AN OPEN-SOURCE SOLUTION FOR TLE BASED ORBIT DETERMINATION

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ABSTRACT

Earth orbital space suffers from the ever increasing count of space objects, including operational satellites and space debris. Space system operations rely on the management of vast catalogs of objects to avoid any damaging collision. NORAD (North American Aerospace Defense Command) and NASA (National Aeronautics and Space Administration) both maintain a database for a large quantity of orbiting objects. Data are stored as Two Line Elements (TLE) and used along with specific analytical propagation models. Operation centers need Orbit Determination methods to accurately compute conjunctions and collision probabilities. With more and more flying objects, computations must be fast enough to ensure satellite safety. Mixing Orbit Determination and TLE analytical propagation models appears to be an effective way to grant security in space. This paper presents an open-source solution for an Orbit Determination method based on TLE propagation models. The method was implemented and validated inside the Orekit space mechanic library. It was then confronted with a classical numerical Orbit Determination on a GNSS test case.

Keywords: Orbit Determination; TLE; Open Source; Orekit; Automatic Differenciation; Analytical Orbit Propagation.

1. INTRODUCTION

Orbit Determination is a technique used to estimate the state vector of a space object from a first guess and a set of observable measurements. The state vector contains the orbital elements, the propagation dynamic parameters and the measurement biases. Determined orbit is meant to be as accurate as possible and accessible within the shortest computation time. Those two paradigms become even more challenging with the ever increasing number of space objects orbiting the Earth. Orbit Determination is especially necessary for debris tracking and collision probability computation [1]. Thus, this technique shall be able to quickly take care of large object collections.

Numerical Orbit Determination is widely used. It needs a numerical orbit propagator which accurately computes the orbital perturbations on a space object by equation of motion numerical integration. It reaches significant precision level with realistic force models, but requires high computation time. Analytical orbit propagators rather sacrifice accuracy to benefit computation speed. They employ a set of empirical equations for modeling a space object dynamic instead of a numerical integration of the equation of motion. This kind of orbit propagators can be adapted to be employed in an Orbit Determination process. This kind of application is suitable to address space surveillance topics, where fast orbit estimation is required.

TLE data are a widespread way to represent an orbit. They are generated and freely released by the NASA and the NORAD for each Earth orbiting object bigger than a softball ball. A TLE is a set of mean orbital elements that locates an object in space with a few kilometer-accuracy [2]. Even though quite imprecise, they remain deployed in myriad space mechanic applications [3] [4]. This kind of data also requires specific propagation models to be used properly. The realization of an Orbit Determination application based on TLE orbit propagators looks promising. It was chosen to implement the process with both the Simplified General Perturbation 4 (SGP4) and the Simplified Deep-space Perturbation 4 (SDP4) orbit propagation algorithms. Those analytical propagation models are empirical and only take into account the main orbital perturbations as presented in Figure 1.

A cornerstone of an Orbit Determination algorithm development is the state transition matrix computation. This mathematical object, regrouping state vector partial derivatives, requires an accurate computation and validation. It becomes important for short determination arcs and very accurate observations. Automatic differentiation is able to compute those partial derivatives by applying chain rule. This method prevents the complex task to establish and validate all model derivatives. State transition matrix terms are computed directly from model evolution equations, instead of creating a new differential equation to integrate as it is done in Orekit numerical or semi-analytical orbit determination [5] [6]. Moreover, automatic differentiation is used to create SGP4/SDP4 compliant TLEs from a state vector.

We tested the analytical orbit determination performances

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using real GPS satellite data. A network of six IGS (International GNSS Service) stations is used. Only pseudo range measurements were considered. Estimated orbits were compared with IGS precise products. Station reference positions were retrieved from SINEX file. We also compared the analytical orbit determination method with the Orekit's numerical method. Final estimated orbit accuracy and computation time were investigated. It appeared that computation time is slightly increased with analytical method while determined position accuracy on the orbit shares the same the magnitude order.

