DETERMINING ORBITS FOR COLLISION PREDICTION USING DYNAMICAL SYSTEMS THEORY

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ABSTRACT

Increase in debris density is a threat to the stability of the orbital environment. To alleviate this threat, collision prediction and debris removal must be addressed. The current orbit and collision prediction mechanisms are inaccurate. They attempt prediction using covariance matrices, debris density, atmospheric evolution models, Hamiltonian equations among others. This paper focuses on predicting collisions using dynamical systems theory and attempts to increase the accuracy of prediction.

This paper models orbits with Cartesian ephemeris. Unlike the existing models such as SGP4 and OMOP, this paper approaches orbit determination from a wider perspective and attempts more accurate solution by developing mathematical models using chaos theory tools (Lyapunov Characteristic Exponent, Attractors among others). Dynamical systems theory describes the temporal evolution of a system. The deterministic model described in this paper will reduce the net requirement for orbital data. Simulation of the model supports the hypothesis.

1 INTRODUCTION

Earth Orbits (LEO, MEO and GEO) have more than 170,000,000 space debris – every satellite in space is now vulnerable to collision with atleast 13000 debris objects of varying sizes [1][6]. With space debris accumulating in the Earth orbits, the chances for the occurrence of a Kessler Syndrome are on the rise.

Space agencies all over the world are continuously tracking the trackable space debris and developing removal strategies to evade any possible Kessler Syndrome scenarios. Evasion of such scenarios require accurate orbit prediction and collision prediction [2]. Unfortunately, the current collision prediction and orbit determination mechanisms are not accurate enough [3]. The inaccuracy of orbit prediction mechanisms is evident from the Iridium – Kosmos collision. This was a collision between an active and an inactive satellite. These satellite – satellite collisions occur at much lower rates compared to satellite – debris or debris – debris collisions. Collisions can also happen between rockets and debris. While collision avoidance maneuvers can be executed, excess course changes will result in wastage of resources [4].

Accurate collision prediction has two significant components – data collection and orbit determination. The accuracy of these two components hugely influences the accuracy of the result, sometimes even exponentially.

Why current orbit determination strategies are not accurate enough?

1) There are several complex factors (atmospheric drag, lunisolar gravitational effects, solar radiation pressure and Earth's structure among others) influencing each other and the orbit up there – this causes the difference between expected orbit calculations and real values
2) Two Line Elements (TLEs) are not accurate enough – they do not represent the values of the osculating orbit [5] and therefore are not sufficient for calculations, causing inaccuracies of hundreds of arcseconds
3) Accurate orbital elements are not available for certain objects
4) Atmospheric drag is highly unpredictable and is influenced by several factors, including the mass of the particles
5) Position errors of even 3-4 metres can change the collision probability and subsequently determine if avoidance maneuvers have to be performed or not. [10][5]

Space Debris orbit determination and collision prediction models have not effectively tapped the potential of deterministic chaos in evaluating collision probability.

Recent force models in the LEO, MEO and the GEO have indicated that most orbit determination errors happen in the LEO [10]. The factors which contribute significantly to the uncertainty are the atmospheric drag (in LEO) and the solar radiation pressure. The errors caused due to geopotential models have been reduced by the advancement of technology [12]. Errors in relatively short data arcs are from the observational errors, regardless of the value of the error, whilst for long data arcs the uncertainty can be traced to the errors in the dynamic models [11]. This paper addresses the latter, by attempting to reduce errors in the dynamic models, especially the larger orbital uncertainties in the LEO.

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This paper focuses on orbit prediction and determination, and methods to address challenges in prediction when minimum required data is already available. This paper also focuses on limiting the quantity of required data through changes in the mathematical modeling structure, introducing dynamical system theory in orbit prediction.

This paper discusses a dynamic model to evaluate the probability for a debris or satellite to encounter a close approach or collision with target debris.

2 RESEARCH QUESTION

How to determine orbits and predict collisions between space objects and identify rocket launch paths, even with low accuracy data?

3 HYPOTHESIS

Modeling with dynamical systems theory and chaos theory predicts space debris collisions and close approaches with better accuracy in comparison with current collision prediction models (especially concerning the Lower Earth Orbit).

4 CHAOS THEORY & DYNAMICAL SYSTEMS THEORY IN ORBIT DETERMINATION

4.1 Chaos Theory

Chaos, as a dynamical phenomenon, can be described by the sensitive dependency on initial conditions and unpredictability of evolution of an orbit in the phase space [13].

Chaos theory is a concept developed in the late 20th century that concerns with the behavior of highly sensitive, unpredictable and random systems [7].

4.2 Dynamical Systems Theory

Dynamical systems theory describes the temporal evolution of a system. Dynamical systems can be deterministic or stochastic. Chaotic systems are always deterministic [15]. Deterministic systems display low dimensional chaotic behavior – short term prediction accuracy is the highest. Stochastic systems display high dimensional chaotic behavior. Deterministic systems have only one future state regardless of the varying initial states and influencing factors [8][14].

Edward Lorenz elaborates on how a butterfly flapping its wings in one place can disproportionately influence the weather somewhere else. This, called Butterfly effect describes the sensitivity to initial conditions [9]. Chaos theory distinguishes chaotic and normal movement.

Orbits in space are influenced by their initial states. This effect is more pronounced in orbits closer to the Earth (< 600 km) where atmospheric drag is stronger. At these altitudes, even relatively smaller errors and uncertainties in orbital state vectors or dynamic models will lead to an error of several meters in the resultant position vectors. Advancement in technology still leads to comparatively smaller, but yet large errors in orbit determination [5]. This paper has attempted to abate these errors by using dynamical systems theory and chaos theory.

