COMPARISION BETWEEN MULTIPLE BARE TETHERS AND SINGLE BARE TETHER FOR DEORBITING SATELLITES

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ABSTRACT

As per the Inter-agencies Space Debris Coordination Committee (IADC) guidelines, the satellites in Low Earth Orbits (LEO) must have an orbital lifetime of fewer than 25 years. Tethers used in previous missions have been subjected to damage due to their extremely large lengths. Hence a multi-tether design with each tether shorter in length is proposed which could thus reduce the probability of damage to tethers. This paper compares the working of multiple bare short electrodynamic tethers versus a single long bare tethered system on a satellite in Low Earth Orbit (LEO) for end-of-life de-orbiting to combat the further formation of space debris. The paper shows the use of several shorter tethers, each equally long, placed in parallel orientation on one face of the satellite. It also upholds the fact that the same drag force can be produced using a multiple tether system like that of a single long tether. A similar comparison considering the de-orbiting time of both the systems is presented along with the effects of initial induced voltage and current. Optimum spacing distances between the multiple short tethers to obtain the maximum efficiency from each are also discussed in the paper.

Keywords: Multiple tether; Passive de-orbiting mechanism; End of life satellites; Low Earth Orbit; Lorentz force; Induced EMF; De-orbiting time; Orbit Motion Limited theory; Spacing; Debye Sheath; Sheath thickness; Quasi neutral plasma.

1. INTRODUCTION

Orbital debris is one of the major issues in the field of space technology. It comprises uncontrollable objects in space. Any body or particle that is uncontrollable or has been left defunct constitutes space debris. This includes paints, gas, tools, as well as destroyed rockets and satellites. They not only pose a major threat to the existing assets in space but also have a chance of affecting future missions too. According to the National Space Society (NSS), there are about 22,000 Earth-orbiting debris pieces that are larger than 10cm size, around 7,00,000 fragments between 1cm and 10cm range, and the number of tiny bits that are smaller than 1cm exceeds 100 million [1].

Orbital debris persists in space due to the lack of onboard de-orbiting mechanisms that could dispose of the satellite at the end of the mission. These abandoned satellites have no control over their trajectories and hence can lead to collisions resulting in the generation of more debris. Inter-Agency Space Debris Coordination Committee (IADC) has made guidelines that any object put into Low Earth Orbit (LEO) should not have an orbital lifetime for more than 25 years post the end of its useful life [2]. Hence several institutions and organizations have proposed and tested different de-orbiting mechanisms [3].

De-orbiting mechanisms can be of two types, active and passive. Passive de-orbiting mechanisms do not require a power supply from the spacecraft, need no monitoring, and also can perform re-entry more effectively. Among

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passive de-orbiting mechanisms, electrodynamic tethers (EDT) are found to be one of the most effective systems. This study focuses on the physics behind electrodynamic tethers while addressing a possible solution for optimization of its length, which is a major issue in the case of using EDTs. The length of single tethers is proposed to be reduced by replacing it with a set of multiple short tethers, without affecting its intended performance.

2. METHODOLOGIES AND THEORY

2.1. Premise

For this study, the magnetic field of the Earth is considered to be constant along the orbit and altitude of consideration. The variation of solar irradiance is ignored as the satellite is within the bounds of the Van Allen belt. The orbits are considered to be equatorial and circular in LEO. The oblateness of the Earth is ignored. The ambient plasma condition in the ionosphere is considered to be quasi-neutral. Only prograde satellites are being considered. The study considers sample microsatellites of mass of 10 and 100kg. If found effective, this mechanism of multiple tethers can be implemented for various classes of satellites. The study looks into the performance of the tether system in orbit and does not consider the structural properties of deployment of the system.
For all equations and expressions taken into consideration for the purpose of this paper, a lack of boldface font denotes the magnitude of the quantity alone.

