decay rate ȧ, directly influencing object lifetimes:

$$\frac{da}{dt} \approx -\frac{1}{2} C_D \frac{A_{cross-sectional}}{m} \rho v^2 \qquad (5)$$

Where C_D is Drag coefficient, $A_{cross\text{-}sectional}$ is Cross-sectional area of the object, m: Mass of the object, ρ is Atmospheric density, and v is Orbital velocity

In Geostationary Orbit (GEO), atmospheric drag is negligible, and gravitational perturbations and solar radiation pressure are dominant, influencing station-keeping requirements and debris persistence [2]. Objects in GEO are typically moved to a disposal orbit at mission end, and while immediate collision risk is minimal, the long-term stability is affected by gravitational perturbations causing orbital drift:

$$\Delta v_{drift} = \int \left(a_{solar\ radiation} + a_{gravitational\ perturbation}\right) \qquad (6)$$

Where $a_{solar\ radiation}$ is acceleration due to solar radiation pressure, $a_{gravitational\ perturbation}$ is acceleration from gravitational perturbations (e.g., luni-solar perturbations)

Small Debris Impacts: Additionally, environmental factors include small debris and micrometeoroids that degrade spacecraft surfaces, optics, and solar panels, shortening spacecraft operational lifetimes. These impacts, although smaller in scale, accumulate over time, contributing negatively to spacecraft performance and operational efficiency, particularly in stable orbits like GEO and MEO [2]. Within the SF model, the environmental parameter A captures these dynamics with nuance. In LEO, atmospheric drag enhances natural debris removal, making A a net mitigating factor. In contrast, GEO lacks significant drag, and while gravitational perturbations and small debris effects introduce challenges, A is effectively neutral or slightly negative in terms of sustainability. Accurately characterizing A through empirical data and environmental modeling is therefore essential. Its proper integration ensures that SF reflects the persistence and decay behavior of orbital objects across regimes. The model's numerical treatment of A is discussed in greater detail in the sections that follow.

The Technology Factor (T) represents the effectiveness of debris mitigation technologies in the Space Footprint model, combining three core components: adoption rate, success rate, and impact factor.

$$T = (Adoption\ Rate) \times (Success\ Rate) \times (Impact\ Factor) \times \frac{Nominal\ Removal\ Time}{Actual\ Removal\ Time} \qquad (7)$$

The adoption rate reflects the proportion of space objects equipped with a given mitigation measure, ranging from 0 to 1. The success rate quantifies the probability that the mitigation technology—such as end-of-life disposal or active debris removal (ADR)—operates as intended. The impact factor captures the relative effectiveness of the technology in reducing collision risks. This factor varies by mitigation type: ADR strategies typically yield values greater than one due to their ability to remove high-risk objects, while post-mission disposal (PMD) practices exhibit non-linear effectiveness. Empirical studies show that PMD adherence only produces significant sustainability gains when compliance exceeds 90%, with limited impact at lower levels.

In addition to performance metrics, the model accounts for temporal efficiency. The effectiveness of a mitigation strategy diminishes significantly if debris removal occurs on long timescales. This is evaluated relative to a nominal removal time, typically 25 years as defined by international guidelines, compared to the actual duration required for mitigation. Rapid removal results in higher T values, while slower strategies reduce overall effectiveness.

A value of T=0 indicates the absence of any mitigation, contributing to a higher SF. Higher values of T signal increasingly effective interventions; however, achieving values above 10 would require near-universal adoption of highly reliable and fast-acting technologies—an aspirational yet currently unattainable benchmark. This non-linear and time-sensitive formulation ensures that T accurately captures both current operational realities and the potential future trajectory of technological advancement, aligning with recent research on debris mitigation and PMD effectiveness.

