STATE VECTOR UNCERTAINTY AND MANEUVER ERRORS: ANALYSIS OF THE EARLY ORBIT AND STATION-KEEPING PHASES OF AN ELECTRICAL SATELLITE

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ABSTRACT

Maneuver uncertainties have a significant impact on an object state vector uncertainty, especially when multiple maneuvers are performed over a short timespan (for example, during post-launch, station-keeping or deorbitation phases using electrical, low-trust maneuvers). They are usually due to nozzle imperfections or faulty propellant combustion, leading to small errors on the magnitude and direction of the thrust. Downstream Space Situational Awareness (SSA) functions — such as cataloging or collision risk assessment — usually require propagating state vector uncertainties over long periods of time, and often assume the uncertainty follows a Gaussian distribution in Cartesian space through the entire propagation.

This paper presents the results of a pre-launch study aiming to assess how long such an hypothesis holds during the early orbit and station-keeping phases of a satellite planning to perform respectively 29 and 9 electrical maneuvers over one day. To that end, the state vector uncertainty computed through a state transition matrix (STM) is compared at different times to the one computed using a Monte Carlo method.

Initial results show that although the state vector uncertainty remains Gaussian during the analyzed time span, special care should be taken when propagating covariances through state transition matrices. Indeed, Monte Carlo results reveal the maneuver direction error can have a significant impact on the overall uncertainty distribution, depending on its value. The greater the direction error, the more the nominal state vector departs from the perturbed samples, causing the uncertainty distribution to lose its Gaussianity and making state transition matrices propagation unsuitable.

Keywords: Uncertainty propagation, Covariance, Maneuver errors, State Transition Matrix, Gaussianity, Electrical propulsion, Monte Carlo.

1. INTRODUCTION

Proper characterization of the orbital state vector uncertainty of an object is a challenge faced during most Space Situational Awareness (SSA) activities, because of the highly non-linear dynamics and long periods of propagation involved. Many methods are available to evaluate the uncertainty and its propagation (for examples, see [3]), some more precise than others depending on the context of the study. Among them, the uncertainty representation through State Transition Matrices (STM) in Cartesian space remain the most used method operationally: it has a low computational cost, and can yield accurate results if the right conditions are met. In particular, this method assumes the propagated uncertainty remains Gaussian during the analyzed time span, which is not always verified.

This paper presents the results of a pre-launch study aiming to assess if STM could be used to monitor the early orbit and station-keeping phases of an electrical satellite performing a large number of low-thrust, not 100%-efficient maneuvers. The purpose of this study is to estimate whether or not the state vector uncertainty distribution remains Gaussian over the period of interest of these operational phases, and if so, to determine how to propagate it accurately in presence of maneuver uncertainty using STM. After generating reference results using Monte Carlo simulations and concluding on the Gaussianity of the uncertainty distribution after one day of propagation, the maneuver errors are modeled as a matrix to be included in the STM propagation, which is then compared to the reference results. In this process, a particularly interesting phenomena is observed: the nominal state vector departs from the center of the Monte Carlo sample distribution, consequence of the impact of the maneuver direction uncertainty.

This paper is arranged as follows. Section 2 introduces the context of this study and the representation of maneuver errors. Section 3 presents the reference results for uncertainty propagation, performed using Monte Carlo simulations for the post-launch and station-keeping phases, and analyses the Gaussianity of its distribution. Section 4 focuses on the maneuvers errors and describes the way they are modeled and combined to the state transition ma-

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trices. Results of the STM propagation are compared to the reference, and the difference between the uncertainty distributions is analyzed in Section 5, where a solution to counter this phenomena is proposed. Finally, Section 6 presents the evolution of the state vector uncertainty over 7 days to test its Gaussianity, in case the STM propagation needs to be extended.

2. STUDY CONTEXT AND MANEUVER ERRORS

The post-launch phase is composed of 29 maneuvers per day, performed at each ascending and descending node of the orbit. Their magnitude is $24\,\mathrm{mm\,s^{-1}}$, and all last $1300\,\mathrm{s}$. The station-keeping phase performs 9 maneuvers per day, alternatively on the ascending or descending node as shown in the Figure 1. The magnitude is $10\,\mathrm{mm\,s^{-1}}$ for the $600\,\mathrm{s}$-long maneuvers and $5\,\mathrm{mm\,s^{-1}}$ for the $300\,\mathrm{s}$-long maneuvers.

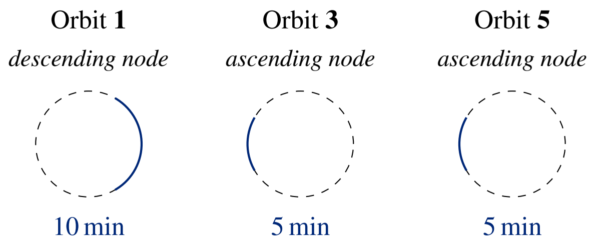


Figure 1: Station keeping maneuvers plan to repeat over the period of interest.

For both early orbit and station-keeping phases, the overall uncertainties consist of the initial state vector uncertainty and the maneuver errors. The initial state vector uncertainty distribution is assumed to follow a normal distribution law $\mathcal{N}_{\mathrm{X}}(0, \sigma_{\mathrm{X}})$ on each coordinate of position (σ in m) and velocity (σ in $\mathrm{m\,s^{-1}}$) in the QSW local orbital frame, and the corresponding standard deviation vector is defined as follows:

$$S = \begin{bmatrix} 5.471 \\ 14.922 \\ 18.418 \\ 7.089 \times 10^{-3} \\ 11.613 \times 10^{-3} \\ 21.052 \times 10^{-3} \end{bmatrix}$$

The correlation between these uncertainty distribution laws is expressed through the following matrix:

$$C = \begin{bmatrix} 1 & -0.011 & -0.046 & -0.474 & -1.0 & 0.015 \\ - & 1 & -0.028 & 0.504 & 0.004 & -0.022 \\ - & - & 1 & 0.007 & 0.047 & 0.05 \\ - & - & - & 1 & 0.475 & -0.073 \\ - & - & - & - & 1 & -0.015 \\ - & - & - & - & - & 1 \end{bmatrix}$$

with "–" expressing the symmetry of the matrix.