2.2. Basics of Electrodynamic Tether

EDT works on the principle of the electromotive force produced due to the motion of the induced current-carrying tether in the earth's magnetic field. The motion of the satellite is from west to east i.e., eastward. The tether is deployed from the satellite in the direction radially pointing towards the Earth. When a conductor is moved in a magnetic field with some relative velocity such that it cuts the field lines, voltage is induced in the conductor as a consequence. The induced voltage in the tether (ψ) can be obtained from the cross product of the velocity of the tether (v) and the magnetic field (B) where the direction will be radially away from the Earth as

$$\psi = \int_0^L (\mathbf{v} \times \mathbf{B}).dl = vBL\sin\theta$$

where L is the length of the tether and θ is the angle between the direction of velocity and the direction of the magnetic field. Hence the current (I) induced due to the induced voltage (ψ) will also be in the direction away from the Earth and is given by

$$I = \frac{\psi}{R_{\text{net}}} = \frac{vBL}{R_{\text{net}}},$$

where R_{net} is the resistance of the tether.
The Lorentz force (F_{t}) produced by the induced current (I) is given by the below expression where its direction will be opposite to that of the velocity of the satellite and thus help in reducing the orbit altitude gradually.

$$F_t = \int_0^L (\mathbf{I} \times \mathbf{B}).dl = BIL\sin\alpha,$$

where α is the angle between the direction of current and magnetic field.
In this method, the EDT can be used to de-orbit a satellite from orbit. The induced voltage produces a current in the tethers, a complete path of current is formed in the ambient plasma with help of contactors, which act as electrons flow medium between the tethers and the ambient plasma. This current in the tether is responsible for producing the required force, the drag force, for de-orbiting.

2.3. Debye Length and Orbital Motion Limited Theory

Debye length is the measure of the net electrostatic effect of charge carriers in a solution and how far its electrostatic effect persists. Debye length is denoted by λ_{d}, it can be defined as the decrease in the magnitude of the electric field by a factor of $\frac{1}{e}$, where e is the charge of the electron.
According to Orbital Motion Limited (OML) theory [4], the orbital-motion-limit regime is attained when the cylinder radius is small enough such that all incoming particle trajectories that are collected are terminated on the cylinder's surface while being connected to the background plasma. In an electrodynamic tether system and for a given mass of tether, the best performance is achieved for a tether diameter chosen to be smaller than 1 electron Debye length, for typical ionospheric ambient conditions from 200 to 2000 km altitude range [5].

2.4. Effect of Quasi Neutral Plasma on Conductors

Consider a region with plasma that has an equal number of electrons and ions. Let a conducting metal wire be placed inside the plasma. It is observed that a sheath gets formed around this conducting surface. The sheath arises because of the following reasons. The electrons usually have a temperature of order equal to or greater than that of the ions, electrons being comparatively lighter than the surrounding ions. Speed of electrons is greater than speed of ions by a factor of $\left[\sqrt{\frac{m_{ion}}{m_e}}\right]$, where m_{ion} is the mass of ion and m_e is the mass of electron.
As the plasma is considered to be electrically neutral, a sample assumption can illustrate that if being neutral, the number of electrons and the number of ions in the plasma could be, say 100. Due to the high speed of the electrons for every 1 ion collision with the surface of the conductor, there would have been 100 electrons that would have collided with the surface. Because of the movement of

electrons and ions, the surface of the tether gets negatively charged. Owing to the electrostatic force of attraction, positive ions get accumulated around the metal surface to balance the negative charge on the surface. This layer of electrically charged particles is called as the Debye sheath.

2.5. Debye Sheath

The Debye sheath is a layer in plasma that has a greater density of positive ions, and hence an overall excess positive charge, that balances out an opposite negative charge on the surface of a material with which it is in contact. The thickness of this layer can be several Debye lengths, a value whose size depends on various characteristics of plasma, like its temperature, density, et cetera [6]. The formula giving the sheath thickness in terms of Debye length (λ_d) and thickness of Child Langmuir sheath(ϵ_d) is as follows:

$$\epsilon_d = \frac{d}{\lambda_d} = \left(\frac{1}{c_1} \ln \left[\sqrt{\frac{m_{\text{ion}}}{2\pi m_{\text{e}}}} \right] \right)^{\frac{3}{4}} \tag{1}$$

where d is the sheath thickness and c_1 is a constant related to density of ions at sheath edge.