The Operational Efficiency (O) parameter quantifies the effectiveness of real-time operational measures, such as collision avoidance maneuvers and coordinated space traffic management, in reducing collision risks and improving orbital sustainability. To accurately reflect real-world conditions, O integrates three clearly defined sub-factors:

$$O = (\text{Implementation factor}) \times (\text{Impact factor}) \times (\text{Observation Quality Factor}) \quad (8)$$

Where, Implementation Rate (IR) represents the proportion of space objects within an orbit actively adopting operational measures, defined as the ratio of the number of objects actively implementing operational measures over the total number of tracked objects (including debris), with $0 \leq IR \leq 1$. Impact Factor (IF) quantifies the relative effectiveness of operational measures employed by the spacecraft population, based on empirical or simulated collision avoidance success rates. An impact factor of 1 means collision risk is completely mitigated for objects applying the measure, while an impact factor of 0 means no effective mitigation. Practically, $0 \leq F \leq 1$ in realistic scenarios, with values close to 1 representing very effective operational systems. Observation Quality Factor (OQF) captures the quality, reliability, and availability of observational data required for effective operational strategies. Good observational data enables precise maneuvers, while poor data severely limits effectiveness. Defined on a scale from 0 to 1: OQF = 1 → Perfect observational capability (complete, accurate, and timely orbital data available). OQF = 0 → No observational capability (operational measures ineffective regardless of their sophistication).

Interpretation and Example - Low O scenario: If only one spacecraft out of 1000 has a perfect collision avoidance system (IR = 0.001), even with IF = 1 and OQF = 1, the resulting is O minimal (O = 0.001), reflecting a limited positive impact. High O scenario: If nearly all objects adopt moderately effective collision avoidance (IR = 0.9, IF = 0.7) with excellent observational data (OQF – 0.95), then: O = 0.9 X 0.7 X 0.95 ~ 0.60. This represents a realistic scenario with a strong impact on reducing collision risk. Impact of poor observational data: Even if all objects have sophisticated avoidance systems (IR = 1, IF = 0.9), poor tracking data (OQF = 0.2) significantly reduces effectiveness: O = 1.0 X 0.9 X 0.2 = 0.18 Highlighting that observational data quality is crucial.

This refined formulation of the Operational Efficiency parameter ensures realistic modeling of current and near-term operational capabilities, accurately reflecting practical orbital sustainability conditions and avoiding overly optimistic interpretations.

Together, these four parameters—Population Size (N), Environmental Influence (A), Technology Factor (T), and Operational Efficiency (O) - form the foundation of the Space Footprint (SF) metric. Rather than evaluating orbital sustainability through isolated factors or simplistic thresholds, the SF metric integrates both strain-inducing and mitigating influences into a single, dynamic indicator. By accurately reflecting evolving conditions in orbital environments, technological capabilities, operational practices, and natural environmental influences, the SF model provides a comprehensive tool for policymakers, satellite operators, and researchers aiming to manage and improve long-term orbital sustainability.

The conceptual framework of the Space Footprint (SF) model is built upon a synthesis of factors that contribute to orbital strain and those that mitigate such strain. On one side, the model captures strain through Population Size (N) and Environmental Influence (A). These parameters quantify, respectively, the sheer number of space objects—whose density can precipitate collision cascades—and the natural forces, such as atmospheric drag and solar radiation pressure, that govern object persistence.

Conversely, the mitigating forces in the SF model are represented by the Technology Factor (T) and Operational Efficiency (O). The Technology Factor captures the effectiveness and adoption of mitigation technologies, including active debris removal and enhanced end-of-life strategies. Meanwhile, Operational Efficiency reflects the role of real-time collision avoidance and space traffic management protocols in reducing orbital risks.

This dualistic approach mirrors the structure found in terrestrial sustainability indicators such as the ecological footprint, which balances resource consumption against regenerative capacity [11]. By integrating both strain and mitigation into a single metric, the SF model not only accounts for the escalating challenges of orbital congestion but also dynamically incorporates the benefits of technological and operational interventions.

3.3. The Initial Space Footprint Equation

The Space Footprint (SF) quantifies orbital sustainability by aggregating the cumulative impacts of human space activities and moderating factors within a dynamically updating mathematical framework. Having established the theoretical definitions and preliminary mathematical expressions of the four key parameters—Population Size (N), Environmental Influence (A), Technology Factor (T), and Operational Efficiency (O)—this section formulates the initial SF equation, serving as a foundational point for subsequent refinement and validation. Mathematically, the SF equation is expressed as:

$$SF = \frac{N \times A}{T \times O} \tag{9}$$

Where N represents the total number of space objects.