In this context, two types of maneuver errors are taken into account. The first one is the magnitude error, which is defined as a percentage of the nominal maneuver magnitude. It is assumed to follow a normal distribution law $\mathcal{N}(0, \sigma_{\mathrm{m}})$, where $\sigma_{\mathrm{m}} = 0.01\|\boldsymbol{V}\|$, meaning that the standard deviation of the magnitude error is $1\,\%$ of the nominal magnitude. This error can be due to propellant random behavior, causing the effective thrust magnitude to be slightly bigger or smaller than the planned magnitude. The second one is the direction error, which is defined as a deviation angle around the nominal thrust direction. It is assumed to follow a normal distribution law $\mathcal{N}(0, \sigma_{\mathrm{d}})$, where $\sigma_{\mathrm{d}} = 5°$ around the nominal direction. This error can be due to imperfections on nozzle manufacturing, causing the effective thrust direction to be deflected from the planned direction.

Considering the maneuver frame with the thrust along the X axis, and with m and α the magnitude and direction perturbations, the maneuver vector $\boldsymbol{V}_{\mathrm{p}}$ subject to these perturbations can be represented as:

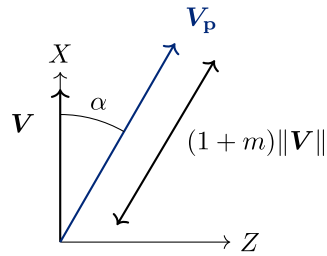


Figure 2: Maneuver errors representation.

The magnitude error is easy to represent, as it is only distributed on one direction, along the maneuver vector. The direction error is harder to perceive since it is defined in 3 dimensions. Indeed, this error is evenly distributed around the nominal maneuver vector $\boldsymbol{V}$: in the Figure 2, the specific direction perturbation represented is a rotation α around the Y axis of the maneuver frame, but it could be any rotation with an axis included in the YZ plane. In other words, the direction error can be represented by a cone around the X axis, of angle σ_{d} the standard deviation of the error normal distribution law as shown on Figure 3.

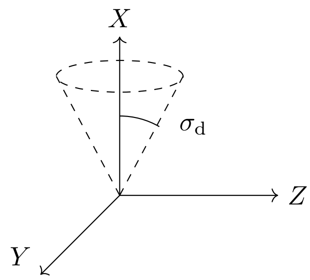


Figure 3: Direction error representation in 3D.

In order to model this perturbation in 3D, which will be necessary later, another variable is introduced: the angle θ represents the even distribution of the perturbation around the maneuver vector (see Figure 4). This variable is defined by a uniform distribution law $\mathcal{U}(0, \pi)$.

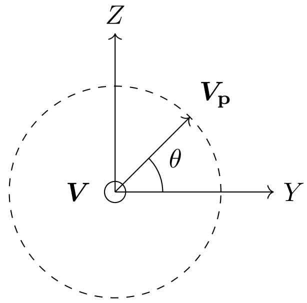


Figure 4: Definition of θ.

Now that the different types of uncertainty taken into account in this case have been defined, they can be propagated for each operational phases in order to determine whether or not the global state vector uncertainty distribution stays Gaussian after one day.

3. REFERENCE RESULTS

The propagation of the state vector uncertainty is studied for the post-launch and station-keeping phases, which are especially interesting due to the high number of electrical maneuvers executed during the period of interest. During these phases, it is necessary to assess collision risk probabilities with other orbiting objects in order to anticipate eventual avoidance maneuvers, which requires propagating the state vector uncertainty over the timespan of each phase. In this case, it would be convenient to use the State Transition Matrices to model the uncertainty propagation and compute collision risks. Given that the application of this method depends on the Gaussianity of the distribution, reference results are necessary to determine the nature of the uncertainty distribution after one day of propagation, and whether or not STM are suitable to represent it. The method selected to provide reference results for the uncertainty propagation is Monte Carlo simulations. It consists in representing every uncertainty as a large number of perturbed states, and propagate these states to obtain the final uncertainty of the object. For both phases, the state vector and maneuvers uncertainties are sampled in 10^5 perturbed states and propagated over one day in the QSW local orbital frame. Figures 5 and 6 show the perturbed samples position in QSW coordinates (in meter) from the final distribution of the early orbit and station-keeping phases.

The normality of the final uncertainty distribution is assessed using the normality test introduced by Henze-Zirkler [2]. In this case, multiple Henze-Zirkler tests are conducted on randomly selected sub-groups of 5000 samples, and the normality of the final distribution is determined by the percentage of groups that were designated