3. DESIGN AND ANALYSIS

3.1. Force for Constant Current

Consider a tether of length L and let voltage induced by this tether be

$$\psi = \int_0^L (\mathbf{v} \times \mathbf{B}).dl,$$

where v is the velocity of the satellite and B is the magnetic field of earth. By integrating, the following equation is obtained,

$$\psi = vBL,$$

where L is the length of a single tether. Now consider a short tether of length $\frac{L}{N}$, keeping the material and the area same.
The induced voltage in this case, ψ' , is given by

$$\psi' = \int_0^{L/N} (\mathbf{v} \times \mathbf{B}).dl,$$

$$\psi' = \frac{vBL}{N},$$

$$\psi' = \frac{\psi}{N}. \tag{2}$$

So the length of the tether affects the voltage induced by the tether.

Force by a Long Tether

The force acting on the tether at time t is

$$F_{\text{t}} = \int_0^L (\mathbf{I} \times \mathbf{B}).dl,$$

which, when integrated gives,

$$F_{\text{t}} = BILsin(\theta),$$

where θ is the angle between direction of current I and magnetic field B. The tether is placed perpendicular to the direction of magnetic field of Earth, therefore $\theta = 90°$ and voltage is given by

$$\psi = \text{I}R_{\text{net}},$$

where R_{net} is the net resistance of the tether.

$$I = \frac{\psi}{R_{\text{net}}},$$

and

$$R_{\text{net}} = \frac{\rho L}{A},$$

where ρ is the resistivity of the tether and A is the cross sectional area of tether.
Therefore force produced by the tether is

$$F_{\text{t}} = \frac{B\psi A}{\rho}. \tag{3}$$

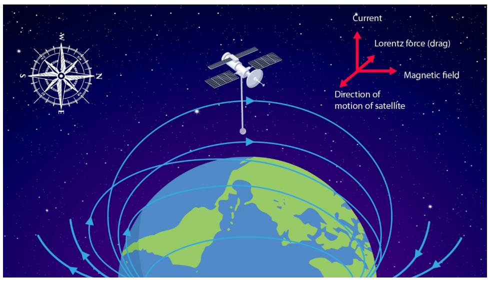

Figure 1. Force on a single long tether.

Force by a Short Tether

Since the tethers are placed parallel to each other and the force is in the same direction, the force F_i for a shorter tether of length $\frac{L}{N}$ is,

$$F_{\text{i}} = \int_0^{L/N} (\mathbf{I} \times \mathbf{B}).dl,$$

where **I** is current and **B** is magnetic field.
Now by integrating,

$$F_{\text{i}} = \frac{B\psi' A}{N\rho} = \frac{F_{\text{t}}}{N} \tag{4}$$

As the induced voltage reduces due to reduce in length, the force also reduces.
But when there are N such short tethers, the force is same as that due to single long tether. Therefore, the total force produced by a single tether is equal to the force produced by multiple tethers whose lengths sums up to the length of the single tether.

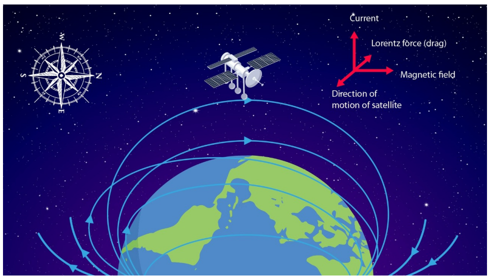

Figure 2. Force on multiple short tethers

3.2. Force due to Changing Current

In most passive methods, the current changes throughout the process of de-orbiting, as a result, the force would too. The force produced is modelled as follows.