This paper presents the development and validation of the TLE orbit determination in the Orekit flight dynamics library. It also demonstrates Orekit capability to build improved TLE with an accuracy of a hundred of meters instead of a kilometer. This promotes developing additional analytical propagation models in order to acquire greater diversity within the open-source library.

2. THE OREKIT TLE ANALYTICAL ORBIT DETERMINATION PROCESS

Orekit is an open-source space flight dynamics library [7]. It is written in Java and provides low level elements for the development of flight dynamics applications. It was first developed by CS GROUP in 2002 as a private library for the company collaborators. In 2008, the library evolved towards an open-source project under Apache v2.0 License [8]. Orekit is now used worldwide, both by academics and industries, to realize space applications, studies and operations.

2.1. The TLE SGP4 and SDP4 propagation algorithms

TLE format was created in the 60s, it gathers mean orbital parameters of a space object. Though, it requires specific algorithms to be analysed and then propagated [9]. A collection of analytical orbit propagators were developed along with this format, seeking fast computations of space trajectories. These models were named Simplified General Perturbations (SGP). Among them, SGP4 and SDP4 are mainly used for TLE manipulations. SGP4 is used for low Earth orbit propagation while SDP4 handles further space objects with terrestrial orbit. If orbit period is below 225 minutes, the low orbit model is to be applied, otherwise, the SDP4 is. These propagation algorithms consider the main perturbation influences on a satellite: first four zonal Earth gravity field harmonics, atmospheric drag and solar radiation pressure. SDP4 adds luni-solar gravity attraction and deep space secular effects. Perturbation models are summed up in Figure 1.

For a given object, TLEs are generated periodically. This refreshment rate along with the use of SGP4 and SDP4 enables to estimate the position of a space object within a kilometer magnitude precision [2]. The aim of the study was to improve the accuracy of this well spread data format, in order to increase its scope.

It is to be noted that atmospheric drag model takes into account a special coefficient called B*. The B* represents the ballistic coefficient of the considered space object. Drag coefficient is usually estimated during Orbit Determination process. Thus, estimating the B* coefficient may also improve a TLE.

TLE analytical Orbit Determination will then aim at estimating the six TLE coefficients that express the orbital state: mean motion, eccentricity, inclination, longitude of the ascending node, argument of periapsis and mean anomaly, in addition of the B*.

2.2. The Batch Least Squares Orbit Determination

The Batch Least Squares algorithm is a classical technique used for operational orbit determination. For a given satellite initial state Y_{t_0} and for an available observation arc, the Batch Least Squares algorithm provides an estimation of the satellite's state such as

$$\hat{Y}_{t_0} = Y_{t_0} + \delta y_0 \qquad (1)$$

The calculation of Equation 1 is done by an iterative process solving the non-linear Equation 2 [10]

$$\delta y_0 = (A^T W A)^{-1} A^T W b \qquad (2)$$

where

A	:	the partial derivatives matrix
W	:	the weighting matrix
b	:	the residual vector

The weighting matrix is initialized, at the beginning of the estimation, by the user. The residual vector is computed, for each measurement, by the difference between the observed and the estimated measurements. Finally, the partial derivatives matrix can be expressed by the product of the observation matrix H by the state transition matrix Φ

$$A = H_{t,t}.\Phi_{t,t_0} \qquad (3)$$

where

$$H_{t,t} = \frac{\partial \rho_t}{\partial Y_t} \qquad (4)$$

$$\Phi_{t,t_0} = \frac{\partial Y_t}{\partial Y_{t_0}} \qquad (5)$$

where ρ_t is an observed measurement at an arbitrary epoch t. In Orekit library, both the observation matrix and the state transition matrix are calculated using the automatic differentiation technique, which is detailed in the next section.

