COVARIANCE DETERMINATION FOR IMPROVING UNCERTAINTY REALISM

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ABSTRACT

The reliability of the uncertainty characterization, also known as uncertainty realism, is of the uttermost importance for Space Situational Awareness (SSA) services. One of the greatest sources of uncertainty for the orbits of Resident Space Objects (RSOs) comes from the uncertainty of dynamic models, which is not normally taken into account during the orbit determination processes. A classical approach to account for these sources of uncertainty is the consider parameters theory, which consists in including parameters in the underlying dynamical models whose variance represents the corresponding uncertainty. However, realistic variances of these consider parameters are not known. This work presents a methodology to infer the variance of the consider parameters, based on the observed distribution of the Mahalanobis distance of the orbital differences between predicted and estimated orbits, which theoretically should follow a chi-square distribution under Gaussian assumption. The methodology is presented in this paper, validated in a simulated scenario and tested in a real operational environment with radar data of the Sentinel-3A satellite.

Keywords: uncertainty realism, covariance realism, space situational awareness, covariance determination, Mahalanobis distance, chi-square distribution.

1 INTRODUCTION

The provision of most of the services in Space Traffic Management (STM) and SSA relies on the proper characterization of the orbital uncertainty. This is known as uncertainty realism, and focuses on the correct representation of the Probability Density Function (PDF) of the orbital state. Uncertainty realism can be reduced to covariance realism under Gaussian assumptions, requiring not only an unbiased estimation but also covariance consistency (correct covariance orientation, shape and size). When these requirements are met, the PDF representing the uncertainty of the system can be fully characterised by its two first moments, gathered in a covariance matrix. The misrepresentation of the uncertainty of a RSO impacts STM and SSA products, being crucial for: RSO cataloguing, collision risk assessment, fragmentation analysis, re-entry prediction, track association, manoeuvre detection or sensor tasking and scheduling, among others.

Many existing Orbit Determination (OD) processes are based on weighted batch least-squares theory and provide the estimation (state and covariance) as the nominal output, given that measurements are sufficient and available. Along this process, the dynamical model defining the motion of the orbiting object is assumed to be deterministic, and only the measurements uncertainty is normally accounted for. The resulting covariance matrix in this estimation process is known as the noise-only covariance [1]. However, one of the main sources of uncertainty during OD and subsequent propagation arises from the errors in the underlying dynamical models, which is typically disregarded. For instance, when the ballistic coefficient is estimated in an OD process, the correlation of its uncertainty with the uncertainty of the rest of the state might not be accounted for during the covariance computation. This will cause the covariance realism to be degrade, since the uncertainty in the ballistic coefficient is spread to the position and velocity components through propagation in time [2].

Therefore, it is customary for SSA and particularly for Space Surveillance and Tracking (SST) purposes to characterize and determine the inherent uncertainty and their effects, which is commonly known as uncertainty quantification. Two fundamental problems can be distinguished for uncertainty quantification: the propagation of uncertainty and the inverse problem (model and parameter uncertainty) [3]. The former one concentrates on how to propagate forward an initially given Probability Density Function (PDF) of a state, accurately and efficiently. This is not the focus of the present work, where linearized propagation techniques are used. The inverse problem, on the contrary, consists in assessing the differences between the observed behaviour of a system and the underlying models and parameters used to represent it. Regarding the uncertainty in the modelling, an alternative is to revisit the deterministic assumption in the equations of motion. A common approach is to account for the model uncertainty with stochastic dynamics or process noise, exploring different stochastic modelling such as Brownian motion, Ornstein-Uhlenbeck or Gauss-

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Markov processes [3]. The other target of the inverse problem is the parameter uncertainty, whose objective is to represent the uncertainty in specific terms of the dynamic or measurement equations. If the uncertain parameter can be observed or estimated, it is possible to include its time evolution in the differential equations of motion. On the contrary, if the parameter is not observable, it can be treated probabilistically by assuming that the parameter follows a certain distribution, such as a Gaussian one. In the end, the goal of the parameter uncertainty modelling is not only the posterior quantification of the errors, but also to represent the relationship between the uncertain parameter and the state variance.

Besides choosing to target the previously described components of the inverse problem separately, there exists a wide variety of techniques whose intention is to cover the complete problem. Process noise methods, which consist in adding an additional noise in the dynamics to account for un-modelled error sources, are gaining confidence over stochastic acceleration methods in the current state of the art since they can account for both dynamic model and parameter uncertainty. Some authors propose to estimate the process noise with a calibration process in order to improve the covariance realism [4]. However, a physically-based derivation of a process noise can be rather challenging, and typical solutions lack the physical meaning of the different sources of the uncertainty. These techniques are typically used for filtering applications rather than in batch processing. Nonetheless, some works describe the computation of a process noise matrix that accounts for the drag uncertainty and include it in the batch least-squares estimation process [5]. Other approaches suggest the use of empirical covariance matrices to include all residuals of the estimation process in the covariance computation, regardless of whether the uncertainty has been modelled or not [6]. This proposal claims to account more accurately for noise variations rather than process noise or consider parameter analysis, at the expense of the physical interpretation of the uncertainty. Finally, filters are also a state of the art technique that allows to retain higher moments of the PDF under analysis by the selection of specific sigma points in pseudo Monte Carlo analysis [7]. The key of these methods is the trade-off between accuracy and computational cost.