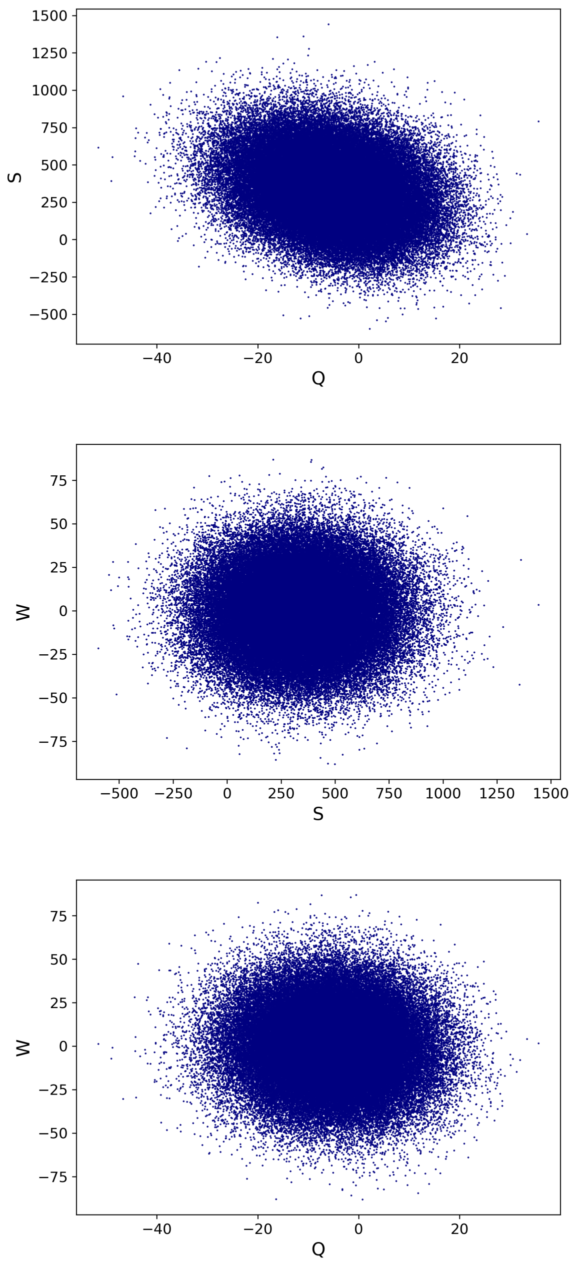


Figure 5: Uncertainties propagated for one day with the Monte Carlo method in QSW (early orbit phase).

as Gaussian according to the Henze-Zirkler test. A high number of groups tested is requested to get an accurate percentage – in this case, the percentage converges for approximately 4000 groups selected. From experience, in the context of this study, the final uncertainty distribution can safely be considered Gaussian if the percentage reaches 85 %. Otherwise, a visual verification is necessary to determine the normality of the final uncertainty. For example, a Gaussian distribution of samples can be defined by its ellipsoid shape, like in Figure 5. After running Henze-Zirkler tests on the final uncertainty in the QSW local orbital frame, the percentage of tests giving a positive result for the Gaussian distribution hypothesis is 90.4 % for the early orbit phase, and 90.6 % for the station-keeping phase. Thus, for both phases, the state vector uncertainty distribution is Gaussian at one day of propagation, and STM should be suitable to model the final distribution. It should be noted that only one date has been tested for Gaussian distribution. The Gaussianity of the state vector final uncertainty doesn't necessarily imply that this uncertainty distribution remains Gaussian at all times between the initial and final date of propagation.

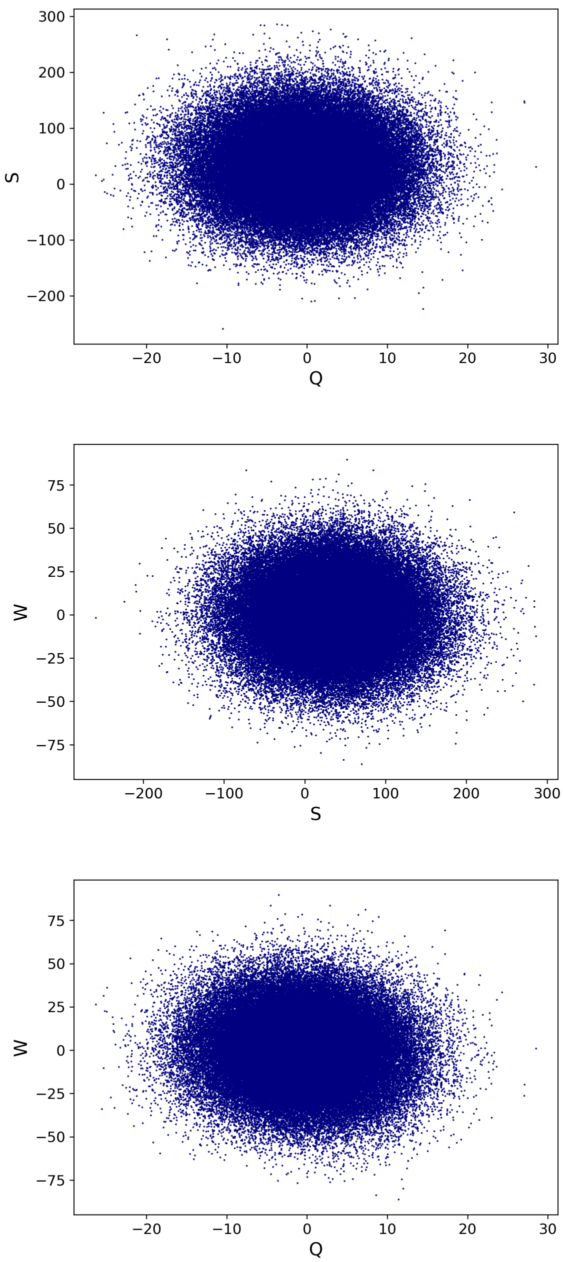


Figure 6: Uncertainties propagated for one day with the Monte Carlo method in QSW (station-keeping phase).

4. MANEUVER UNCERTAINTIES MODELING

In order to use the STM to represent the state vector uncertainty propagation, all uncertainties need to be modeled as matrices. For the rest of this paper, vectors will be written in bold letters (e.g. $\overrightarrow{V} = \boldsymbol{V}$) for ease of reading. The initial state vector uncertainty matrix is defined thanks to the standard deviation vector and correlation matrix given in Section 2. For the maneuver uncertainties, we start from the initial formula proposed by Gates in a technical report from 1963 [1]:

$$\begin{aligned} L &= \mathrm{Var}\left[\boldsymbol{V_\mathrm{p}}\right] \\ &= \mathrm{E}\left[\boldsymbol{V_\mathrm{p}}\boldsymbol{V_\mathrm{p}^{T}}\right] - \mathrm{E}\left[\boldsymbol{V_\mathrm{p}}\right]\mathrm{E}\left[\boldsymbol{V_\mathrm{p}^{T}}\right] \end{aligned}$$

with $\boldsymbol{V_\mathrm{p}}$ the perturbed maneuver vector, and $\mathrm{E}\left[X\right]$ and $\mathrm{Var}\left[X\right]$ the expected value and variance of a variable X. The modeling of maneuver errors following this formula in Gates' report is not adapted to this case, as it is not compatible with the high direction error value considered ($\sigma_\mathrm{d} = 5°$). A different approach will be presented here.