Force on a Single Tether due to Changing Current

By using the principle of the Lorentz force, all the quantities B, I, and L lying perpendicular to each other, their respective magnitudes can be related by

$$F_{\text{t}} = BIL,$$

$$\frac{dF_{\text{t}}}{dt} = BL\frac{dI}{dt}, \tag{5}$$

$$\frac{dI}{dt} = \frac{d\psi}{dt}\frac{1}{R_{\text{net}}}. \tag{6}$$

By Faraday's law of electromagnetic induction, the induced voltage is given by,

$$\frac{d\psi}{dt} = LB\frac{dv}{dt}. \tag{7}$$

By Newton's second law of motion,

$$\frac{dv}{dt} = -\frac{F_{\text{t}}}{m_{\text{tot}}}. \tag{8}$$

From Equations 5, 6, 7 and 8

$$\frac{dF_{\text{t}}}{dt} = -\frac{B^2L^2F_{\text{t}}}{m_{\text{tot}}R_{\text{net}}}.$$

Transposing the terms

$$\frac{dF_{\text{t}}}{F_{\text{t}}} = -\frac{B^2L^2dt}{m_{\text{tot}}R_{\text{net}}}.$$

Integrating on both sides from 0 to t and force F_{t} at time t taking the value F_0 in the initial condition limit,

$$\ln\frac{F_{\text{t}}}{F_0} = -\frac{B^2Lt}{m_{\text{tot}}R_{\text{net}}},$$

$$\frac{F_{\text{t}}}{F_0} = e^{\frac{-B^2L^2t}{m_{tot}R_{net}}}. \tag{9}$$

At $t = t_0$, $F_{\text{t}} = F_0$, $I = I_0$, $\phi = \phi_0$ and $v = v_0$ hence,

$$F_0 = BI_0L,$$

$$I_0 = \frac{\psi_0}{R_{\text{net}}},$$

$$I_0 = \frac{v_0BL}{R_{\text{net}}},$$

$$v_0 = \sqrt{\frac{GM}{r_0}}.$$

Therefore,

$$I_0 = \frac{BL}{R_{\text{net}}}\sqrt{\frac{GM}{r_0}}.$$

Hence Equation 9 becomes,

$$F_{\text{t}} = \frac{B^2L^2}{R_{\text{net}}}\sqrt{\frac{GM}{r_0}}\left[e^{\frac{-B^2L^2t}{m_{tot}R_{net}}}\right]. \tag{10}$$

This is the force produced by a single tether of length N due to changing current.

Force on Multiple Tethers due to Changing Current

By Lorentz force,

$$F_{\text{i}} = \frac{BI_{\text{i}}L}{N},$$

where F_{i} is the Lorentz force and I_{i} is the current in a single short tether.

$$\frac{dF_{\text{i}}}{dt} = B\frac{L}{N}\frac{dI_{\text{i}}}{dt}, \tag{11}$$

$$\frac{dI_{\text{i}}}{dt} = \frac{d\psi_{\text{i}}}{dt}\frac{N}{R_{\text{net}}}, \tag{12}$$

$$\frac{d\psi_{\text{i}}}{dt} = \frac{BL}{N}\frac{dv_{\text{N}}}{dt}, \tag{13}$$

$$\frac{dv_{\text{N}}}{dt} = -\frac{F_{\text{tm}}}{m_{\text{tot}}} = -\frac{NF_{\text{i}}}{m_{\text{tot}}}, \tag{14}$$

where F_{tm} is the force by multiple tethers.
From Equation 11,12,13 and 14

$$\frac{dF_{\text{i}}}{dt} = -\frac{B^2L^2F_{\text{i}}}{m_{\text{tot}}R_{\text{net}}}. \tag{15}$$