Apart from uncertainty quantification methods, there are other techniques conceived to improve covariance realism without focusing on the sources of uncertainty and their modelling. For instance, state representation in mean orbital elements allows the covariance matrix to represent more realistically the uncertainty of Monte Carlo simulations [8]. Other typical representations of the state and covariance in non-linear reference frames that are able to slow down the realism degradation upon propagation are being widely studied, such as in [9].

In an operational environment, operators require simple techniques in order to improve covariance realism since, as previously discussed, the nominal covariance determination methods provide optimistic results. The most common options are: (1) the previously mentioned process noise and (2) scaling techniques, which inflate the covariance by means of scaling factors. Some authors propose the computation of such scaling based on increasing the initial position uncertainty to match the velocity error [10]. Others explore the usage of the Mahalanobis distance of the orbital differences to find the scale factor [11]. However, a common drawback of artificially increasing the covariance is that the physical meaning of the correction is lost, not being able to understand the contributions of each source of uncertainty. These sort of methods are used nowadays in operation centres such as Space Operations Center (CSpOC) [3].

One of the classical approaches for parameter uncertainty in the dynamic equations is the consider parameter theory, whose details can be found in [1] and could be classified as process noise techniques. It consists in extending the state space by including parameters in the dynamic models, such as atmospheric force, solar radiation pressure force or measurement models. These parameters are devised to follow a certain model with its corresponding uncertainty, in the general application, a Gaussian distribution with a null mean (to maintain an unbiased estimation) and a certain variance. This allows to represent unaccounted error sources of the dynamical or measurement models by including the parameter uncertainty. This formulation can be combined with batch estimation or filtering algorithms such as in the Schmidt-Kalman filter [12], [13]. This approach provides the advantage of tracking the effect of the specific uncertain physically-based parameters that are included, as opposed to artificial scaling factors. However, one of the main drawbacks of the consider parameter theory is that realistic variances of such parameters are not normally known, a common problem in process noise methods. Overly optimistic or sized variances may fail to model the uncertainty of parameters in the estimation and subsequent propagation of the covariance, not achieving covariance realism.

The aim of this work is to present a novel methodology to determine the variance of the included consider parameters based on the orbital differences between estimated and predicted orbits. Under Gaussian assumptions, the differences between both orbits projected into de covariance space, i.e. Mahalanobis distance, shall follow a chi-square distribution to achieve covariance realism. Thus, the variance of the consider parameters can be determined by means of a minimization process between the observed

Mahalanobis distance distribution and the expected one, i.e. a chi-square distribution. The work carried out concentrates in Low Earth Orbits (LEO) regimes, where the most relevant uncertainty sources are atmospheric density in the drag force acceleration and the range bias in the radar measurements [14].

A similar analysis based on the consider parameter theory to improve the covariance realism is performed in [15], a precursor work for this study. There, it is proposed to correct the noise-only covariance with a least squares fitting to a so-called observed covariance, this latter being obtained from Monte Carlo orbital differences aggregation. This approach has a main drawback, which is that to compute such observed covariance, orbital differences at distinct orbital positions are mixed from orbit estimated based on different observation scenarios. This issue is mitigated by the normalisation obtained with the Mahalanobis distance concept, which is the cornerstone of the work at hand.

The remainder of the paper is organised as follows: in Section 2 the consider parameter theory is reviewed and the methodology is presented. In Section 3 the process followed for the validation of the proposed methodology is briefly described. Next, the results of the proposed covariance determination methodology are shown in Section 4 for a real scenario, corresponding to the Sentinel 3A satellite. The focus is placed on the physical interpretation of the consider parameter variances obtained and the level of covariance realism enhancement achieved. Finally, Section 5 summarizes the conclusions of this work and the future work to be performed.

2 METHODOLOGY

This section describes in first place the consider parameters theory and its direct effect on the covariance computation. Then, the analysed consider parameters are described. Finally, the concept of Mahalanobis distance and its relation with chi-square distribution for the minimization process is described.

2.1 Consider Parameter Theory in Batch Least-squares algorithm

The complete description of the consider parameter theory, or consider covariance analysis as denoted by some authors, can be found in many references such as [1] or [16]. For brevity, only the final derivation in the nominal batch least-squares process is described next. The expected value of the orbit estimation remains unbiased provided that the consider parameters have null mean and their variances are uncorrelated with the measurement noise. On the contrary, the covariance of the estimation is affected. Recalling the nominal OD process, the *noise-only covariance* is obtained as follows:

$$P^n = (H_x^T W H_x)^{-1} \tag{1}$$

where H_x corresponds to the Jacobian of the observations with respect to the reference state, and W is the weighting matrix containing the confidence of each measurement and the possible correlation among the measurements.