Considering a random variable m from a normal distribution $\mathcal{N}(0, \sigma_\mathrm{m})$, which represents the magnitude error (see Figure 2). The maneuver vector $\boldsymbol{V_\mathrm{p}^\mathrm{m}}$ perturbed by maneuver errors can be formulated as:

$$\boldsymbol{V_\mathrm{p}^\mathrm{m}} = (1 + m)\boldsymbol{V} \tag{1}$$

Considering now two variables α and θ to represent the direction error in 3 dimensions, respectively from a normal distribution law $\mathcal{N}(0, \sigma_\mathrm{d})$ and a uniform distribution law $\mathcal{U}(0, \pi)$ (details in Section 2, and Figures 3 and 4). In the maneuver frame, the maneuver vector $\boldsymbol{V_\mathrm{p}^\mathrm{d}}$ subject to direction error is:

$$\boldsymbol{V}_{\mathbf{p}}^{\mathbf{d}} = \|\boldsymbol{V}\| \begin{bmatrix} \cos\alpha \\ \sin\alpha\cos\theta \\ \sin\alpha\sin\theta \end{bmatrix} \tag{2}$$

When combining magnitude and direction errors, the perturbed maneuver can thus be written as:

$$\boldsymbol{V}_{\mathbf{p}} = (1+m)\|\boldsymbol{V}\| \begin{bmatrix} \cos\alpha \\ \sin\alpha\cos\theta \\ \sin\alpha\sin\theta \end{bmatrix}$$

Knowing that the expected value of a vector is the vector of expected values of its components, the expected value of $\boldsymbol{V}_{\mathbf{p}}$ can be rewritten as:

$$\mathrm{E}\left[\boldsymbol{V}_{\mathbf{p}}\right] = \|\boldsymbol{V}\| \begin{bmatrix} \mathrm{E}\left[(1+m)\mathrm{c}_\alpha\right] \\ \mathrm{E}\left[(1+m)\mathrm{s}_\alpha\mathrm{c}_\theta\right] \\ \mathrm{E}\left[(1+m)\mathrm{s}_\alpha\mathrm{s}_\theta\right] \end{bmatrix}$$

with $\mathrm{c}_x = \cos x$ and $\mathrm{s}_x = \sin x$.

Moreover, if X and Y are two independent variables, the expected value behaves as $\mathrm{E}\left[X+Y\right] = \mathrm{E}\left[X\right] + \mathrm{E}\left[Y\right]$ and $\mathrm{E}\left[XY\right] = \mathrm{E}\left[X\right]\mathrm{E}\left[Y\right]$, which leads to:

$$\mathrm{E}\left[\boldsymbol{V}_{\mathbf{p}}\right] = \|\boldsymbol{V}\| \begin{bmatrix} \mathrm{E}\left[\mathrm{c}_\alpha\right] \\ \mathrm{E}\left[\mathrm{s}_\alpha\right]\mathrm{E}\left[\mathrm{c}_\theta\right] \\ \mathrm{E}\left[\mathrm{s}_\alpha\right]\mathrm{E}\left[\mathrm{s}_\theta\right] \end{bmatrix}$$

knowing that $\mathrm{E}\left[m\right] = 0$ since $m \in \mathcal{N}(0, \sigma_\mathrm{m})$.

Considering the formula $\mathrm{e}^{i\alpha} = \cos\alpha + i\sin\alpha$, and that if $\alpha \in \mathcal{N}(0, \sigma_\mathrm{d})$:

$$\mathrm{E}\left[\mathrm{e}^{i\alpha}\right] = \mathrm{e}^{\frac{-\sigma_\mathrm{d}^2}{2}}$$

then the expected value of the cosine and sine of α are:

$$\begin{aligned} &\mathrm{E}\left[\mathrm{e}^{i\alpha}\right] = \mathrm{E}\left[\cos\alpha\right] + i\mathrm{E}\left[\sin\alpha\right] \\ &\mathrm{E}\left[\cos\alpha\right] = \mathrm{e}^{\frac{-\sigma_\mathrm{d}^2}{2}} \\ &\mathrm{E}\left[\sin\alpha\right] = 0 \end{aligned}$$

Finally, the expected value of the dispersed vector $\boldsymbol{V}_{\mathbf{p}}$ can be expressed as:

$$\mathrm{E}\left[\boldsymbol{V}_{\mathbf{p}}\right] = \|\boldsymbol{V}\| \begin{bmatrix} \mathrm{e}^{\frac{-\sigma_\mathrm{d}^2}{2}} \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

and the product of the expected values of $\boldsymbol{V}_{\mathbf{p}}$ and $\boldsymbol{V}_{\mathbf{p}}^T$ is:

$$\mathrm{E}\left[\boldsymbol{V}_{\mathbf{p}}\right]\mathrm{E}\left[\boldsymbol{V}_{\mathbf{p}}^T\right] = \|\boldsymbol{V}\|^2 \begin{bmatrix} \mathrm{e}^{-\sigma_\mathrm{d}^2} & 0 & 0 \\ - & 0 & 0 \\ - & - & 0 \end{bmatrix}$$