2.3. Computing the state transition matrix with automatic differentiation

Automatic differentiation is a set of techniques to avoid the analytical calculation of the derivatives of long equations. It relies on the fact that every computer program is decomposed as a sequence of elementary arithmetic operations (i.e. addition, subtraction, etc.), elementary functions (i.e; exp, sin, cos, etc.), and control flow statements [11]. The calculation of the derivatives is accurate to the precision of the computer system. For instance, if Y_i denotes an orbital element (e.g. the eccentricity of the orbit), automatic differentiation gives all Y_i derivatives with respect to any parameter (i.e. orbital, dynamic, or measurement parameters) by using only the analytical expression of Y_i. The partial derivatives are stored in an array where the first element is the value of the parameter and the other elements are its partial derivatives, as represented in Equation 6.

$$[Y_i \quad \partial Y_i/\partial Y_1 \quad \partial Y_i/\partial Y_2 \quad \cdots \quad \partial Y_i/\partial Y_6] \tag{6}$$

As a result, automatic differentiation is used to automatically differentiate all the simplified equations of SGP4 and SDP4 algorithms in order to build the state transition matrix needed by the orbit determination process. Automatic differentiation is also used to calculate the observation matrix H. Indeed, during the measurements estimation, the partial derivatives are calculated simultaneously with the value of the measurements thanks to automatic differentiation technique.

3. TLE ORBIT DETERMINATION RESULTS

Once the method is implemented, it remains to be tested against real data. GNSS (Global Navigation Satellite System) satellites are interesting because of the availability of measurements and precise ephemeris. Data are freely provided by the IGS. A satellite was chosen to simulate a full orbit determination test case, with wisely selected measurement stations along orbit ground track. Then, a robustness test was performed on several GNSS satellites, with more or less degraded conditions.

3.1. GPS IIR-M 6 test

In order to validate the process on a real operational case, satellite GNSS IIR-M 6, NORAD ID 32711, is studied. Considered measurement arc and stations are given on Figure 2. First guess of the Orbit Determination process is the following *Space-Track.org* TLE:

```
1 32711U 08012A 16044.40566026 -.00000039 00000-0 00000+0 0 9991
2 32711 55.4362 301.3402 0091577 207.7302 151.8353 2.00563580 58013
```

IGS SP3 high precision ephemeris are considered as the reference for the spacecraft position [12]. The measurements are also taken from IGS products. They allowed to gather 8211 pseudo-range measurements on a 5h30m arc [13].

Three configurations are tested. Firstly, only the six orbital parameters are estimated during the Orbit Determination process. Then, the B* coefficient is added to the set of estimated parameters. A more classical numerical Orbit Determination is finally performed for comparison. The force models of the latter are fitted on SDP4. During the computation, drag coefficient is also estimated. In all three cases, ionospheric and tropospheric pseudo range biases are also estimated [14].
The analytical method computations return two new TLEs, that can be immediately propagated by usual SGP4/SDP4 algorithms. B* is not estimated to produce the first, however, it is in the second. They are given thereafter:

```
1 32711U 08012A 16044.40566026 -.00000039 00000-0 00000-0 0 9992
2 32711 55.4358 301.3401 0091593 207.7154 151.8480 2.00564373 58012
```

```
1 32711U 08012A 16044.40566026 -.00000039 00000-0 -13229+5 0 9994
2 32711 55.4358 301.3400 0091600 207.7008 151.8629 2.00564891 58018
```

The process clearly evolved orbital parameters. When estimated, the B* coefficient value seems far too high. It is usually few 10^{-1} at most against 10^4 in this case. Propagation will show either if this estimation is appropriate or overestimated.

Relevant figures about the outputs of the different runs are provided in Table 1. This test highlights that analytical TLE Orbit Determination provides a significant enhancement for the spacecraft position knowledge compared to the initial TLE. Indeed, estimated TLEs are about an order of magnitude more accurate than the original.

Measurement residuals are presented in Figure 3. As expected, dispersion is greater with SDP4 usage, yet, the calculations still manage to oncentrate the residuals around zero. This reflects the correct behavior of the Batch Least Square while using analytical propagation. Analytical method remains less accurate than the numerical. However, with respect to the batch least square number of state evaluations, the analytical Orbit Determination is faster than the numerical.