By transposing the terms,

$$\frac{dF_{\text{i}}}{F_{\text{i}}} = -kdt$$

Where

$$k = \frac{B^2 L^2}{m_{\text{tot}} R_{\text{net}}}.$$

Integrating both sides from F_{i0} to F_{i} and 0 to t

$$\ln \frac{F_{\text{i}}}{F_{i0}} = -kt,$$

$$F_{\text{i}} = F_{i0} e^{-kt},$$

$$F_{i0} = \frac{BLI_{i0}}{N},$$

$$I_{i0} = \frac{v_0 B \frac{L}{N}}{\frac{R_{\text{net}}}{N}},$$

$$F_{i0} = \frac{B^2 L^2 v_0}{N R_{\text{net}}} = \frac{F_0}{N}.$$

Therefore,

$$F_{\text{tm}} = \frac{F_0 e^{-kt}}{N} N.$$

But there are N such tethers and their individual forces act on the same satellite in the same direction. Therefore,

$$F_{\text{tm}} = F_{\text{t}}. \tag{16}$$

Thus all calculations henceforth will have the bearing for both single tethered systems as well as multiple tethered systems, as the de-orbiting time by a single tether and multiple tethers are the same.

3.3. Distance between Adjacent Tethers

Consider a condition where two conductors are placed in the plasma in proximity of each other, a positive ion sheath is formed over the conductors. When the spacing between the two conductors is less than the minimum required spacing, the net positive ions around each conductor reduces as there will be interference with the sheath of adjacent conductors. This results in the distribution of positive charges between the interfering sheaths. The reduction of positive ions in the sheath will in return reduce the number of electrons on the sheath-contact surface of the conductor resulting in the reduction of the current in the individual conductor. Hence, if each of the tethers is placed outside the sheath radius of its adjacent tether, there will not be any reduction of induced current. Maintaining a minimum spacing of twice the sheath radius of the individual tether will prevent any loss of performance, as there will be no interference between the sheath of adjacent tethers. The table 1 is one such example.
Here oxygen ions have been taken where its mass is

$$m_{\text{ion}} = 16.022 \times 10^{23},$$

The constant

$$C_1 = 1.36.$$

Thus from Equation 1,

$$d = 3.39\lambda_{\text{d}}.$$

The debye lengths for different altitudes as calculated by Wijnans, A.M in the paper " Bare Electrodynamic Tape Tether Experiment onboard the Delfi-1 University Satellite" [7] were used to calculate the sheath thickness and spacing distance for tethers at different altitudes as shown in Table 1.

Table 1. Table relating Debye length, sheath radius and spacing distance with different altitudes

Altitude (km)	λ_{d} (mm)	Sheath thickness d(mm)	Spacing distance(mm)
650	7.25	24.57	49.14
700	8.25	27.967	55.93
750	8.5	28.815	57.63
800	8.5	28.815	57.63
850	9.75	33.052	66.104
900	12.25	41.527	83.054
950	13.75	46.612	93.224
1000	16	54.24	108.48

3.4. De-orbiting Time due to Changing Current

By Lorentz force,

$$\frac{dF_{\text{t}}}{dt} = BL\frac{dI}{dt}, \tag{17}$$

$$\frac{dI}{dt} = \frac{d\psi}{dt}\frac{1}{R_{\text{net}}}. \tag{18}$$

By Faraday's law of electromagnetic induction, induced voltage is,

$$\frac{d\psi}{dt} = LB\frac{dv}{dt}. \tag{19}$$

By Newton's second law of motion,

$$\frac{dv}{dt} = -\frac{F_{\text{t}}}{m_{\text{tot}}}. \tag{20}$$

From 17, 18, 19 and 20

$$\frac{dF_{\text{t}}}{dt} = -\frac{B^2 L^2 F_t}{m_{\text{tot}} R_{\text{net}}}.$$

By transposing terms,

$$\frac{dF_{\text{t}}}{F_{\text{t}}} = -\frac{B^2 L^2 dt}{m_{\text{tot}} R_{\text{net}}}.$$

