MEGA-CONSTELLATIONS – A HOLISTIC APPROACH TO DEBRIS ASPECTS

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ABSTRACT

Mega-constellations with thousands, and even tens of thousands, of satellites in low Earth orbit (LEO) are becoming a reality. This paper synthesizes previous work addressing various aspects of how these constellations drive evolution of the debris environment, providing a holistic approach.

A model taking account of the changing debris environment during the orbital lifecycle of a constellation is used to estimate expected time to Kessler Syndrome (self-sustaining collision cascade) and identify a "safe space" boundary in the parameter space. It is shown that with the appropriate parameter choices it is possible to safely deploy large constellations, and that conversely with poor choices, the Space Age could come to an inglorious end.

1 INTRODUCTION

Mega-constellations with thousands, and even tens of thousands, of satellites in low Earth orbit (LEO) are becoming a reality ([1] and [2]). This paper synthesizes previous work addressing various aspects of how these constellations drive evolution of the debris environment, providing a holistic approach.

Constellations are appropriately analysed over their orbital lifecycles. Large constellations are incrementally deployed, and satellites are replenished as they fail, reach end-of-life, or are replaced with more capable models. This replenishment can be reasonably modelled to continue until the constellation is no longer economically viable. The result is a continuing process of orbit raising and phasing, and a combination of active and passive deorbiting.

Mega-constellation operators will receive millions of conjunction warnings each year [3]. Every time an operator does not manoeuvre a satellite in response to a low probability conjunction warning, there is a non-zero collision risk that depends on the space situational awareness (SSA) accuracy. Additionally, every time an operator does manoeuvre, there is another non-zero probability that the manoeuvre will result in a collision. In both cases, with millions of conjunction warnings each year, even six sigma events can become likely.

Satellites that cannot manoeuvre, cannot avoid. Loss of manoeuvrability can result from failures of satellite sub-systems in the manoeuvre chain or from collision with small objects that disable these subsystems. Both can be mitigated with sub-system redundancy, and small objects collision can be further mitigated with shielding. Operational techniques, such as initiating deorbit immediately after the (N – 1)-th failure with N-th redundancy, can be used to improve the effective satellite reliability.

Recent reports [4] estimate that there are about 28,210 debris objects regularly tracked by Space Surveillance Networks. The numbers of debris objects estimated by statistical models and their potential effect on active satellites are shown in Tab. 1.

Table 1. Effect of Satellite Collision with Debris

Object Size	Number in Orbit [11]	Effect of Collision on Active Satellite
>10 cm	34,000	Catastrophic
1 cm to 10 cm	900,000	May be catastrophic May render it non-maneuverable
1 mm to 1 cm	128 million	May render it non-maneuverable

The debris environment naturally evolves over time as objects decay, active satellites become non-manoeuvrable (passive), and new objects are created by collisions between debris objects. Satellite collisions with large objects are typically catastrophic, fragmenting the objects and causing a step increase in the debris population. In addition, upper stages and dispensers associated with the initial and replenishment launches add to the population.

A model is developed taking account of the changing debris environment during the orbital lifecycle of a

Proc. 8th European Conference on Space Debris (virtual), Darmstadt, Germany, 20–23 April 2021, published by the ESA Space Debris Office
Ed. T. Flohrer, S. Lemmens & F. Schmitz, (http://conference.sdo.esoc.esa.int, May 2021)

constellation. The model is used to calculate the expected time to Kessler Syndrome (a self-sustaining collision cascade first postulated by NASA's Donald J. Kessler in 1978 [5]) and identify a "safe space" boundary in the parameter space.

It is shown that with the appropriate parameter choices it is possible to safely deploy large constellations, and that conversely with poor choices, the Space Age could come to an inglorious end.

2 MODEL

A toy model, one that is deliberately simplistic with many details removed to explain a mechanism concisely, is developed to explore the dependence of time to Kessler Syndrome on several key parameters. The assumptions employed to create this model are discussed in Section 2.1. Using these assumptions, the estimated collision rate of passive objects is derived in Section 2.2. This is used in Section 2.3 to develop and evaluate a differential equation for the evolution of the number of non-manoeuvrable (passive) objects. Finally, the evolution equation is used to find the time to Kessler Syndrome in Section 2.4.

2.1 Assumptions

The first simplifying assumption is that the collision rate of the non-manoeuvrable (passive) LEO objects can be modelled using the kinetic theory of gases. Passive objects are assumed to be uniformly spaced in the LEO volume, between 200 km and 2,000 km altitudes. References [6], [7], [8], [9], [10], [11], [12], and [13] provide examples of the kinetic theory of gases applied to estimating debris collision rates.