Stability of the algorithm is assessed studying Root Mean Square error along the Orbit Determination steps. They are given for all the three runs in Figure 4. Indeed, RMS evolution is regular, and does not present any monotony variation. The orbital state clearly always evolve toward a better estimation of the measured object.

To complete the study and ensure that the new TLEs are truly closer to real orbit, propagation comparison is performed. Once again, the reference orbit is taken from IGS SP3 ephemeris. Propagation does not exceed the 13th of February 2016 because of discontinuities between two SP3 files [15]. Results are presented in Figure 5. Clearly, orbit knowledge is improved not only at the Orbit Determination epoch, but on 20-hour time span. Considered that a TLE is usually generated every orbit, *i.e.* about 12 hours in a GNSS case, the satellite position accuracy can always be improved. Minimum position error is achieved at median measurement epoch, it reaches few ten meters against about 600 meters for the original TLE.

3.2. Expansion and robustness

A final robustness test is performed. It consists in several TLEs analytical Orbit Determinations on different GPS satellites, using the same set of measurement stations and arc than the GPS IIR-M 6 case. Therefore, measurements may not be suited for all orbits. The aim is to appreciate how stiff the process is while dealing with data of various quality. In all cases, it is chosen not to estimate the B* coefficient but still tropospheric and ionospheric pseudo range biases. Results are provided in Table 2. In a nutshell, 9 out of 14 test cases show a position accuracy enhancement after TLE Orbit Determination is performed. Most part of the improvements are beyond 50% gain in position precision. Failing cases correspond to degraded Orbit Determination conditions. Indeed, either the computation epoch is out of measurement arc (G09, G19), or the number of measurements is quite low (G15, G22). However, the algorithm still works out for some cases with same issues, such as G02 or G06. It is not obvious to conclude about the robustness, and precisely characterize how inputs shall be to perform the analytical TLE Orbit Determination. Still, the method perfectly acts with regular Orbit Determination conditions, *i.e.* significant number of measurements well spread around computation time.

4. CONCLUSION

The presented tests demonstrate that the analytical Orbit Determination based on the SDP4 model truly improves the accuracy of public TLE. Witnessed enhancement is up to an order of magnitude on a GPS satellite position accuracy at Orbit Determination epoch, and even more when propagated. Moreover, the method produced a real TLE that can be used as any NASA and NORAD data. Orekit is even able to build a corresponding OMM (Orbit Mean Message) to address other user needs.

5. FUTURE WORK

Computation time is yet to be improved for the method to be useful for massive catalog management. As a matter of fact, the most part of the calculation time, with numerical or analytical method, is spent in frame conversion. They are performed for each measurement at each Batch Least Square step. This explains that with a lot of measurements, analytical method is hardly faster than numerical. Orekit community is about to solve this issue, then the previous test cases should be run again to experience the computation time gain.

Orbit Determination using an analytical propagator coupled with automatic differentiation still seems to be a promising method to manage huge space object catalogs. Implementing it successfully within the Orekit library with the TLE models encourages to extend the method to other analytical propagators. Though, Eckstein-Hechler or Brouwer-Lyddane models could also be used to perform fast and adapted Orbit Determinations. This work would enlarge Orekit capabilities in term of Orbit Determination and its applications.

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TABLES AND FIGURES

Table 1. TLE analytical and numerical Orbit Determination results on GPS IIR-M 6 case. Errors are given at Orbit Determination epoch, which is also first guess TLE time.

	Only orbital parameter estimation	B* and orbital parameter estimation	Numerical orbit determination
First guess error with respect to reference (m)	914.4	914.4	914.4
Result error with respect to reference (m)	111.2	130.9	64.2
Position improvement	87.2%	85.7%	93.0%
Residual mean (m)	3.52	2.20	-1.9e-3
Residual standard deviation (m)	7.23	3.95	1.10
State evaluation count	5	9	4
Computation time (s)	36.6	65.3	32.9

Table 2. TLE analytical and numerical Orbit Determination results on multiple GPS satellite cases. Errors are given at Orbit Determination epoch, which is also first guess TLE time. Green colored lines correspond to cases with position accuracy enhancement compared to the initial TLE. In red, Orbit Determination does not lead to a better position knowledge.