A encapsulates environmental influences, computed using empirical environmental models:

$$A = \frac{1}{1+D} \textit{ in LEO}, \quad \textit{and } A = 1 + G + S \textit{ in GEO} \tag{10}$$

With D as normalized atmospheric drag coefficient (reflecting orbital decay rates), G is gravitational perturbations factor, and S is solar radiation pressure influence factor.

The A parameter thus inherently adjusts SF values based on orbit-specific environmental dynamics, effectively distinguishing between the natural debris-clearing processes in LEO and the persistent debris environment in GEO.

T, the Technology Factor, represents technological effectiveness in debris mitigation:

$$T = \log_{10}(1 + \alpha \times \sigma \times \mu) \tag{10}$$

Where, α is adoption rate of mitigation technologies (0 to 1), σ is success rate of implemented technologies (0 to 1), μ is relative impact factor, representing the effectiveness and timeliness of debris removal or mitigation techniques (0 to typically around 10, higher values indicating greater effectiveness).

O, the Operational Efficiency parameter quantifies operational and observational effectiveness:

$$O = 1 + (\beta \times \gamma \times \delta) \tag{11}$$

Where β is implementation rate of operational strategies (0 to 1), γ is operational effectiveness factor (reflecting the quality and reliability of operations, typically 0 to 1), and δ is observational data quality factor, reflecting availability and reliability of debris tracking data (typically 0 to 1).

Interpreting the Initial SF Equation: This initial SF equation is structured to reflect both direct human-induced strain on the orbital environment (through parameters N and A) and mitigating factors resulting from human innovation and operational practices (through T and O). By design, a higher numerator (large populations or unfavorable environmental conditions) increases SF, signaling heightened sustainability risks. Conversely, a higher denominator (reflecting advanced technological adoption and efficient operational strategies) reduces the Space Footprint, indicating improved sustainability conditions.

This initial formulation deliberately emphasizes simplicity, providing clarity and facilitating initial scenario analyses. However, it also acknowledges potential complexities and nuances, including non-linear dynamics: the logarithmic and normalization functions ensure diminishing returns, realistically capturing the limitations and incremental benefits of technologies and operations, orbit-specific adaptability with differences between orbital regimes (e.g., LEO vs. GEO) are inherently accounted for through parameter, and realistic constraints with capping and bounding of parameters prevent unrealistic scenarios (e.g., immediate universal technological adoption), ensuring physical plausibility.

The next section will systematically test and validate this initial formulation across multiple orbital environments and scenario analyses, progressively refining it to enhance accuracy, robustness, and predictive reliability.

3.4. Initial Calculation for LEO

To demonstrate the application of the Space Footprint (SF) indicator, we conducted an initial baseline calculation specifically tailored for Low Earth Orbit (LEO), using empirical and observational data representative of current conditions. The following parameter values were derived from established public databases and hypothetical yet realistic modeling assumptions:

Population Size (N): Using approximations from current object population data from ESA's DISCOS, NASA’s Orbital Debris Program Office, and MASTER, we obtained:

$$N = N_{active} + N_{defunct} + N_{debris} \quad (12)$$

$$= 4{,}500 + 5{,}000 + 30{,}000 = 39{,}500$$

Environmental Influence (A): The Environmental Influence factor was derived using a simplified linear model to account for the current solar cycle phase (0.8, representing high solar activity and thus increased atmospheric drag), which accelerates object decay in LEO:

$$A = 1 + solar\ cycle\ phase = 1 + 0.8 = 1.8 \quad (13)$$

Technology Factor (T): Assuming a technological adoption trajectory beginning approximately five years ago (e.g., introduction of improved post-mission disposal technologies in 2020), we calculated T using an exponential adoption rate model. With an estimated adoption rate of approximately 70% after five years, a success rate of 90%, and a standard impact factor of 1.0, we obtained:

$$T_{raw} = adoption\ rate \times success\ rate \times impact\ factor \approx 0.7 \times 0.9 \times 1.0 = 0.63 \quad (14)$$