Then, the *consider covariance* results in:

$$P^c = P^n + (P^n H_x^T W)(H_c C H_c^T)(P^n H_x^T W)^T \tag{2}$$

where H_c is the Jacobian of the observations with respect to the consider parameters and C is a diagonal matrix containing the consider parameters variance, where no correlation between them is assumed:

$$C = \begin{pmatrix} \sigma_{c1}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{cn}^2 \end{pmatrix} \tag{3}$$

A more compact form of Eq. 2 can be defined as follows:

$$P^c = P^n + K^T C K \tag{4}$$

with:

$$K = P^n (H_x^T W H_c); K \in \mathbb{R}^{n_c} \times \mathbb{R}^{n_x} \tag{5}$$

being n_c and n_x the number of consider parameters and the state vector dimension, respectively.

Therefore, the consider covariance is obtained as the noise-only covariance plus a covariance correction, which depends on the consider parameter variances. The goal of the work at hand is to determine the values of C so that the consider covariance realism is improved.

2.2 Analysed Consider Parameters

Two different consider parameters have been analysed representing the error of the atmospheric drag force model and of the range bias calibration. The classical drag force equation with the applied consider parameter is defined as:

$$\vec{a}_{drag} = -\frac{1}{2}\rho\frac{C_D A}{m} v_{rel}^2 \frac{\vec{v}_{rel}}{|\vec{v}_{rel}|}(1 + c_{AE}) \qquad (6)$$

where ρ is the atmospheric density, C_D the drag coefficient, A the cross-sectional area, m the object mass and $\vec{v}_{rel}$ is the relative speed of the object with respect to the atmosphere. Finally c_{AE} is the applied consider parameter representing the error of the model, which is assumed to follows Gaussian distribution of the form:

$$c_{AE} \sim N(0, \sigma_{AE}^2) \qquad (7)$$

where σ_{AE}^2 is the variance of the parameter. The objective of this consider parameter is to absorb and model the error in the atmospheric density and ballistic coefficient, containing C_D, mass and cross-sectional area uncertainty.

The second consider parameter is the range bias error, included to represent possible errors in the measurements model and calibration process. In this case, the measurement model is modified as follows:

$$z^* = z + c_z \qquad (8)$$

where z represents the range measurement and c_z is the range bias error consider parameter. Analogously:

$$c_z \sim N(0, \sigma_z^2) \qquad (9)$$

with σ_z^2 representing the variance of the consider parameter. Note that Eq. 8 could represent any other measurement retrieved from a sensor, which in the case of a typical radar for SST purposes corresponds to: azimuth and elevation angles, range and range rate.

2.3 Mahalanobis Distance with Consider Covariance

The Mahalanobis distance is a well-known statistical metric that describes how far a state is from a certain reference, projected into the covariance space [17], this is:

$$d_M^2 = \left(\boldsymbol{x} - \boldsymbol{x_{ref}}\right)^T \left(\boldsymbol{P} + \boldsymbol{P_{ref}}\right)^{-1} \left(\boldsymbol{x} - \boldsymbol{x_{ref}}\right) \qquad (10)$$

where the covariance of both the state and the reference must be considered. However, the covariance of the reference is generally several orders of magnitude smaller, allowing to neglect it from the computations. As will be further detailed, precise orbits from GNSS data are used as reference in part of the present work, allowing to neglect their covariance in the Mahalanobis distance computations. For the sake of clarity, the reference covariance is omitted in the following equations.

Combining Eq. 4 with Eq. 10:

$$d_M^2 = \Delta\boldsymbol{x}^T(\boldsymbol{\Phi}(\boldsymbol{P}^n + \boldsymbol{K}^T\boldsymbol{C}\boldsymbol{K})\boldsymbol{\Phi}^T)^{-1}\Delta\boldsymbol{x} \qquad (11)$$

where $\Delta\boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x_{ref}}$ and $\boldsymbol{\Phi}$ is the State Transition Matrix (STM). Since the objective of this work is not limited to the covariance realism improvement at estimation epoch, but also during the typical propagation arcs performed in SST, a propagation by means of the state transition matrix is performed, as applied operationally in most SST services. More complex and accurate uncertainty propagation methods are out of the scope of this work, though the linearized orbital propagation shall remain valid for the propagation arcs analysed in this work, having checked the required Gaussian behaviour along the validation and real case scenarios. Besides, it is important the propagation of the full covariance matrix, since the effect of the estimated parameters covariance will affect significantly the position and velocity covariance evolution. For instance, the uncertainty in the drag coefficient must be considered if the proper evolution of the position uncertainty in the velocity direction is to be computed, since this component is strongly affected.

2.4 Consider Parameter Variance Determination

Under linear and Gaussian assumptions, this is, when the differences between the state and the reference are normally distributed and the covariance is representative of such distribution (i.e. realistic), the squared Mahalanobis distance should follow a chi-square distribution, whose detailed characteristics may be found in [18]. Eq. 10 allows us to evaluate the Mahalanobis distance at any propagation epoch by computing the orbital differences between a predicted orbit and a reference. Thus, if a sufficient number of predicted orbits is available, it is possible to look for the consider parameter variances included inside the matrix

C so that the squared Mahalanobis distance population resembles the theoretical chi-square distribution. Hence, computing C is reduced to a minimization process, where the optimization variables are the variances of the consider parameters.

The minimisation process consists in comparing the Cumulative Distribution Function (CDF) of the observed squared Mahalanobis distances with the chi-square CDF of as many degrees of freedom as the ones included in the Mahalanobis distance computation.