The expected value of the product of the vectors $\boldsymbol{V}_{\mathbf{p}}$ and $\boldsymbol{V}_{\mathbf{p}}^T$ is formulated as:

$$\mathrm{E}\left[\boldsymbol{V}_{\mathbf{p}}\boldsymbol{V}_{\mathbf{p}}^T\right] = M \begin{bmatrix} \mathrm{E}\left[\mathrm{c}_\alpha^2\right] & \mathrm{E}\left[\mathrm{c}_\alpha\mathrm{s}_\alpha\mathrm{c}_\theta\right] & \mathrm{E}\left[\mathrm{c}_\alpha\mathrm{s}_\alpha\mathrm{s}_\theta\right] \\ - & \mathrm{E}\left[\mathrm{s}_\alpha^2\mathrm{c}_\theta^2\right] & \mathrm{E}\left[\mathrm{s}_\alpha^2\mathrm{c}_\theta\mathrm{s}_\theta\right] \\ - & - & \mathrm{E}\left[\mathrm{s}_\alpha^2\mathrm{s}_\theta^2\right] \end{bmatrix}$$

with $M = \mathrm{E}\left[(1+m)^2\right]\|\boldsymbol{V}\|^2$.

Since $\theta \in \mathcal{U}(0, \pi)$, the expected values of the cosine and sine of this variable follow $\mathrm{E}\left[\cos\theta\sin\theta\right] = 0$, and $\mathrm{E}\left[\cos^2\theta\right] = \mathrm{E}\left[\sin^2\theta\right] = \frac{1}{2}$. With the same argumentation as before, and with:

$$\begin{aligned} \mathrm{E}\left[\cos\alpha\sin\alpha\right] &= 0 \\ \mathrm{E}\left[\cos^2\alpha\right] &= \frac{1}{2}\left(1 + \mathrm{e}^{-2\sigma_\mathrm{d}^2}\right) \\ \mathrm{E}\left[\sin^2\alpha\right] &= \frac{1}{2}\left(1 - \mathrm{e}^{-2\sigma_\mathrm{d}^2}\right) \end{aligned}$$

this expression can be simplified to:

$$\mathrm{E}\left[\boldsymbol{V}_{\mathbf{p}}\boldsymbol{V}_{\mathbf{p}}^T\right] = N \begin{bmatrix} 2(1+P^2) & 0 & 0 \\ - & (1-P^2) & 0 \\ - & - & (1-P^2) \end{bmatrix}$$

with $N = \frac{1}{4}(1+\sigma_\mathrm{m}^2)\|\boldsymbol{V}\|^2$ and $P = \mathrm{e}^{-\sigma_\mathrm{d}^2}$.

The matrix representing the combined maneuver errors in the maneuver frame can finally be defined as:

$$L = \begin{bmatrix} 2N(1+P^2) - P & 0 & 0 \\ - & N(1-P^2) & 0 \\ - & - & N(1-P^2) \end{bmatrix}$$

This matrix is added to propagated uncertainty at the mid-thrust date, simulating a contribution as an impulse maneuver (unlike the Monte Carlo simulations which take into account the fact that the maneuvers are continuous). Tests have been run on the Monte Carlo results to confirm that, in this case, the maneuver type has no significant impact on the overall distribution, but the proper inclusion of the maneuver errors matrix in the STM propagation remains an area for improvement. The state vector uncertainty propagated by STM with contribution of the maneuver errors for the early orbit phase is shown in Figure 7. The comparison with the reference results shows that the Gaussianity of the uncertainty distribution is not a sufficient condition for STM results to accurately match the reference results. The sample distributions are similar, but a slight offset from the Monte Carlo results can be observed. The same phenomena occurs for the station-keeping phase, with a smaller shift between the distributions.

5. NOMINAL STATE VECTOR SHIFT

The offset phenomena observed in both operational phases is due to the maneuver direction error. Indeed,

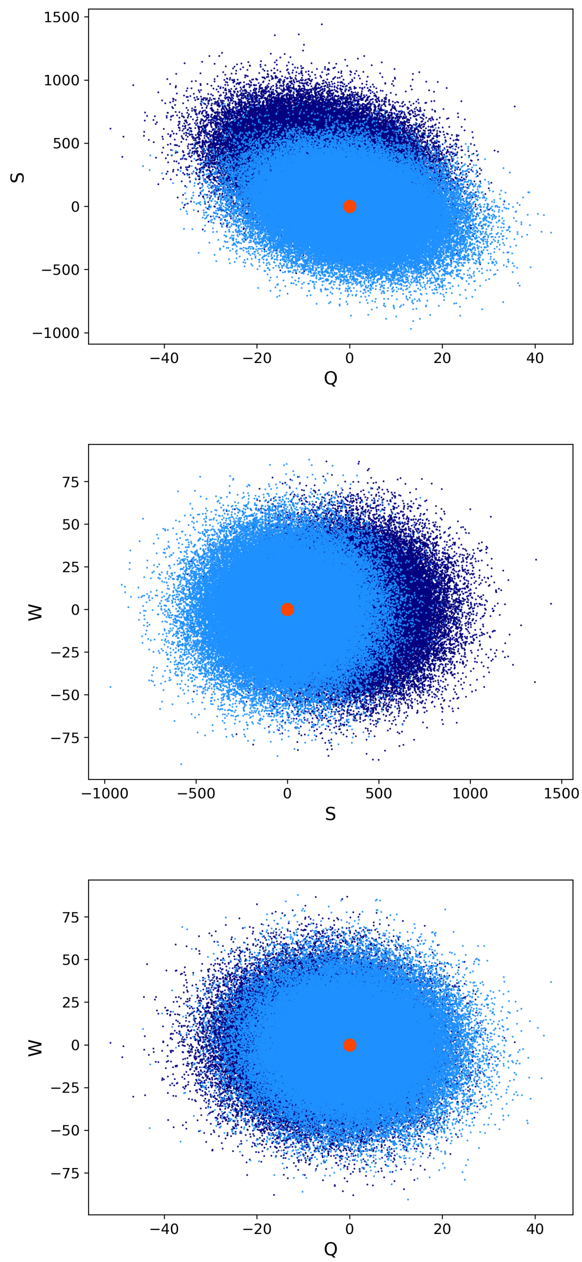


Figure 7: Uncertainties propagated for one day with Monte Carlo (dark blue) and STM (light blue) methods in QSW (early orbit phase).