The second simplifying assumption is that the number of manoeuvrable (active) LEO satellites is constant. It is assumed the mega-constellations will be replenished as satellites fail (become passive) or as they deorbit.

The third simplifying assumption is that the passive objects are fungible, they all have the same properties. Specifically, all passive objects are modelled as having the same hard body radii and the same orbital velocities.

As a final simplifying assumption, the natural decay of passive objects is ignored.

2.2 Estimated collision rate (collisions/s)

Using the kinetic theory of gases, the collision rate (collisions per second) of non-maneuverable (passive) objects, R_C, is given by Eq. 1.

$$R_C = N \cdot D \cdot v \cdot \sigma = \frac{v \cdot \sigma}{V} \cdot N^2 \qquad (1)$$

Where:

N is the number of non-maneuverable (passive) objects in LEO.

$D = N/V$ is the passive object density (km^{-3}), where $V = 4\pi/3 \cdot (R_2^3 - R_1^3)$ is the LEO orbital volume (km^3), and R_1 and R_2 are min and max LEO orbital radii (km). Using the assumed 200 km to 2,000 km bound for LEO, the orbital volume is 1.27×10^{12} km^3. "Space is big. You just won't believe how vastly, hugely, mind-bogglingly big it is", from [14]. However, as discussed in Section 3, it may not be big enough to support safe operation of all mega-constellations.

$v = \sqrt{2\mu/R}$ is the average relative object velocity (km/s), where $\mu = 398{,}600$ (km^3/s^2) is the Earth's gravitational parameter, and $R = (R_1 + R_2)/2$ is the average object orbital radius (km). The factor of 2 in the radical accounts for the relative velocity of two object with trajectories crossing at 45°. Using the assumed 200 km to 2,000 km bound for LEO, the average object orbital radius is 7,478.14 km, and the average relative object velocity is 10.3 km/s.

$\sigma = 4\pi r^2$ is the collision cross-section (km^2), where r is the object's hard body radius (km). A typical value for mega-constellation satellites is a 0.005 km (5 m) hard body radius, giving a typical value of 0.000314 km^2 for the collision cross-section.

2.3 Evolution of Number of Non-Manoeuvrable Objects

Three parameters (α, β, and γ) are introduced to simplify the equations. The collision parameter, α, is defined by Eq. 3. It has units of per object per second.

$$\alpha = \mathrm{k} \cdot \frac{v \cdot \sigma}{V} \qquad (3)$$

Where k is the number of passive objects created by each collision. Using the values from Section 2.2 and assuming k = 1,000 objects/collisions, a typical value is $\alpha = 2.55 \times 10^{-12}$ (objects^{-1}s^{-1}).

The active object to passive object conversion parameter, β, is defined by Eq. 4 and has units of objects per second.

$$\beta = N_A \cdot R_F \qquad (4)$$

Where N_A is the stable number of active satellites, and R_F

is the conversion rate of active satellites to passive objects per second. With a stable population of 100,000 active satellites and a 1% per year failure rate, β = 1,000 objects/year = 3.2×10^{-5} objects/s.

The initial number of passive objects, γ, is defined by Eq. 5.

$$\gamma = N_0 \tag{5}$$

Where N_0 is the initial number of passive objects in LEO. Potential values for this parameter are discussed in Section 1, especially in Tab. 1.

Using these parameters with the simplifying assumptions (Section 2.1) and the estimated collision rate model, Eq. 1, a first-order non-linear differential equation with initial condition is constructed, Eq. 6, to model the evolution of the number of passive objects, N(t). The equation is parametrized over the three-dimensional (α, β, γ) space using the definitions in Eqs. 3-5.

$$\frac{dN}{dt} = \alpha \cdot N(t)^2 + \beta, \quad N(0) = \gamma \tag{6}$$

The number of passive objects as a function of time, Eq. 7, is obtained by integrating Eq. 6.

$$N(t) = \sqrt{\frac{\beta}{\alpha}} \cdot \tan\left(\tan^{-1}\left(\sqrt{\frac{\alpha}{\beta}} \cdot \gamma\right) + \sqrt{\alpha \cdot \beta} \cdot t\right) \tag{7}$$

2.4 Time to Kessler Syndrome

The time at which the number of passive objects becomes infinite, the time to Kessler Syndrome, T_K, is obtained from Eq. 7 as shown in Eq. 8.