Applying logarithmic scaling for diminishing returns gives:

$$T = \log(1 + 9 \times T_{raw}) = \log(1 + 9 \times 0.63) \approx 1.797 \quad (15)$$

Operational Efficiency (O): Operational Efficiency was modeled based on the assumed current average implementation rate of operational practices (e.g., collision avoidance maneuvers) at 60% with a baseline impact factor of 1.0, resulting in:

$$O_{raw} = adoption\ rate \times impact\ factor = 0.6 \times 1.0 = 0.6 \quad (16)$$

With similar logarithmic scaling, we obtained:

$$O = \log(1 + 9 \times O_{raw}) = \log(1 + 9 \times 0.6) \approx 1.856 \quad (17)$$

Integrating these values into the Space Footprint equation:

$$SF = \frac{N \times A}{T \times O} = \frac{39{,}500 \times 1.8}{1.797 \times 1.856} \approx 21{,}309.34 \quad (18)$$

This initial baseline LEO calculation establishes a reference point for the current state of orbital sustainability, representing present-day conditions including population size, environmental conditions influenced by solar activity, and realistic assumptions regarding technological and operational mitigation efforts.

The value obtained (SF ≈ 21,309.34) serves as a baseline from which subsequent analyses—including edge cases, breaking scenarios, scenario simulations (e.g., large constellations, mitigation failures, environmental shocks), and comparative assessments across other orbital regions (e.g., MEO, GEO) will be conducted, as detailed in the following sections.

4. STRESS-TESTING THE FORMULA: BREAKING POINTS & REFINEMENTS

The initial Space Footprint (SF) formula provided a structured way to quantify orbital sustainability. However, upon applying it to different scenarios, we identified critical breaking points that required refinement. This section systematically explores these weaknesses and introduces modifications to ensure SF remains robust across diverse orbital conditions.

4.1 Breaking Point #1: When T or O Become Too Large

The initial Space Footprint (SF) equation assumes that higher technology effectiveness (T) and operational efficiency (O) inherently reduce sustainability risks. However, when subjected to hypothetical scenarios where these factors approach their theoretical upper bounds, unintended behaviors arise. For example, if T and O approach idealized conditions (e.g., universal adoption of flawless collision avoidance and debris removal capabilities), the mathematical structure of the SF formula could unrealistically approach zero, suggesting negligible sustainability risks even with significant orbital populations (N).

To explore this, a targeted stress test was conducted using baseline values for the Population Size (N = 39,500) and Environmental Influence (A = 1.8), while artificially setting T and O to idealized extremes (T_raw = 1.0, O_raw = 1.0). Applying the logarithmic scaling used in the SF formulation:

$$T_{scaled} = \log(1 + 9 \times T_{raw}) \quad (19)$$

$$O_{scaled} = \log(1 + 9 \times O_{raw}) \quad (20)$$

resulted in capped values (T ≈ 2.303, O ≈ 2.303). Under these extreme conditions, the SF was recalculated as:

$$SF_{extreme} = \frac{N \times A}{T \times O} = \frac{39{,}500 \times 1.8}{2.303 \times 2.303} \quad (21)$$

$$\approx 13{,}400.65$$

This test highlights a critical insight: even under optimal technological and operational conditions, significant populations (N) can still yield notable SF values. This underscores that perfect operational scenarios are practically unattainable in real-world conditions due to existing uncontrollable objects (e.g., legacy debris and defunct spacecraft).