Therefore, the covariance determination process can be divided in three main steps:

1. For a sufficient amount of orbits, perform an OD process to obtain the noise-only covariance and the components of matrix $\boldsymbol{K}$. Then propagate the estimated state to obtain the predicted orbit and the state transition matrix.
2. For each predicted orbit, compute the orbital differences at any desired propagation epoch comparing against a reference orbit.
3. With all the data from the orbits population to construct Eq. 10, obtain the consider parameters variance that minimizes the differences between the observed squared Mahalanobis distance distribution and the chi-square distribution.

An example of the minimization process can be seen in Fig. 2, where the obtained squared Mahalanobis population resembles the chi-square distribution with the optimum consider parameter variances.

3 VALIDATION

To test the covariance realism improvement capabilities of the proposed methodology, a validation campaign in a simulated environment was carried out. Progressively more realistic cases were performed, sharing a common scheme that is summarized below:

1. Starting from a reference state vector of a LEO RSO, an orbit propagation is performed with a high fidelity dynamical model. This defines the reference orbit.
2. Monte Carlo iterations:
   a. From the reference state, propagate 7 days backwards including a perturbation in the dynamic model, particularly in the aerodynamic force. The perturbations follows a Gaussian distribution with certain variance.
   b. From such perturbed orbits, tracks are simulated from a ground-based sensor with similar visibility capabilities and accuracy as a real operational case. The range bias perturbation is introduced in this step.
   c. ODs are performed with the simulated tracks, with a determination arc of 7 days. The estimated state is then propagated from the estimation epoch (t_0), corresponding to the last measurement, obtaining the predicted orbit and the state transition matrix.
3. Once the complete population of predicted orbits is obtained, the covariance determination methodology previously explained is applied.

Table 1 shows the most relevant results of the validation process.

Table 1: validation tests cases summary

Id	**Analysis epoch**	**Orbit comparison**	**Input Perturbation**	**Consider Parameter Result**
1	t_0	predicted vs reference	None	None
2	t_0	predicted vs reference	$\sigma_{AE} = 25\%$	$\sigma_{AE} = 24.5\%$
3	t_0	predicted vs reference	$\sigma_{RB} = 20m$	$\sigma_{AE} = 18.2m$
4	$t_0, \Delta t$	predicted vs reference	$\sigma_{AE} = 35\%$	$\sigma_{AE} = 34.1\%$
5	$t_0, \Delta t$	predicted vs reference	$\sigma_{AE} = 5\%$ $\sigma_{RB} = 20m$	$\sigma_{AE} = 4.83\%$ $\sigma_{RB} = 20.1m$
6	$t_0 + N$ days, Δt	predicted vs reference	$\sigma_{AE} = 35\%$	$\sigma_{AE} = 34.6\%$
7	$t_0 + N$ days, Δt	predicted vs estimated	$\sigma_{AE} = 30\%$ $\sigma_{RB} = 20m$	$\sigma_{AE} = 30.1\%$ $\sigma_{RB} = 17.7m$

Where Δt denotes that the OD arc of each Monte Carlo iteration is shifted 1 day in absolute time, as would be the case in an operational environment. The results shown in Table 1 correspond to a population of 480 Monte Carlo samples. This number was selected to resemble the amount of data that was available for the real scenario. Along the different cases, the proposed methodology was tested when computing the Mahalanobis distance at different epochs (estimation or relative propagation epochs) and perturbations. In the last validation case, a different orbit is used for the computation of the orbital differences. For each predicted orbit under analysis, an estimated orbit obtained from an OD process using measurements contained during the propagation arc of the predicted orbit is used.

Case 1 showed that in the absence of model errors, the noise-only covariance is a realistic representation of the uncertainty, directly obtaining a chi-square distribution without requiring any consider parameter correction. This can be seen in Figure 1. On the contrary, the noise-only covariance was found to lack realism when perturbations were included. Case 2 and 3 prove that the covariance determination method is able to retrieve the input perturbation at estimation epoch. In the case of the aerodynamic force perturbation, the proposed simulation scheme caused the perturbation to be absorbed in the drag coefficient in the OD process, therefore requiring some propagation time to appreciate the effect on the perturbation in the orbit ephemeris. Cases 4 and 5 show that the proposed method is able to retrieve the input perturbation when a sliding window simulation scheme is applied, also determining both consider parameter variances simultaneously. In Case 6, the performance of the proposed method when computing the orbital differences at relative propagation epochs is tested, being able to retrieve precisely the consider parameter variances only using the position components of the state vector for the Mahalanobis distances computation. Finally, Case 7 demonstrated the possibility of using as reference for the orbital differences computation an orbit estimation whose determination arc includes the propagation epoch under analysis. In this case, the covariance of the estimated orbit used as reference must also be included in the Mahalanobis distance computations to account for the estimation uncertainty. The Mahalanobis distance distribution of this case can be appreciated in Figure 2, with the obtained consider parameters variance, being able to recover the chi-square behaviour and the input perturbations using the operationally feasible orbit as reference.

Overall, the validation cases showed a satisfactory performance of the proposed consider parameter covariance determination, being able to retrieve the input perturbations up to a 12% accuracy, even in the case when both consider parameters are estimated simultaneously. Additional tests performed confirmed that increasing the number of samples used in the Monte-Carlo simulation allow to reduce this 12% to much lower values.