PRN Identifier	Orbit Determination Epoch	Number of measurements	Residual Mean (m)	Residual Standard Deviation (m)	Inital TLE Error wrt IGS sp3 Ephemris(m)	Determined TLE Error wrt IGS sp3 Ephemeris (m)	Position Accuracy Gain (%)
G01	13/02/2016 13:11:25	3010	1,5	15,9	299,5	597,8	0,0
G02	13/02/2016 05:41:31	3729	3,0	6,7	2990,1	918,0	69,3
G05	13/02/2016 09:06:51	3941	0,3	7,8	2279,6	510,7	77,6
G06	13/02/2016 05:21:37	6284	1,4	30,7	2864,6	1822,1	36,4
G07	13/02/2016 09:44:09	8211	3,5	7,2	914,4	111,2	87,8
G08	13/02/2016 11:36:49	4268	8,1	45,1	1903,6	1660,5	12,8
G09	13/02/2016 07:37:14	9892	90,0	201,5	2947,0	3784,0	0,0
G11	13/02/2016 14:31:22	1882	0,0	2,0	616,5	251,4	59,2
G15	13/02/2016 11:57:42	441	-0,1	0,9	3163,4	14902,9	0,0
G16	13/02/2016 16:40:54	5253	12,4	32,5	1683,6	1117,9	33,6
G17	13/02/2016 08:49:14	2560	0,0	7,4	2504,9	773,9	69,1
G19	13/02/2016 21:36:13	2317	0,0	5,4	563,2	2064,4	0,0
G20	13/02/2016 10:43:47	1215	0,4	2,5	1230,2	410,3	66,6
G22	13/02/2016 14:18:32	1246	-1,7	23,5	3201,9	8437,7	0,0

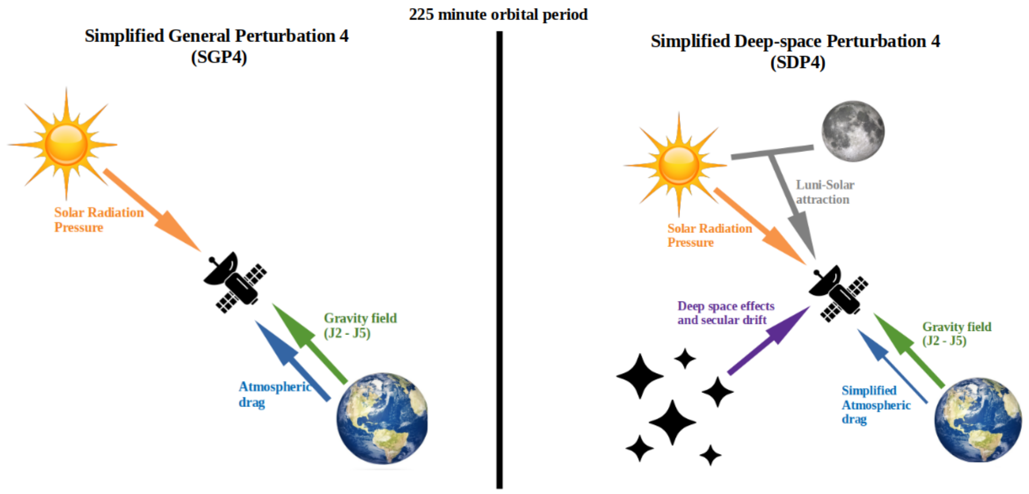

Figure 1. SGP4 and SDP4 analytical propagation force models. SGP4 is to be used when object orbital period is lower than 225 minutes. SDP4 should be applied either.