considering the formulas of the perturbed maneuver vector from magnitude error (Equation 1) and from direction error (Equation 2), the average perturbation on the maneuver vector for each error are:

$$\boldsymbol{V} - \mathrm{E}\left[\mathbf{V_p^m}\right] = \mathbf{0}$$

$$\boldsymbol{V} - \mathrm{E}\left[\mathbf{V_p^d}\right] = \|\boldsymbol{V}\| \begin{bmatrix} 1 - \mathrm{e}^{\frac{-\sigma_\mathrm{d}^2}{2}} \\ 0 \\ 0 \end{bmatrix}$$

Note that the average perturbation caused by the direction error on the maneuver vector is not zero when $\sigma_\mathrm{d} \neq 0$, which explains why the state vector propagated with no perturbation does not correspond to the average of the Monte Carlo perturbed samples. By nature, the uncertainty propagated through the STM method is centered on the unperturbed state vector, meaning that, in this case, it cannot be used for collision risk assessment since it doesn't properly represent the uncertainty distribution.

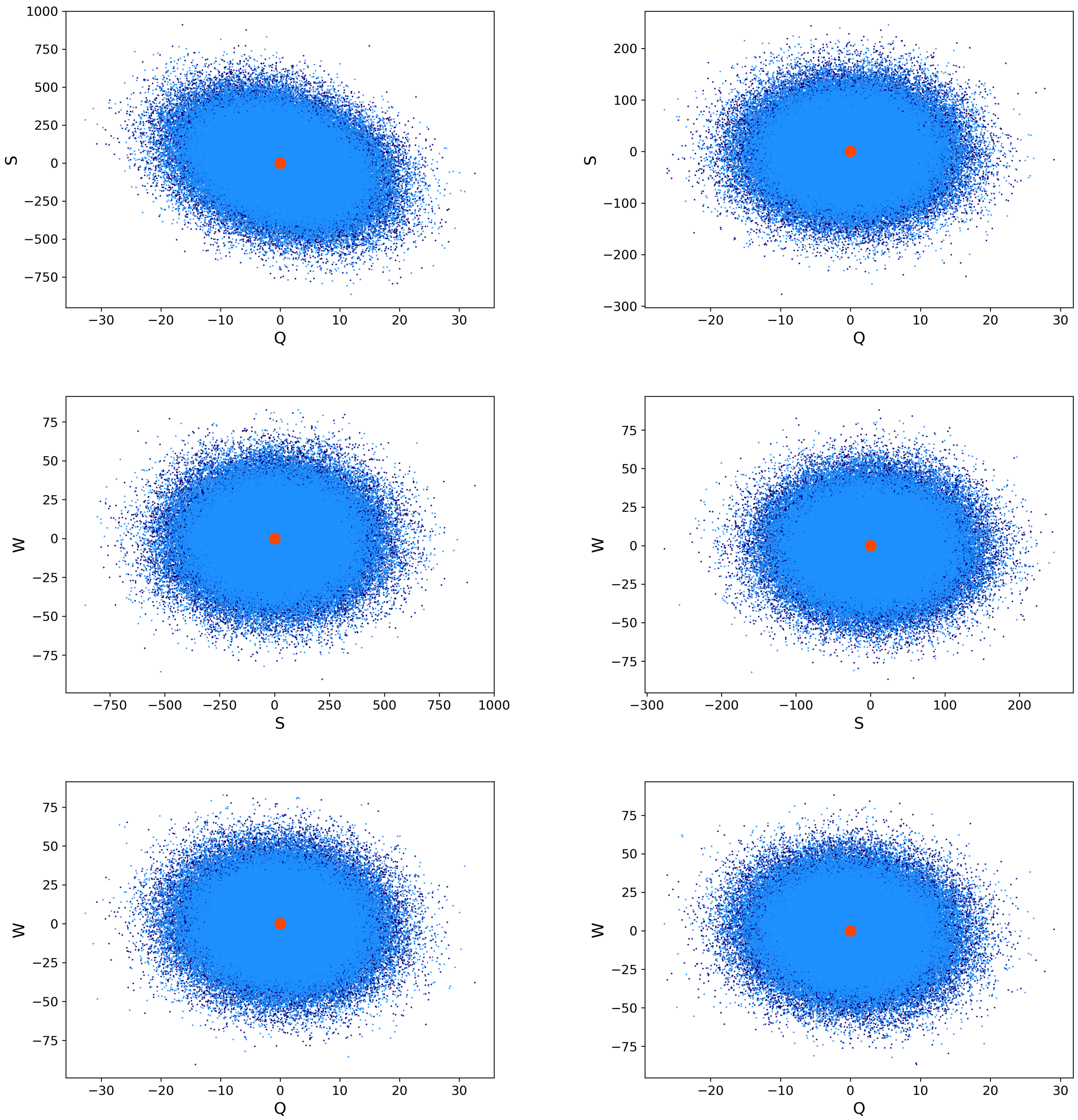
To counter this problem, the idea is to create a *shifted* nominal state vector, centered in the Monte Carlo particules cloud at all time during the propagation. The shifted nominal state vector is propagated with maneuvers taking into account the average perturbation (that is, with a thrust vector equal to $\mathrm{E}\left[\boldsymbol{V}_\mathbf{p}\right]$, see Equation 3), instead of the planned maneuvers. Figure 8 shows the results of STM propagation combined with the shifted nominal state vector: the propagated samples match the Monte Carlo simulations in QSW.