Furthermore, this finding demonstrates the importance of incorporating logarithmic or other nonlinear scaling for parameters T and O. Without such scaling, the SF model might mistakenly portray a near-zero sustainability risk in highly optimistic scenarios, thereby misleading policymakers or stakeholders into complacency. This refinement ensures that the SF indicator remains physically meaningful, effectively capturing real-world complexities in orbital sustainability management

4.2 Breaking Point #2: Small N, Large SF

The next breaking-point scenario evaluates the behavior of the Space Footprint (SF) formula when the orbital object population (N) becomes extremely small. While the baseline scenario for Low Earth Orbit (LEO) typically involves thousands of objects, certain regions or hypothetical future conditions could yield very small populations. In this scenario, we reduce the population size to a minimal number (e.g., N=5) while holding all other baseline parameters constant to test the sensitivity and robustness of the SF formula.

In this scenario, the object population is reduced to an edge case of N = 5, while all other parameters—A = 1.8, T = 1.797, and O = 1.856—remain at baseline levels. This setup tests the sensitivity of the SF model under minimal congestion conditions. The resulting Space Footprint calculation yields:

$$SF_{small\ N} = \frac{N \times A}{T \times O} = \frac{5 \times 1.8}{1.797 \times 1.856} \quad (22)$$

$$\approx 2.6938$$

The SF value of approximately 2.69 aligns with expectations for a near-empty orbital regime, reflecting minimal congestion and negligible collision risk. Importantly, the formula handles this small-population edge case without distortion, scaling naturally with decreasing N. This confirms that no additional adjustments or normalization are required—the baseline SF equation remains stable and accurate even under minimal population conditions

4.3 Breaking Point #3: A = 0 (No Environmental Influence)

This scenario explores the theoretical edge case where environmental influence is absent (A = 0). While physically implausible—since factors like atmospheric drag or radiation pressure are always present—it provides a useful boundary test for assessing the SF model's mathematical stability. To isolate the effect of A, all other parameters are held constant at their baseline LEO values: N = 39,500, T = 1.797, and O = 1.856. The SF calculation for this extreme condition is:

$$SF_{A=0} = \frac{N \times A}{T \times O} = \frac{39{,}500 \times 0}{1.797 \times 1.856} = 0 \qquad (23)$$

Setting A=0 yields an SF value of zero, a mathematically stable and logically consistent outcome. While not physically realistic—since some level of atmospheric drag or radiation pressure always exists—the scenario confirms that the SF model gracefully handles theoretical boundary conditions without error or undefined behavior.

Importantly, this result does not suggest the absence of sustainability concerns. Rather, it underscores the critical role of environmental forces in shaping orbital dynamics. Without them, the SF model loses its interpretive value in assessing real-world sustainability. Thus, while no structural adjustments to the formula are required, this test reinforces that environmental influence is a non-negotiable component of any realistic orbital sustainability assessment.

4.4. Scenario Analysis: Evaluating the Space Footprint under Varied Conditions

To fully assess the robustness and dynamic behavior of the Space Footprint (SF) model, we conduct a series of scenario analyses. These scenarios are designed to evaluate the SF under a range of conditions—from current operational norms to extreme events—thus offering insights into the model's sensitivity and its practical implications for orbital sustainability.

4.4.1. Baseline Scenario

The Baseline Scenario models current conditions in Low Earth Orbit (LEO), using empirically grounded inputs to reflect the present state of orbital sustainability. The population size parameter N is set to 39,500, representing the total number of tracked objects—including active satellites, defunct spacecraft, and debris larger than 10 centimeters. Environmental influence A is assigned to a value of 1.8, corresponding to typical levels of atmospheric drag under moderate solar activity. The technology factor T is 1.797, reflecting moderate adoption of debris mitigation strategies such as post-mission disposal, while the operational efficiency O is 1.856, indicative of current collision avoidance practices and emerging traffic coordination measures. Together, these inputs yield a calculated Space Footprint (SF) that serves as a reference point for assessing sustainability trends in subsequent scenarios. Applying these inputs, the baseline calculation of the Space Footprint (SF) yields:

$$SF_{LEO} = \frac{N \times A}{T \times O} = \frac{39{,}500 \times 1.8}{1.797 \times 1.856} \approx 21{,}309.34 \qquad (24)$$

This baseline SF value provides a quantitative reference, reflecting contemporary orbital strain balanced by current mitigation efforts. As a comparative benchmark, it is instrumental for understanding deviations in sustainability under altered future scenarios.