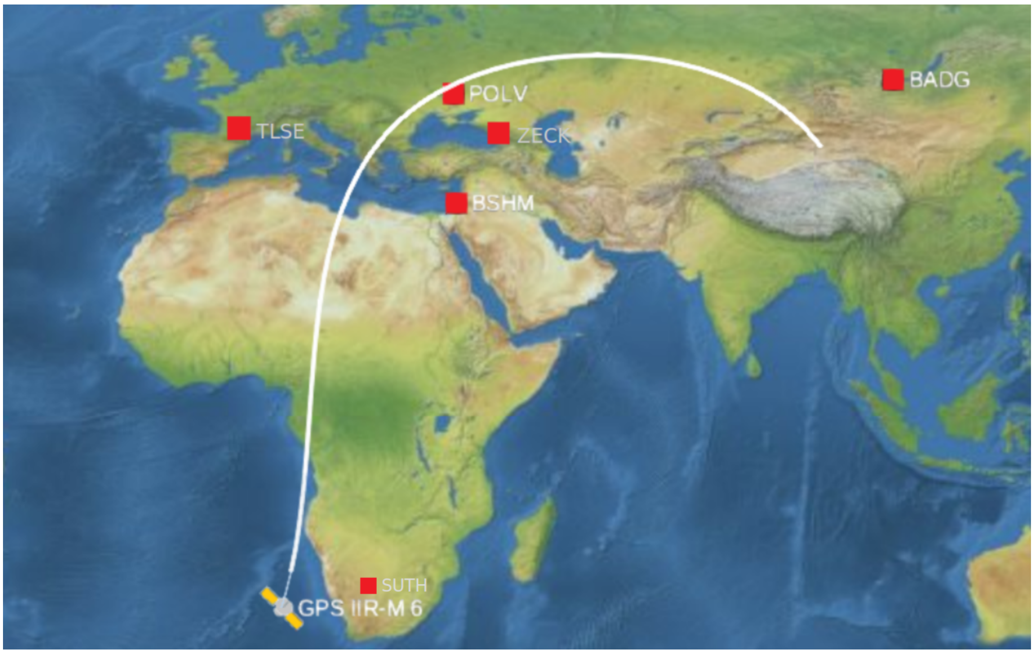

Figure 2. GPS II R-M 6 ground track during measurement arc. Satellite goes from West to East. The arc starts on the 13th of February 2016 at 08:30:00 UTC and lasts 5h30min. Ground stations used for the Orbit Determination are also represented.