With the correct modeling of maneuver errors, and with the use of the *shifted* nominal state vector, results yielded by the STM propagation are consistent with the reference results and can be used for computations performed during operations, like collision risk assessment.

6. ADDITIONAL ANALYSIS

In case the state vector uncertainty needs to be propagated for more than one day (e.g. if orbit restitution is not available every day), its evolution is analyzed every day up to 7 days. The reference results of this propagation are presented on Figure 10. After 7 days of propagation, Henze-Zirkler tests are run on the final uncertainty distribution of both operational phases. For the station-keeping phase, the Gaussianity of the state vector uncertainty is maintained after 7 days, as shown on Figure 9b. However, due to a higher number of maneuvers and perturbations, the uncertainty distribution for the early orbit phase looses its Gaussianity at 4 days of propagation.

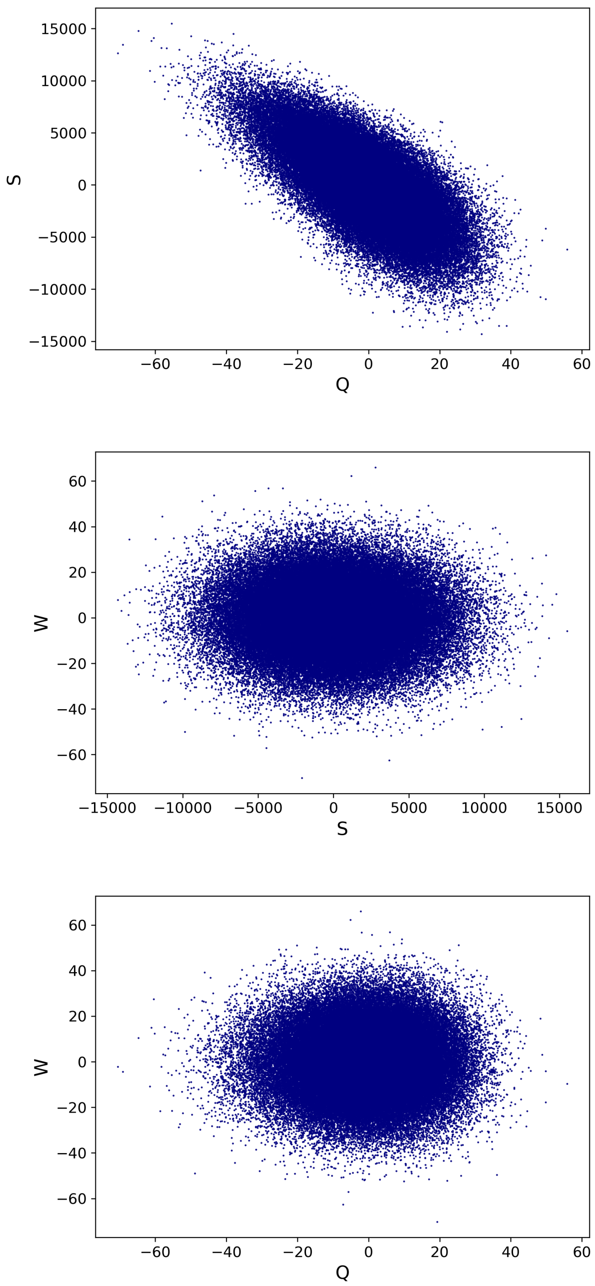
As long as the uncertainty distribution remains Gaussian, and with the use of the *shifted* nominal state vector, the STM propagation is suitable represent the uncertainty and compute collision risk probabilities (results are shown on Figure 10). As mentioned in Section 3, the Gaussianity of the state vector uncertainty distribution has been tested for only a few dates (each day for



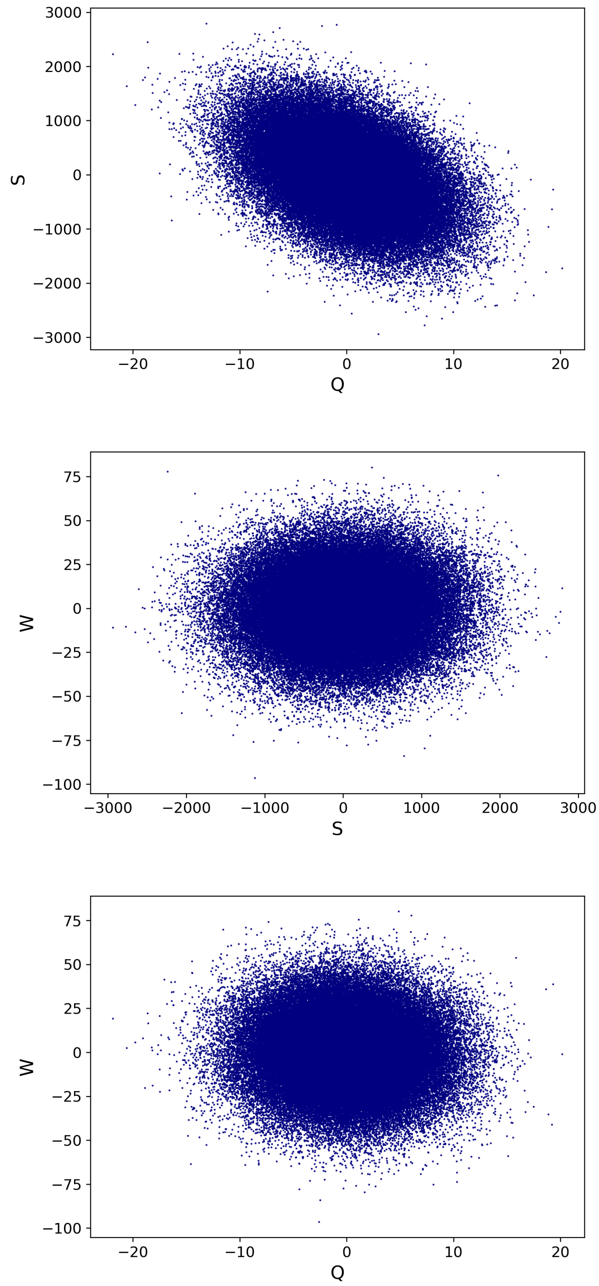
(a) Early orbit phase.