These results were obtained using a structured MATLAB implementation, which incorporates empirically derived values and documented assumptions, establishing reproducibility and traceability for subsequent scenario comparisons.

4.4.2 Large-Constellation Scenario

This scenario evaluates the sustainability implications of large-scale satellite deployments such as those planned by emerging mega-constellation initiatives, including SpaceX's Starlink and OneWeb. These systems aim to place tens of thousands of satellites into Low Earth Orbit (LEO), dramatically increasing the orbital population. To model this growth, we assume an exponential increase in the total number of objects, with the population size N rising at an annual rate of 8%, from a baseline of 39,500 in 2025 to over 184,000 by 2045—mirroring current projections for commercial constellation growth (SpaceX, 2022). The environmental influence A is held constant at 1.8, reflecting stable atmospheric drag conditions throughout the period. Two mitigation trajectories are considered. In the first, mitigation efforts fail to keep pace with deployment, with both the technology factor T and operational efficiency O fixed at half their baseline values, representing underinvestment in sustainability measures. In the second, an enhanced mitigation scenario assumes gradual improvement in both T and O, reaching their logarithmic saturation levels (approximately 2.30) by 2045, simulating the adoption of advanced technologies and stricter operational standards. Together, these assumptions allow us to assess how different mitigation pathways interact with exponential population growth to influence the long-term sustainability of LEO.

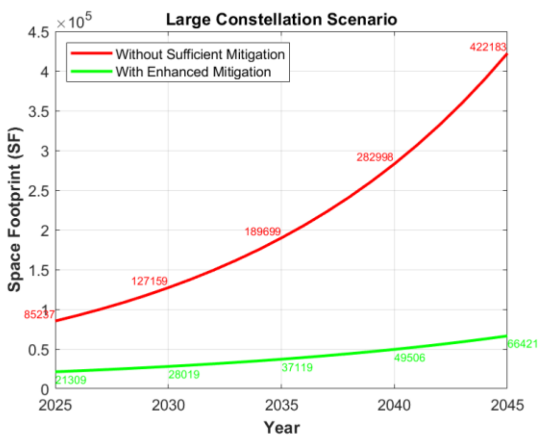

Figure 3: Projected Space Footprint (SF) growth under different mitigation conditions.

Figure 3 illustrates the projected evolution of the Space Footprint (SF) from 2025 to 2045 under two contrasting mitigation scenarios. In the absence of sufficient mitigation, represented by the red curve, SF grows exponentially—from approximately 85,000 in 2025 to over 400,000 by 2045, reflecting a nearly fivefold increase. This steep escalation underscores the unsustainable trajectory resulting from poor management of rapidly expanding satellite populations. In contrast, the green curve models enhanced mitigation, where steady improvements in technology adoption and operational coordination significantly slow SF growth. Under these conditions, SF increases more moderately, from about 21,300 in 2025 to roughly 49,500 by 2045, demonstrating that effective mitigation can maintain sustainability within manageable bounds.

4.4.3. Enhanced Mitigation Scenario

This scenario examines the impact of improved mitigation strategies—specifically technological advancements (T) and enhanced operational efficiency (O)—on the Space Footprint (SF) from 2025 to 2045. Figure 4 presents the comparative impacts of independently improving either T or O, and their combined effect. This scenario builds upon the baseline SF established earlier and models a forward-looking trajectory where mitigation efforts improve over time. It assumes a rapid enhancement in the Technology factor (T), driven by aggressive advancements in active debris removal, reliable end-of-life disposal, and improved satellite design. In parallel, Operational Efficiency (O) improves more gradually, reflecting realistic progress in coordinated collision avoidance and space traffic management systems.