Integrating on both sides from $t_0 = 0$ to t and taking force $F_{\text{t}} = F_0$ when $t = t_0$, we obtain

$$\ln F_{\text{t}} - \ln F_0 = -\frac{B^2 L^2 (t - t_0)}{m_{\text{tot}} R_{\text{net}}},$$

$$F_{\text{t}} = F_0 e^{-kt}$$

where the constant $k = \frac{B^2 L^2}{m_{tot} R_{net}}$.

$$F_0 = BLI_0,$$

$$I_0 = \frac{\psi_0}{R_{\text{net}}},$$

$$\psi_0 = v_0 BL,$$

$$F_0 = \frac{B^2 L^2 v_0}{R_{\text{net}}},$$

$$v_{\text{t}} = v_0 + \frac{F_0 e^{-kt}}{k m_{\text{tot}}} - \frac{F_0}{k m_{\text{tot}}}.$$

As $F_0 = k v_0 m_{\text{tot}}$,

$$r_{\text{t}} = r_0 - \frac{F_{\text{t}}}{k^2 m_{\text{tot}}},$$

the rate of work being done due to the Electromagnetic tether system E_{mag}

$$\frac{dE_{\text{mag}}}{dt} = F_{\text{t}} v_{\text{t}} + \frac{dF_{\text{t}}}{dt} \int v_{\text{t}} dt.$$

$$Power = \frac{F_{\text{t}}^2}{k m_{\text{tot}}} - k F_{\text{t}} r_0 + \frac{F_{\text{t}}^2}{k m_{\text{tot}}}.$$

Integrating on both sides from $t_0 = 0$ to t, so F from F_0 to F_{t} and r from r_0 to r_{t}

$$W_{\text{t}} = -F_{\text{t}} r_0 - \frac{F_{\text{t}}^2}{k^2 m_{\text{tot}}} + F_0 r_0 + \frac{F_0^2}{k^2 m_{\text{tot}}},$$

$$W_{\text{t}} = F_0 r_0 (1 - e^{-kt}) + \frac{F_0^2}{k^2 m_{\text{tot}}} (1 - e^{-2kt}).$$

By substituting $F_0 = k v_0 m_{\text{tot}}$,

$$W_{\text{t}} = k v_0 m_{\text{tot}} r_0 (1 - e^{-kt}) + \frac{k^2 v_0^2 m_{\text{tot}}^2}{k^2 m_{\text{tot}}} (1 - e^{-2kt}),$$

$$W_{\text{t}} = k v_0 m_{\text{tot}} r_0 (1 - e^{-kt}) + \frac{k^2 v_0^2 m_{\text{tot}}^2}{k^2 m_{\text{tot}}} (1 - e^{-2kt}),$$

$$W_{\text{t}} = v_0^2 m_{\text{tot}} [\frac{k r_0}{v_0} (1 - e - kt) + (1 - e^{-2kt})]. \quad (21)$$

We know that the work done gravitationally can be found as

$$W_{\text{grav}} = (\frac{GM m_{\text{tot}}}{r_{\text{t}}^2} - \frac{m_{\text{tot}} v_t^2}{r_{\text{t}}}) r_{\text{t}} - (\frac{GM m_{\text{tot}}}{r_0^2} + \frac{m_{\text{tot}} v_0^2}{r_0}) r_0,$$

$$W_{\text{grav}} = \frac{GM m_{\text{tot}}}{r_{\text{t}}} - m_{\text{tot}} v_{\text{t}}^2 - \frac{GM m_{\text{tot}}}{r_0} + m_{\text{tot}} v_0^2.$$

And as the initial orbital velocity at the orbit of radius from the centre of the Earth r_0 is $v_0 = \sqrt{\frac{GM}{r_0}}$,

$$W_{\text{grav}} = \frac{GM m_{\text{tot}}}{r_{\text{t}}} - m_{\text{tot}} v_{\text{t}}^2,$$

$$W_{\text{grav}} = v_0^2 m_{\text{tot}} [\frac{r_0}{r_{\text{t}}} - 1]. \quad (22)$$

$$W_{\text{t}} = W_{\text{grav}},$$

$$\frac{r_0}{r_{\text{t}}} - 1 = \frac{k r_0}{v_0} (1 - e^{-kt}) + 1 - e^{-2kt}. \quad (23)$$

By solving this equation some inferences can be obtained.