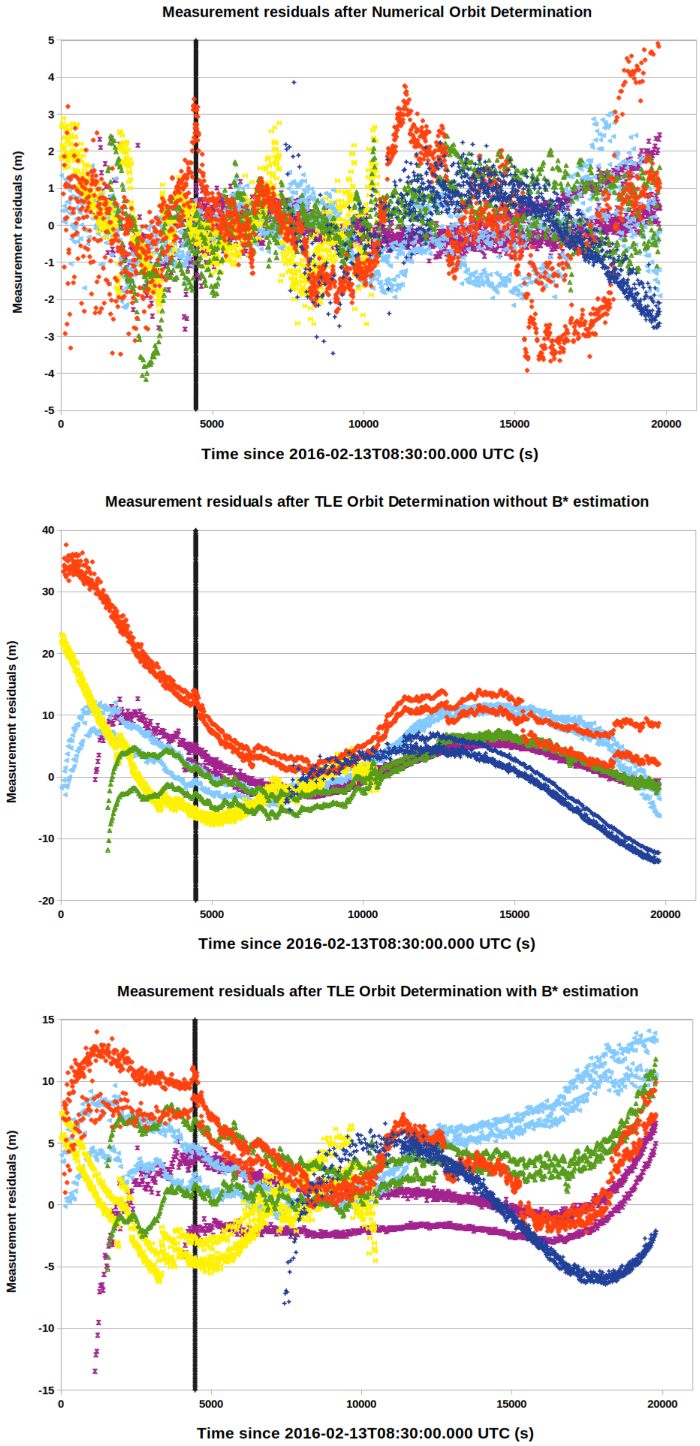

Figure 3. Measurement residuals after numerical (at the top) and analytical TLE Orbit Determination (both at the middle and the bottom). A GNSS satellite was considered along with related pseudo range measurements.
Analytical Orbit Determination was perform twice. Firstly estimating only orbital parameters (at the middle). Then, the B model parameter was also estimated (at the bottom).*
Stations: BADG in blue, BSHM in red, POLV in green, SUTH in yellow, TLSE in light blue, ZECK in purple.
Black vertical line is Orbite Determination first guess epoch.