(b) Station-keeping phase.

Figure 8: Uncertainties propagated for one day with Monte Carlo (dark blue) and STM (light blue) methods and *shifted* nominal position (orange) in QSW.



(a) Early orbit phase.



(b) Station-keeping phase.

Figure 9: Uncertainties propagated for 7 days with Monte Carlo method in QSW.

7 days). The fact that the distribution can be considered Gaussian for two consecutive dates doesn't necessarily mean that it is Gaussian at all times between these dates.

7. CONCLUSION

Although results presented here are specific to this particular study, they show the state vector uncertainty propagation with STM in Cartesian space should be used with caution, especially when maneuver uncertainties are considered. It is necessary to ensure the modeled uncertainty distribution remains Gaussian, but not sufficient: particular attention should be paid to the representation of maneuver errors and the propagation of the nominal state vector. Otherwise, the accumulation of maneuver uncertainties causes the STM propagated uncertainty to depart from the reference results, which is corrected here by the use of the *shifted* nominal state vector, taking into account the average perturbation of maneuvers.

Future work will consist in extending the Gaussianity analysis to the complete period of interest, instead of testing single dates as presented here. Indeed, in order to use the STM to perform collision risk computations at all times during the propagation, it is essential to verify that the modeled uncertainty distribution stays Gaussian during the whole timespan.

8. ACKNOWLEDGMENTS

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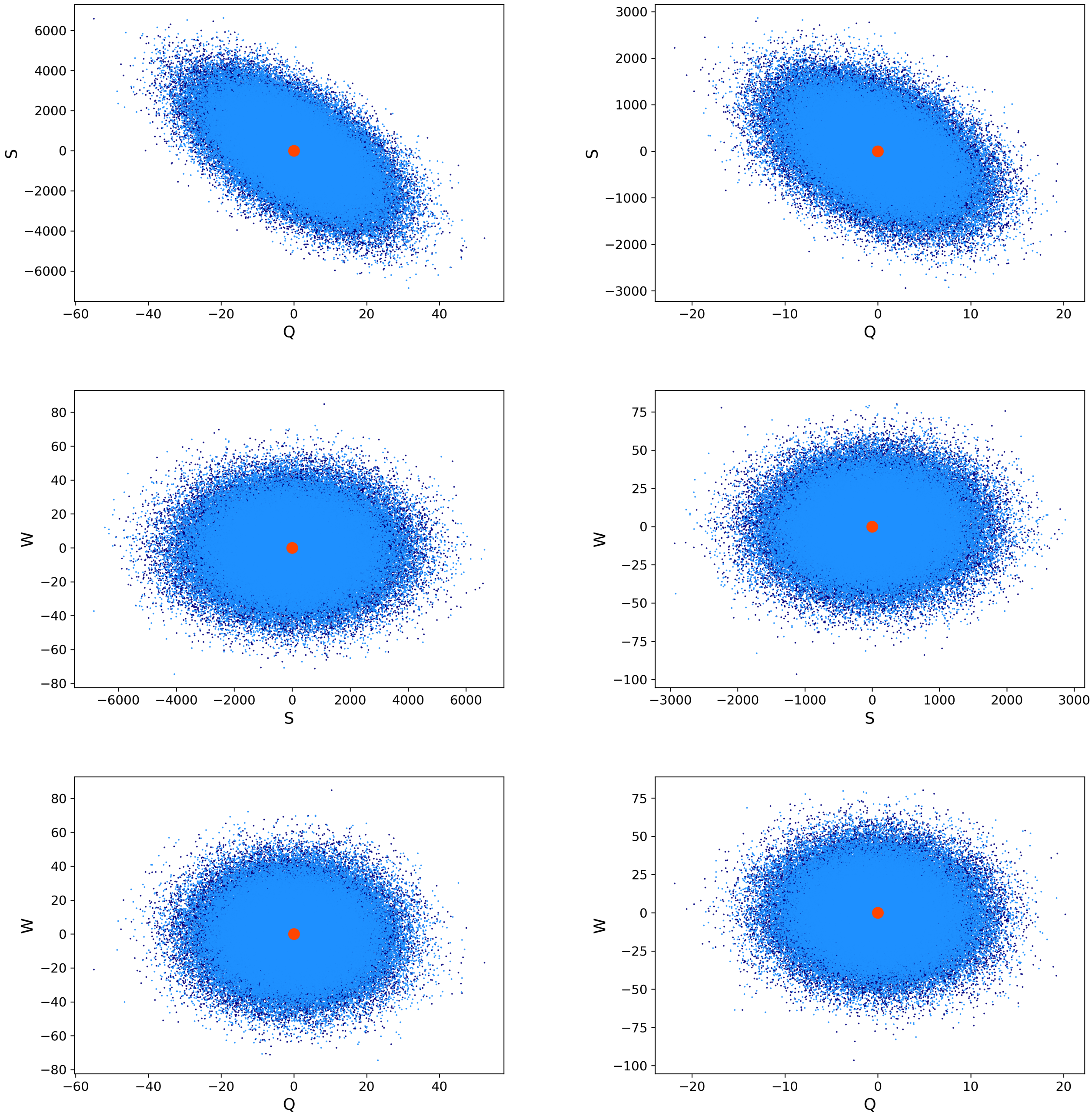

(a) Early orbit phase (4 days propagation).

(b) Station-keeping phase (7 days propagation).

Figure 10: Uncertainties propagated with Monte Carlo (dark blue) and STM (light blue) methods and *shifted* nominal position (orange) in QSW.