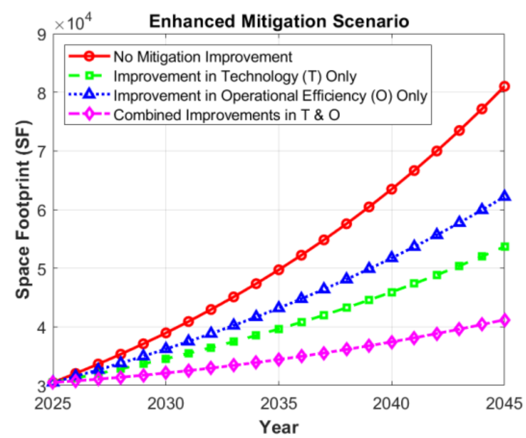

Figure 4: Impact of Enhanced Mitigation Strategies on Space Footprint (SF) Growth (2025-2045)

Figure 4 presents the impact of different mitigation trajectories on SF growth. In the absence of any mitigation improvement, the red curve shows SF increasing exponentially, driven solely by the rising satellite population. When only technological advancements are implemented (green), SF growth slows significantly, highlighting the crucial role of innovation in debris mitigation and sustainable satellite design. Improvements in operational efficiency alone (blue) have a more modest effect, suggesting that while beneficial, they are insufficient on their own to counteract rapid congestion. The most favorable outcome appears in the combined scenario (magenta), where coordinated advances in both technology and operations lead to a substantial reduction in SF growth, demonstrating the synergistic benefits of integrated mitigation strategies.

4.4.4. Worst-Case Scenario: Collision Cascade

The Worst-Case Scenario models the long-term onset of a collision cascade, representing the gradual realization of the Kessler Syndrome—a self-sustaining cycle in which satellite breakups lead to growing debris density and further collisions. Rather than an immediate collapse, this scenario unfolds over decades or centuries, during which mitigation efforts become increasingly ineffective. The object population N is assumed to grow exponentially as each collision generates more debris. Environmental factor A is held high at 1.9, reflecting the persistence of debris in the absence of natural removal mechanisms. Meanwhile, both the Technology factor T and Operational Efficiency O degrade over time, constrained by policy inertia, financial limitations, and technological barriers that prevent a timely or adequate response.

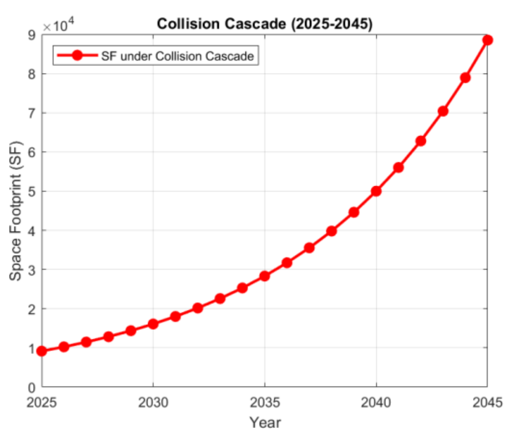

Figure 5: Projected Space Footprint (SF) under a Collision Cascade Scenario (2025-2045).

Unlike a sudden catastrophic collapse, this scenario evolves gradually over decades, allowing mitigation efforts to respond dynamically. However, as with climate change—where some damage is already inevitable—certain orbital regions may already be on an irreversible path toward congestion due to accumulated fragmentation and delayed mitigation. Although the technology (T) and operational efficiency (O) parameters remain functional, their effectiveness diminishes over time under mounting financial, technological, and policy constraints. This raises a critical question: how long can mitigation realistically delay systemic degradation? Our simulation suggests that once debris density crosses a certain threshold, even continuous mitigation efforts fail to arrest the decline in orbital sustainability.