Analytical Solution

$$\frac{r_0}{r_{\text{t}}} - 1 = \frac{k r_0}{\sqrt{\frac{GM}{r_0}}} (1 - e^{-kt})) + 1 - e^{-2kt} \quad (24)$$

To get analytical solution let us do the following simplifications

$$e^{-kt} = x \;\; and \;\; e^{-2kt} = x^2 \quad (25)$$

Here we know that for sample object 1,

$$k = \frac{(B^2 L^2)}{(m_{\text{tot}} \times R_{\text{net}})} = \frac{((2.5 \times 10^{-5})^2 \times (10 \times 10^3)^2)}{(100 \times 10^3)}$$

$$k = 6.2510^{-7} \quad (26)$$

taking

$$M_{\text{Earth}} = 6 \times 10^{24} kg \quad (27)$$

By using Equation 25, 26 and 27, we can modify Equation 24 as follows

$$\frac{r_0}{r_{\text{t}}} - 1 = \frac{6.25 \times 10^{-7} r_0}{\sqrt{\frac{(6.673 \times 10^{-11}) \times (6 \times 10^{24})}{r_0}}} (1 - x) + 1 - x^2$$

As for the purpose of this paper, the final altitude we wish to reach would be 200km above the Earth's surface. Let us take an example of the initial altitude above the Earth's surface to be 1000km. In this case, r_0 would be taken to be $7.4 \times 10^6 \; m$. Substituting and solving,

$$x^2 + x \times (6.245 \times 10^{-4}) + (0.8794124)$$

Solving the quadratic equation we get,

$$x_1 = 0.937457$$

$$x_2 = -0.9380822$$

Discarding the negative root x_2 as invalid and proceeding with the positive root x_1, we obtain de-orbiting time t from t=0 at initial altitude as $t = 10333.587041$ s
Graphing these solutions for the range of orbits of our paper on MATLAB, we obtain the following results as shown in Figure 6.

Table 2. An example of 3 different objects and their specifications

Object	Mass (in kg)	Length (in km)	Total resistance of tether(in kΩ)
Object 1	100	10	1
Object 2	100	1	1
Object 3	10	10	1

4. RESULTS AND CONCLUSIONS

Graphs were plotted for the following sample data for three separate objects as shown in Table 2.

It can be observed that the de-orbiting time, as a result of the force produced by the tethers, decreases, from higher altitudes to 200km above the Earth's surface.
It can also be observed that the multiple tethers does not affect the de-orbiting time as the total force produced multiple tethers, F_t, is same as that of a single tether and even the velocity v_t expression remains the same.
The Energy is only dependent on the above mentioned variables and other common arbitrary and pre-specified constants r_0, m_{tot}, etc. which are taken to be equal for comparison purposes owing to same initial conditions. Notice that the Keplerian orbit being considered from 200km altitude initially, yields zero de-orbiting time, as it is already there at the required resultant position. The deorbiting time for different objects from Table 2 at different altitudes is shown in Table 3.
The de-orbiting time was plotted using MATLAB, against the X axis of altitude for object 1 as shown in Figure 3, for object 2 as shown in Figure 4 and for object 3 as shown in Figure 5. The comparision is made on a logarithmic scale as shown in Figure 6

Table 3. Table relating the time that each object would take to de-orbit from its own orbit to 200km altitude, for given characteristics.