Another relevant comment from the validation cases is that the velocity terms were discarded from the Mahalanobis distance computations due to unstable behaviour after 3 days of propagation. Small errors in the propagation accumulated in the along-track component of the velocity, whose expected accuracy was orders of magnitude smaller than the other components.

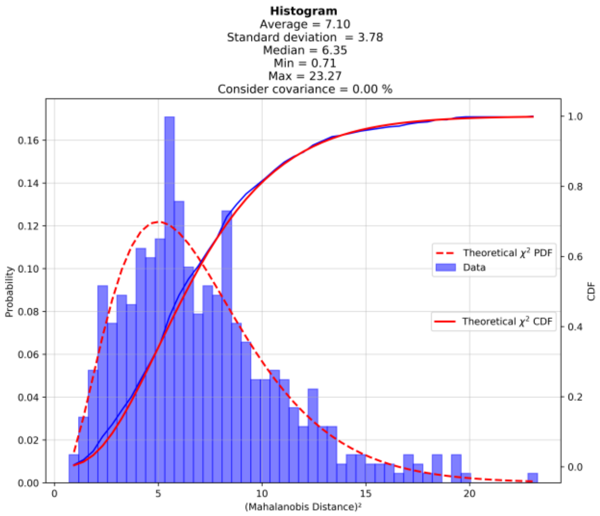

Figure 1: Case 0 – squared Mahalanobis distance distribution at estimation epoch without perturbations.

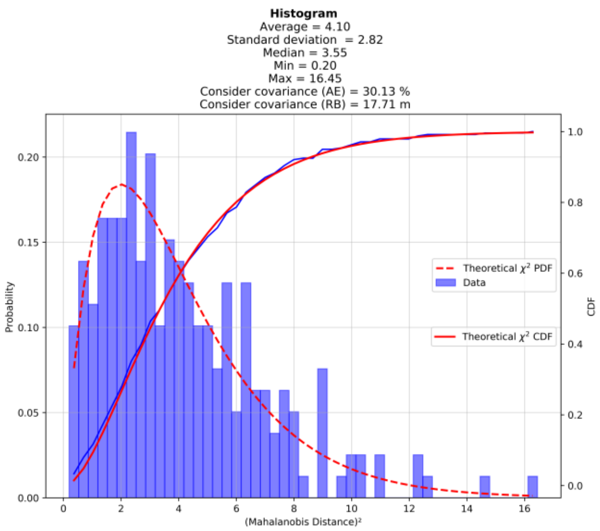

Figure 2: Case 7: Consider Parameters covariance results at $t_0 + 7$ days, considering position and Cd, with an input perturbation of $\sigma_{AE} = 30\%$, $\sigma_{RB} = 20m$.

4 REAL SCENARIO RESULTS

The Sentinel 3A LEO satellite has been used as a realistic scenario to test the proposed methodology. Measurements from two different radars covering from 01/01/2019 up to 01/05/2020 are used. Moreover, Precise Orbit Determination (POD) ephemeris of Sentinel 3A are used as the reference orbits, estimated from GNSS data. The accuracy of these orbits are of the order of 1 cm, allowing to discard their covariance in the Mahalanobis distance computations. Additionally, the manoeuvres of the Sentinel 3A are publicly available at [19].

ODs in batches of 7 days are performed, each one having a 1 day shift. Each estimated state is propagated 14 days. After discarding outliers and ODs affected by a manoeuvre, a population of 315 ODs with their corresponding orbital propagations are available. The sensor calibration process yielded the expected accuracy and biases in Table 2:

Table 2: sensors calibration and bias

	Residuals RMS		Bias	
	Sens 1	Sens 2	Sens 1	Sens 2
Range [m]	27.4	15	-7.4	-4.7
Range-rate [mm/s]	212	5460	-82	35
Azimuth [mdeg]	460	400	-240	150
Elevation [mdeg]	160	270	-2.7	3.2

Additionally, the assumption of Gaussianity for the orbital differences was tested using Michael's normality test, whose details may be found in [20] and [21]. According to [22], Michael's normality test is one of the best suited tests for orbital differences analysis due to a more powerful tail outlier rejection. It was observed that the cross-track component failed to pass the Gaussianity test after several days of propagation. This is shown in Figure 3, where several points of the distribution lay out of the acceptance limits (red line).

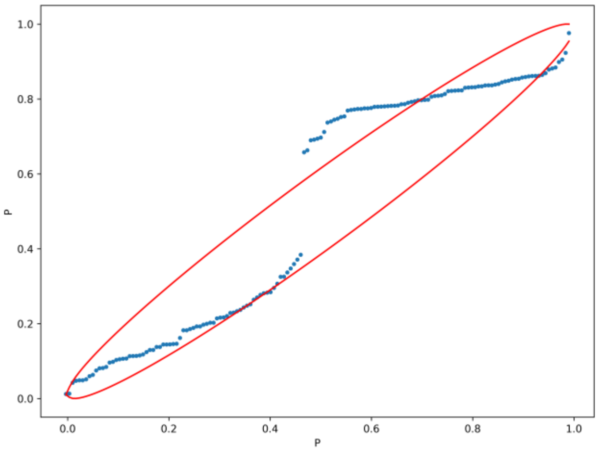

Figure 3: Michael's Gaussianity tests result for cross-track component at $t_0 + 7$ days.