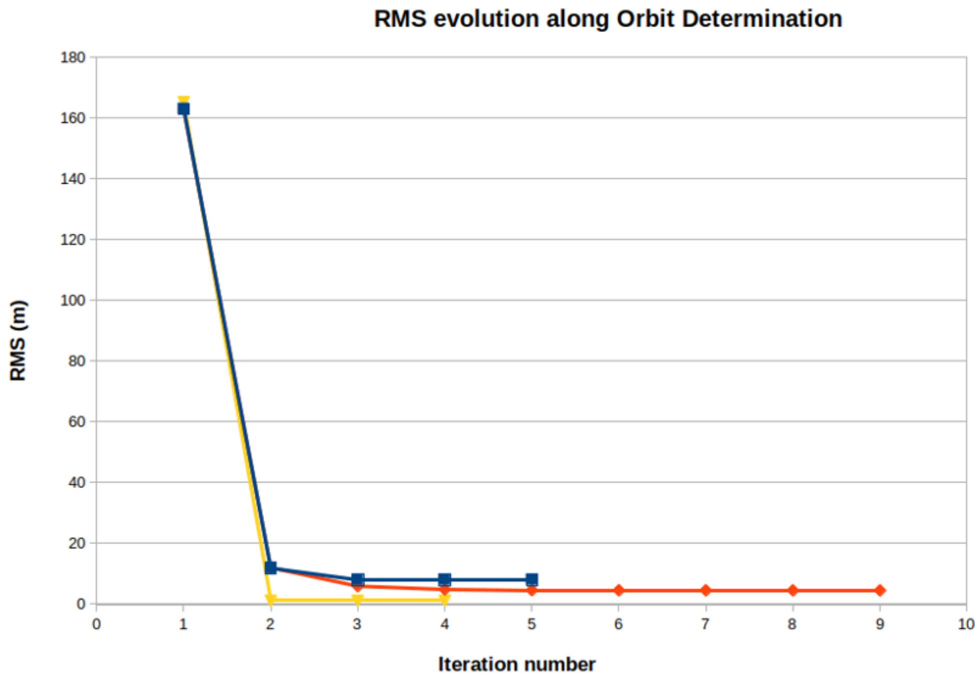

Figure 4. Estimated state Root Mean Square error along Orbit Determination iteration.
In yellow, numerical Orbit Determination
In blue, TLE Orbit Determination without B estimation*
In red, TLE Orbit Determination with B estimation*

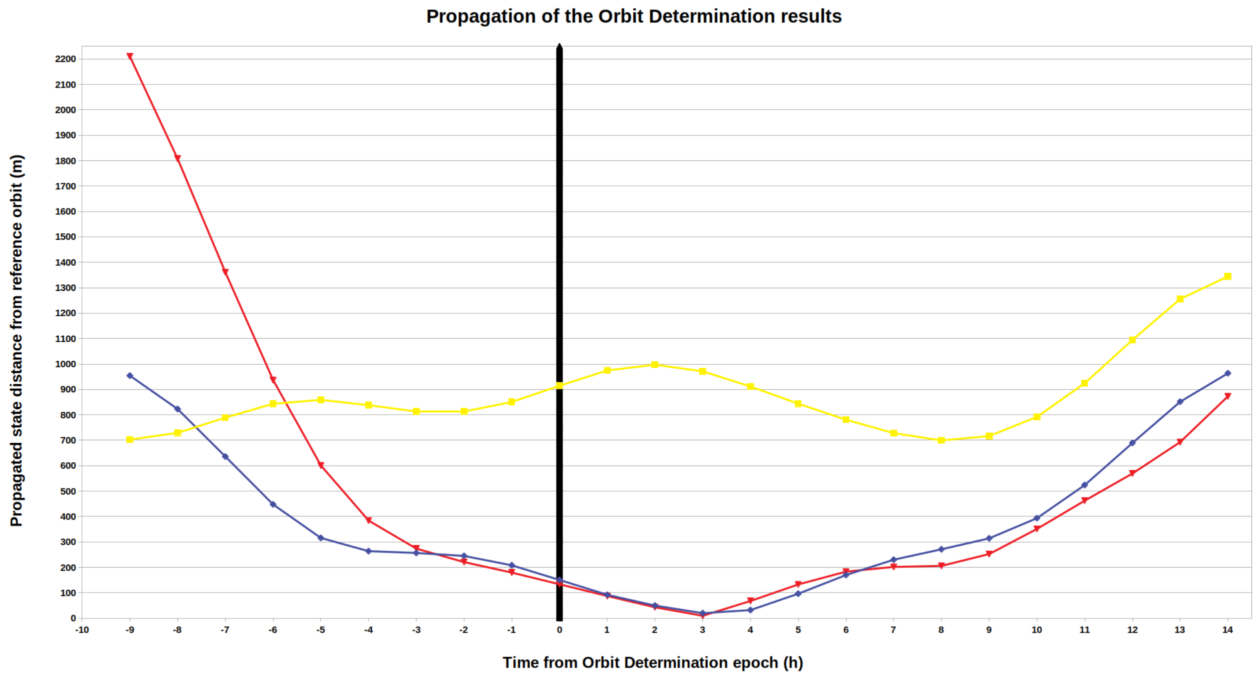

Figure 5. SDP4 analytical propagation of GPS II R-M 6 original and determined TLEs. First guess epoch is 2016/02/13 09:44:09 UTC. Reference orbit is ISG SP3 ephemeris product for the 13th of March 2016.
In yellow, original TLE
In blue, determined TLE without B estimation*
In red, determined TLE with B estimation*

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