As shown in Figure 5, the SF value increases exponentially with rising collision frequency, signaling a worsening sustainability trajectory. Though mitigation continues, its declining influence cannot offset the runaway debris growth. This highlights the urgent need for early and aggressive intervention. The instability modeled here is not in the SF metric itself but in the orbital environment, which becomes progressively less viable for safe operations. In this context, SF functions not as a rigid threshold, but as an early-warning indicator—allowing stakeholders to identify sustainability risks before they become irreversible

The stress-testing and scenario analysis presented in this section affirm the robustness and flexibility of the Space Footprint (SF) model in capturing a wide range of orbital sustainability conditions. Controlled breaking point experiments validated the model's ability to handle extreme parameter variations, revealing both its resilience and the need for certain refinements—such as capping T and O to prevent unrealistically low SF values. Scenarios involving unchecked population growth, such as large-constellation deployments, produced steep rises in SF, highlighting the necessity of proactive mitigation. Conversely, high mitigation effectiveness significantly stabilized SF but depended on broad industry adoption and regulatory enforcement. The theoretical case of zero environmental influence further underscored the indispensable role of natural orbital forces in maintaining sustainability. Collectively, these analyses demonstrate the model's sensitivity to technological, operational, and environmental inputs. Future research should aim to validate these findings through historical case studies and real-world datasets to enhance the model's predictive precision.

5. CONCLUSIONS AND NEXT STEPS

The Space Footprint (SF) model has been developed as a structured framework to quantify orbital sustainability by integrating object population (N), environmental influences (A), mitigation technologies (T), and operational efficiencies (O). Through rigorous formulation, stress-testing, and scenario analysis, the model has demonstrated its ability to capture the dynamic nature of orbital sustainability and provide meaningful insights into the effectiveness of various mitigation strategies.

The findings presented in this study validate the robustness of the SF equation across a range of breaking points and stress scenarios. The model successfully accounts for both current and projected space activity, highlighting the implications of large constellations, regulatory interventions, and catastrophic collision cascades. The scenario analyses reveal that while mitigation measures can effectively counteract population growth, there exist thresholds beyond which sustainability becomes increasingly difficult to maintain. In particular, the Large-Constellation Scenario underscores the urgency of scaling mitigation efforts in tandem with satellite deployments, while the Collision Cascade Scenario emphasizes the long-term risks of insufficient early intervention. Meanwhile, the Regulatory Intervention Scenario provides a clear demonstration of how policy enforcement can significantly curb sustainability risks, reinforcing the role of global governance in space traffic management.

Despite its strengths, the SF model is not without limitations. Some key factors, such as the contributions of sub-centimeter debris, the cumulative effects of long-term environmental changes, and the precise scaling of mitigation technologies, require further empirical validation. Additionally, while the model's stress tests have shown resilience under extreme parameter conditions, real-world data calibration is necessary to refine its predictive accuracy.

Moving forward, the next major step is empirical validation. Historical orbital events—such as the 2009 Iridium-Cosmos collision, the 2007 Chinese ASAT test, and past large-constellation decay trends—offer opportunities to compare SF projections against observed sustainability shifts. This validation effort will improve the parameterization of N, A, T, and O, particularly in refining the real-world scaling behavior of mitigation measures.

Another crucial area for future work involves integrating environmental shock events into the SF framework. While this study has qualitatively examined the influence of solar cycles, atmospheric drag fluctuations, and extreme space weather events, further research is needed to quantify their impact on debris persistence and overall orbital sustainability. Developing a probabilistic model that accounts for solar maximums, solar storms, and long-term atmospheric variability would significantly enhance the SF model's ability to assess sustainability under dynamic environmental conditions.

Beyond these refinements, the SF model has the potential to serve as a real-time sustainability metric for space traffic management. By integrating live space object tracking data from sources like DISCOS, Space-Track, and commercial SSA services, the SF model could provide real-time sustainability assessments for different orbital regions. Additionally, Monte Carlo simulations could be employed to analyze parameter uncertainties, assess probabilistic collision risks, and refine long-term SF projections. These advancements would elevate the SF model from a theoretical framework to a practical decision-support tool for policymakers, satellite operators, and international space agencies.

In conclusion, the Space Footprint model presents a rigorous and adaptable approach to measuring orbital sustainability in an era of rapid space expansion. While this study has laid a strong theoretical foundation, empirical validation, scenario diversification, and real-time implementation represent the next frontiers in SF development. By addressing these next steps, the SF model could become an integral component of global space governance, providing a quantifiable and scalable approach to sustaining Earth's orbital environment for future generations.

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