Thus, the cross track component is discarded from the Mahalanobis distance computations, remaining the along track and normal components of the position.

4.1 Covariance Determination Results

The purpose of the proposed covariance determination methodology is to improve the covariance realism not only at estimation epoch, but at the widest possible range of propagation days, as would be required for cataloguing or collision risk assessment services. For this reason, the orbital differences required for the computation of the Mahalanobis distance are computed at several relative epochs (from $t_0 + 4$ up to $t_0 + 8$ days, in half day intervals), including all the orbital differences in the same minimisation process. The objective is to obtain consider parameter variances that are representative of the complete propagation period. The optimum consider parameter variances for the Sentinel 3A data are shown in figures 4-6. Bear in mind that, under this configuration, each estimated/predicted orbit pair provides 9 different Mahalanobis distances, obtaining a higher population for the minimisation.

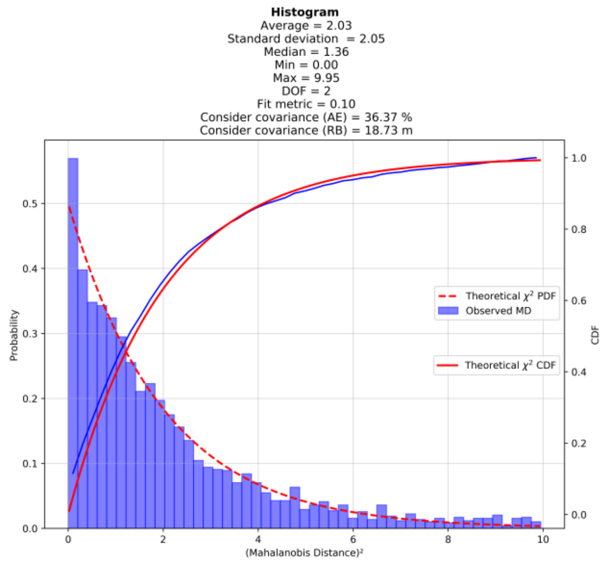

Figure 4: optimum consider parameter variances for Sentinel 3A in the region [$t_0 + 4 - t_0 + 8$ days], using POD orbits as reference.

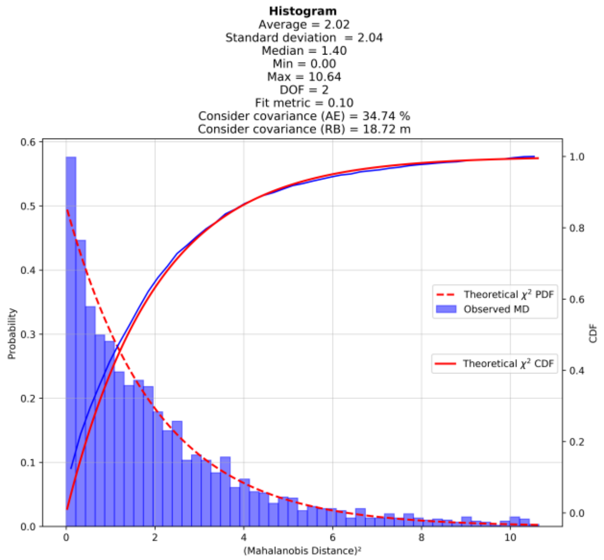

Figure 5: optimum consider parameter variances for Sentinel 3A in the region [$t_0 + 4 - t_0 + 8$ days], using estimated orbits as reference.

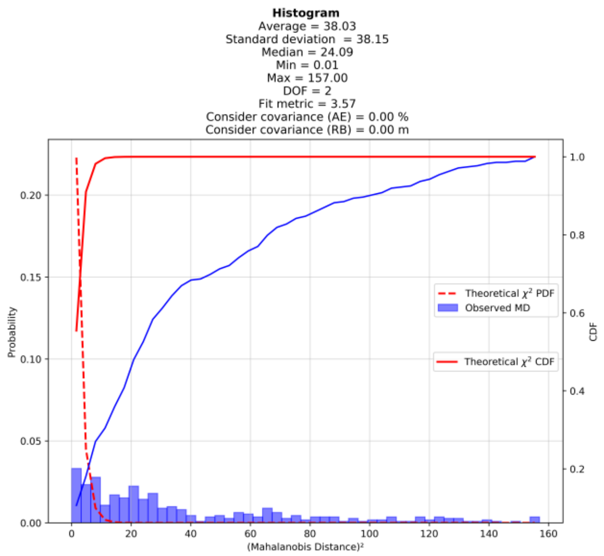

Figure 6: squared Mahalanobis distance distribution for Sentinel 3A using Noise-only covariance (no consider parameter).

Figure 6 depicts how the noise-only covariance is not able to characterise the orbital uncertainty when real data is used, causing the squared Mahalanobis distance distribution to be far from the chi-square behaviour. This is yet another indicator of the presence of errors in the dynamical model and that the noise-only covariance is an optimistic representation of the uncertainty, since large squared Mahalanobis distances are observed when normalizing the orbital differences.