Altitude (in km)	Object-1 time (in s)	Object-2 time (in s)	Object-3 time (in s)
200	0	0	0
300	12210	1221000	1218
400	24610	2462000	2455
500	37210	3722000	3711
600	50000	5001600	4986
700	63000	6302500	6283
800	76220	7624800	7600
900	89660	8969400	8939
1000	103300	10337000	10300

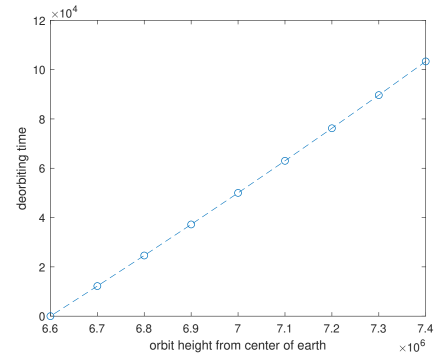

Figure 3. Time of de-orbiting versus altitude object 1

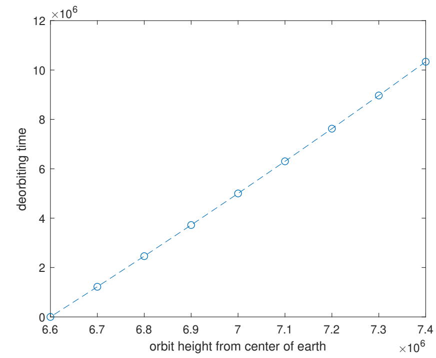

Figure 4. Time of de-orbiting versus altitude object 2

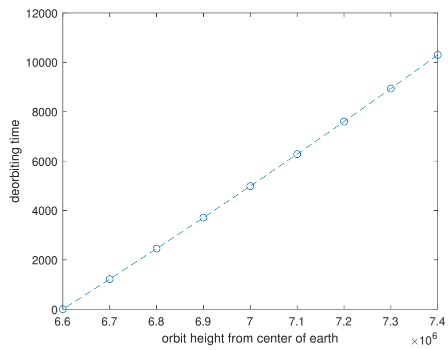

Figure 5. Time of de-orbiting versus altitude for object 3

From the graphs, we can see that the nature of variation of de-orbiting time with initial altitude is similar in nature to that of the other objects.

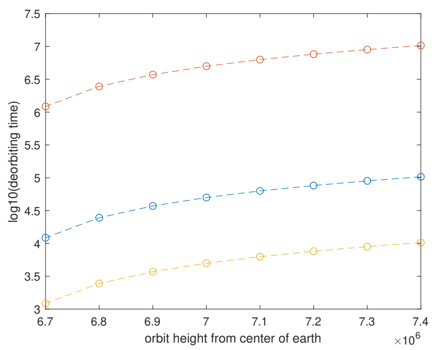

Figure 6. Time of de-orbiting versus altitude for all 3 objects in logarithmic scale

5. PRESUMPTIONS OF THIS STUDY AND FUTURE SCOPE

From past papers and by observing the properties of multiple conductors in quasi-neutral plasma, this paper uses the Child Langmuir equation to calculate the thickness of the sheath around multiple tethers and has theoretically proposed the concept of optimum distance between two tethers. Experimental results should verify this proposal and conclude the optimum distance between the tethers. The sheath to sheath interactions is also open for experimental verification, which can provide greater insight into the behavior of plasma itself. A setup can be demonstrated to develop and verify short multiple tethers, whose length sums up to be equal to the length of a single long tether can produce the same force can also be verified. Apart from the experimental analysis of the deployment mechanics required for multiple tethers, the circuitry of the contactors can begin the era of multiple tethers for future de-orbiting mechanism. This paper extensively works with the consideration of micro-satellites for analysis, but, if efficient enough, the concept of multiple tethers can be extrapolated to all classes of satellites.

6. CONCLUSION

The paper provides an introductory analysis of the concept of using multiple tethers rather than a single long tether to produce the required drag force to de-orbit a satellite.
Introducing the concept of optimum distance between the tethers to avoid any interactions between the plasma sheaths of the multiple tethers, the multiple tethers must be placed with appropriate spacing to produce the same electrodynamic drag forces as a long tether to de-orbit the satellite within the same de-orbiting time with a single large tether. The plasma sheath to sheath interactions, the theory of which would need to be developed further, has been excluded from consideration.

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