$$T_K = \frac{\pi - 2\,tan^{-1}\left(\sqrt{\alpha/\beta} \cdot \gamma\right)}{2\sqrt{\alpha \cdot \beta}} \tag{8}$$

3 RESULTS

Eqs. 7 and 8 are explored over ranges straddling the typical values discussed in Section 2.3. The range for α, the collision parameter, is 10^{-18} to 10^{-12} (objects^{-1}s^{-1}). For β, the active object to passive object conversion parameter, the range is 1 to 10,000 new objects per year created from failures of active satellites (3.2×10^{-8} to 3.2×10^{-4} objects/s). Two values for γ, the initial number of passive objects parameter, are considered, 20,000 objects and 200,000 objects.

Fig. 1 shows the time history (Eq. 7) of the number of passive objects for various combinations of the α and β parameters. For each α-β pair, curves are plotted for two values of γ, the initial number of passive objects, 20,000 and 200,000 objects. It is seen that in each case, reducing the number of initial non-manoeuvrable objects from 200,000 (solid line) to 20,000 (dashed line) significantly increases the time to Kessler Syndrome.

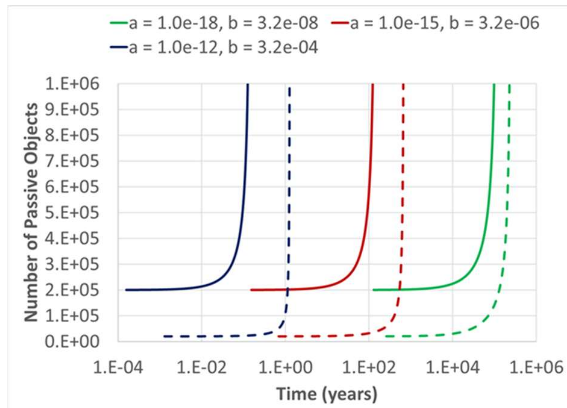

Figure 1. Time History of Number of Passive Objects for Various Combinations of Parameters, Showing the Sensitivity to the Initial Number of Passive Objects (dashed lines are for 20,000 initial objects and solid lines for 200,000)

Using Eq. 8, Fig. 2 shows contours of constant time to Kessler Syndrome plotted with α on the x-axis from 10^{-18} to 10^{-12}, β on the y-axis from 1 to 10,000 satellite failures per year, and for γ = 20,000 initial passive objects. The contours range from 2 years on the right side of the plot to 100,000 years on the lower left side.

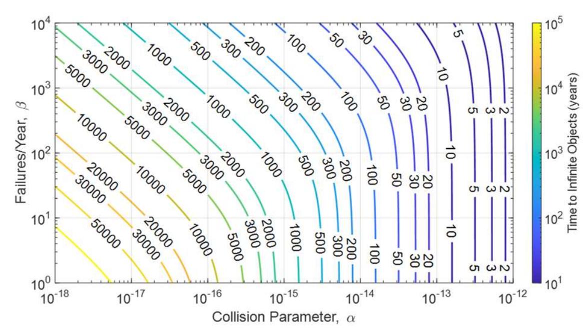

Figure 2. Contours of Constant Time to Kessler Syndrome over Parameter Space with 20,000 Initial Passive Objects

In the regions where a Kessler Syndrome is not imminent (roughly the left half of the plot) it is seen that reducing the failure rate of active satellites significantly increased the time before a Kessler Syndrome will occur.

4 CONCLUSIONS AND FUTURE WORK

A toy model has been used to show: 1) that reducing the number of initial passive objects significantly increases the time to Kessler Syndrome, and 2) that reducing the number of active satellites that fail each year also significantly increases the time to Kessler Syndrome, when it is not imminent.

The key for delaying a Kessler Syndrome, or perhaps even avoiding it, is for mega-constellation operators to minimize the number of non-manoeuvrable satellites (passive satellite) in orbit. Loss of manoeuvrability can result from failures of satellite sub-systems in the manoeuvre chain or from collision with small objects that disable these subsystems. Mitigating these with sub-system redundancy and shielding is important, but perhaps more impactful is operational mitigation. For example, deorbiting satellites promptly when indicated by reliability models.

Future work will refine the toy model by 1) modelling orbital decay of passive objects, 2) modelling different classes of passive objects, and 3) partitioning LEO into orbital shells and individually modelling each shell with coupling between shells.

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