Figure 4 and Figure 5 show the computed consider parameter variances that are required for the squared Mahalanobis distance distribution to resemble the Chi-square one, found during the minimisation process. In the former case, POD orbits are used as reference for the orbital differences computations. In the latter case, for each predicted orbit under analysis, an estimated orbit computed with measurements in the same range as the predicted one is used for the comparison. The accuracy of these orbits are in the range of several meters, thus their uncertainty must be included in the Mahalanobis distance computation. It is important to point out that the optimum consider parameter variances are similar in both Figure 4 and Figure 5. This is a relevant result, since it implies that representative results can be obtained without the use of external information such as POD orbits, which in most operational situations are not available.

The computed consider parameter variances are in realistic ranges, in line with the expected uncertainty of the modelled parameters according to other studies. The obtained drag force consider parameter standard deviation representative of the propagation arc is of 34.74%. According to [14] and [23], the uncertainty in the atmospheric density model ranges from a 10-20% for the model used (NRLMSISE-90), and the uncertainty in the ballistic coefficient presents even greater variability. Regarding the range bias, the obtained consider parameter standard deviation is of 18.72 m. Not only the observed range bias can be accounted in these results (see Table 2), but also biases in other measurements such as the range-rate are partially absorbed in the applied model of the consider parameter. To characterize the exact sources of each uncertainty, additional consider parameters in the measurement models or other forces would have to be included.

The computed variances are obtained using measurements during 1.5 years approximately. Though the optimization aims to represent the modelled uncertainty for the complete period, seasonal variations of the noise are not observable, being a major drawback of the proposed method. However, along this work, further studies about the minimum population required for a sufficiently accurate consider parameter variance determination indicated that around 1 month of measurements would suffice for a 10% accuracy, being able to capture seasonal variations of the uncertainty as expected, for instance, in solar indexes. Additionally, the proposed method can be applied sequentially, adding the information of newly estimated orbits as they arrive and updating the consider parameter variances in an operational schedule.

As previously stated, the consider parameter theory supports an arbitrary number of additional parameters. The proposed methodology has been shown to perform satisfactorily at minimizing two consider parameters simultaneously. Nonetheless, for an increasing number of parameters, the possible correlation between them should be carefully analysed, being this one of the assumptions of the proposed methodology.

4.2 Covariance Containment

Under Gaussian assumptions, if the squared Mahalanobis distance follows a Chi-square distribution, improvement in the covariance realism would be directly achieved. However, covariance containment tests such as the one proposed by [10] provide further physical insight and visual representation of the proposed methodology effectiveness. To evaluate if the covariance is representative of the orbital differences, the Mahalanobis distance can be used as a metric to see the amount of points that lay inside a k-σ ellipsoid and compare it against the theoretical expected fraction for a Gaussian distribution of the same number of degrees of freedom.

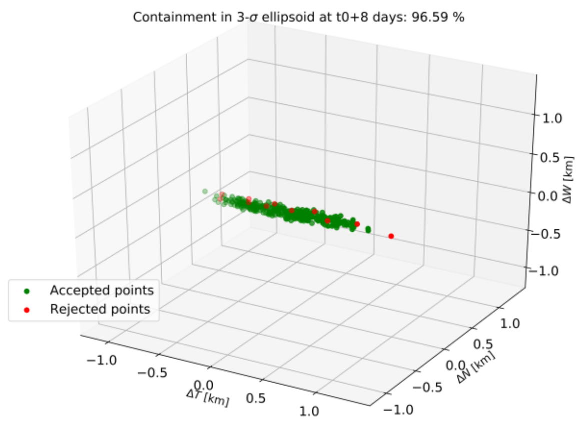

Figure 7: 3-σ covariance containment at $t_0 + 8$ days with the consider covariance

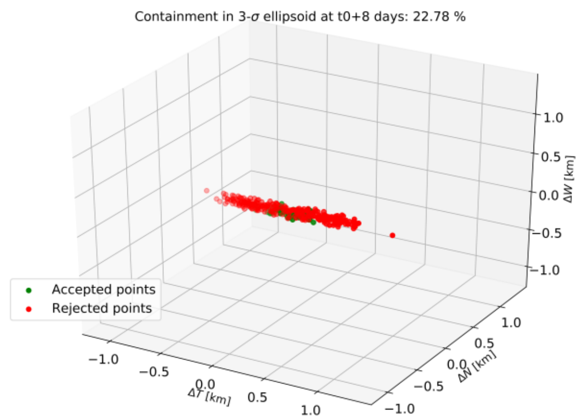

Figure 8: 3-σ covariance containment at $t_0 + 8$ days with the noise-only covariance

Table 3: covariance containment tests results for the complete propagation arc, using the optimum consider parameters of Figure 5 ($\sigma_{AE} = 34.74\%$, $\sigma_{RB} = 18.72m$)

	Containment tests [%]							
	Noise-only covariance				Consider covariance			
Time	**1-σ**	**2-σ**	**3-σ**	**4-σ**	**1-σ**	**2-σ**	**3-σ**	**4-σ**
t0+2	4.58	14.82	33.69	49.6	30.49	57.97	81.32	93.41
t0+3	5.52	15.75	30.39	45.58	32.78	74.71	94.72	100
t0+4	4.19	12.29	27.65	42.74	37.97	78.26	96.81	100
t0+5	3.43	11.71	21.71	36.86	38.62	86.17	100	100
t0+6	4.05	13.01	25.14	38.73	37.35	82.83	96.08	100
t0+7	2.93	11.14	23.46	35.78	40	91.34	100	100
t0+8	3.25	11.54	22.78	35.8	34.98	84.83	96.59	100
t0+9	4.24	11.21	21.21	33.03	41.98	93.52	100	100
t0+10	2.8	13.66	24.22	36.65	36.36	78.57	94.16	100
Average	3.89	12.79	25.58	39.42	38.18	85.07	97.66	100.00
Theoretical	**39.3**	**86.5**	**98.9**	**99.97**	**39.3**	**86.5**	**98.9**	**99.97**

Figure 7 and Figure 8 represent as green dots those orbital differences that lay inside the 3-σ ellipsoid and in red the opposite, where the difference between both figures is the covariance used for the Mahalanobis distances computation. Figure 7 shows a remarkably larger amount of accepted differences. This is a consequence of a better characterization of the covariance due to the included consider parameter in the aerodynamic force, elongating the covariance in the along-track direction. This allows to improve the covariance containment as compared with the Noise-only covariance in Figure 8.

Applying this same analysis to different propagation epochs, always using the determined optimum consider parameter variances of Figure 5, the results of Table 3 are obtained. To measure the realism of the covariance, the containment results are compared against the theoretical containment values that a multivariate Gaussian distribution should show. These theoretical results are obtained from [24], corresponding to a multivariate Gaussian distribution of 2 degrees of freedom (along track and normal components). A colour scale has been used to highlight the results, where colours are similar the closer each value is to its theoretical sigma result. It is easily appreciated the worse covariance realism of the noise-only covariance. On the contrary, the proposed covariance determination method provides a consider covariance that notably improves the realism. For instance, the average 3-σ containment of the noise-only covariance is of 25.58%, whereas the consider covariance provides an average containment of 97.66%, very similar to the theoretical objective of 98.9%.

Note that the average results are computed without $t_0 + 2$ days results or before. For short propagation epochs, the estimation error is still comparable to the propagation error and the effect of the consider parameter correction on the along track and normal components is not fully dominant, reducing the covariance realism improvement. Close to the estimation epoch, the covariance correction of the consider parameters of the present analysis is principally observed in the drag coefficient variance, with a small contribution from the range bias consider parameter in all terms. It is not until some propagation is elapsed that the effect of the consider parameters is appreciated in the position uncertainty. Therefore, the covariance realism enhancement in position is yet not completely developed at those early propagation days. In [15], a more profound analysis of the contribution of the aerodynamic force and range bias consider parameters in the consider covariance was conducted, reaching similar conclusions.

Finally, the containment results of Table 3 present oscillations around the theoretical values, finding for instance higher values at $t_0 + 7$ and $t_0 + 9$ days. This behaviour is expected since the applied consider parameter variances were found by simultaneously optimising the complete region $[t_0 + 4, t_0 + 8]$. Thus, the containment at specific epochs oscillate and slightly overly sized covariance matrices are obtained. Even though it is possible to estimate the optimum consider parameter variance at each specific epoch independently, the operational goal of the proposed methodology is to improve the covariance realism of the widest possible propagation region with a single correction of the covariance.

5 CONCLUSIONS AND FUTURE WORK

The proposed covariance determination method shows a remarkable improvement of the covariance realism in a real case scenario. This has been demonstrated with covariance containment analysis, where it is seen how the noise-only covariance is corrected to resemble the expected theoretical containment of a Gaussian behaviour. The two consider parameters included in the process, namely the aerodynamic force and range bias parameters, are able to represent the major uncertainty sources of the LEO satellite Sentinel 3A, not only allowing an improvement in the covariance realism but also providing realistic quantification of the uncertainty and retaining the physical interpretation. Nonetheless, other consider parameters should be analysed to obtain a closer representation of the specific sources of error and analyse the most relevant contributors. The addition of other parameters in the measurements model, such as the range-rate bias, could provide deeper insight on the effect that the uncertainty of each kind of measurement has in the estimation.

The proposed methodology shows promising results for its operational applicability. First, matching results have been obtained in the real case scenario when using POD orbits as reference or using estimated orbits from the same measurements set, indicating that the proposed covariance determination process is feasible without using any external data apart from the sensor measurements. However, the necessity of a sufficiently large population of estimated/predicted orbits prevents the obtained consider parameter variances to capture seasonal error variations. Further analysis on the amount of orbits required for an accurate consider parameter variance computation is required. Nonetheless, preliminary analysis indicate that the methodology is suitable for monthly noise variations.

Future research activities will be directed towards the analysis of the proposed methodology in other orbital regimes such as Geostationary Orbits, taking into account other relevant sources of uncertainty such as in the solar radiation pressure model. The sources of non-Gaussianity observed in the cross-track orbital differences and their possible effect on the methodology should be further studied. Finally, other interesting analyses suggested by the authors are the time-correlation of the model errors instead of pure Gaussian models.

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