Orbits in space are evidently deterministic. But the prediction of deterministic systems cannot be done perfectly due to reduced accuracy of initial data / modeling structure [15]. Therefore, we need tools to assess the system’s sensitivity to initial conditions. This simultaneously predicts the object's future trajectory by evaluating how the system progresses even with the presence of these anomalies. This paper focuses on two major trajectories whose uncertainties influence the debris positions by the largest margin. Trajectory refers to a collection of states of the system which it follows. The trajectories considered in this paper are:

1) The Atmospheric drag (including the molecular drag) exerted on the spacecraft and
2) Solar radiation pressure

The dynamical tools this paper uses to improve the accuracy of these trajectories are:

i) Phase Space: It is the space of all possible states of a system. It contains the instantaneous description of the system with respect to the values of the different states which influences the system’s characteristics. Phase space is a perfect tool for the dynamical model since it helps analyze and determine the dynamic trajectory of the object’s state values. Since the state values are plotted on phase space, their continuation or future trajectory can be predicted easily by observing previous patterns, analyzing attractors (values in the phase space around which trajectories usually converge – if a disturbance is created in the trajectory near the attractor, its course will mostly not change, unlike in other phase space values), utilizing the Lyapunov Characteristic Exponent among others.
ii) Lyapunov Characteristic Exponent (LCE): The rate of divergence of two infinitesimally close or neighboring trajectories in phase space, LCE, is an indicator of the sensitivity to initial conditions of a system [15] [16] [17]. LCE finds average divergence of these trajectories in the state space values. It is a measure of how chaotic the system is. A system is usually

defined through its maximum Lyapunov exponent.

iii) Lyapunov time: Measure of the predictability of the system. It is the time taken by the system to forget its past states. If the time exceeds the Lyapunov time, accurate prediction cannot take place due to the effect of chaos. It is quantified as the inverse of the Lyapunov exponent of a system.

4.3 Collision Probability and Convergence in Phase Space

Collision probability is the probability of two random objects to collide and in this case, it is either satellite – satellite or a satellite – debris or debris – debris collision. It must be noted that in this paper dynamical systems theory and chaos theory are applied to supplement the current modeling structures (i.e. covariance matrices etc.) – to find the deviation in the predicted future trajectories.

The dynamic model deploys the LCE and Lyapunov time in the phase space to determine convergence and collision probability. For this purpose, this paper considers these following trajectories:

i. The instantaneous plot of original values of states collected
ii. The plot of a random phase line which is infinitesimally close to the original trajectory at a certain point of time (from which prediction must occur)

The modeling structure is illustrated in Fig. 1.

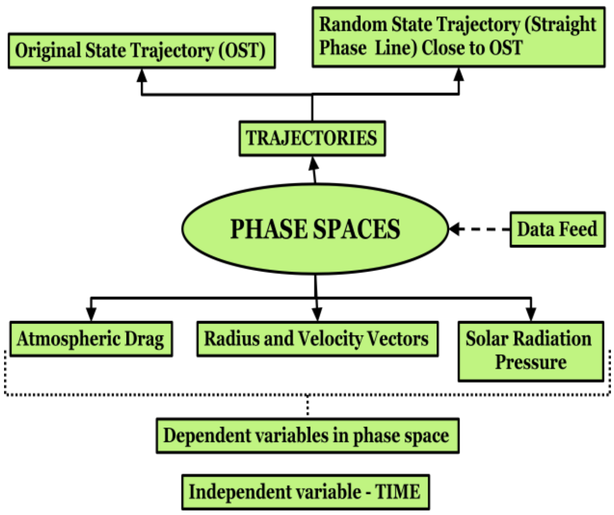

Figure 1: Detailed outline of modeling structure

Since the collision probability is computed for a pair of objects, analysis will be made by comparing the trajectories of both the objects in a radius-time phase space.

While the LCE and the Lyapunov time factor will help determining the orbit (until the system reaches Lyapunov time), the collision probability is the convergence in phase space values of the two orbits (in radius-time plots).

5 MODEL DEVELOPMENT

This paper considers the following phase spaces, the respective quantities and time being the axes of phase space:

1. Radius - Velocity - Time
2. Radius - Time
3. Atmospheric drag acceleration - Radius - Velocity - Time
4. Atmospheric drag acceleration - Time
5. Solar radiation pressure - Time

Following are the steps for developing the model:

i) Introduction of new data analysis systems to suit the requirements of this orbit determination model – analyzing the model in the perspective of dynamical systems theory – collecting data relevant to the model (chaos theory can supplement other models to minimize the errors in data collection, but for this improved accuracy, we will need a chaos-based interpretation of the data).
ii) This data will be plotted in the respective phase space graphs for both bodies
iii) Chaoticity and LCE of these phase space trajectories will be evaluated (these calculations will initially be performed in the state spaces involving Atmospheric Drag and Solar Radiation Pressure – AD and SRP)
iv) Lyapunov Time, LCE and Chaoticity will supplement non-chaotic calculations (unless these calculations are completely inaccurate and insufficient) and will be used to predict the development of the trajectory
v) These calculations will identify a certain area of the phase space as possible future values – consisting of several points – each representing one possible value of AD or SRP
vi) The range of the net acceleration ($\vec{a}$) on the body will be identified
vii) $\text{Range}_{\vec{a}}$ will identify the $\text{Range}_{\vec{r},\vec{v}}$. This predicted range of the radius and velocity will be plotted in the phase space
viii) This range in state space will cover an area
ix) The collision probability will be the proportion of convergence of the areas of the range plots of both the objects

5.1 Data Collection and Analysis

Data collection and analysis is not within the current scope of this paper as this paper focuses on formulating a model for orbit determination.

5.2 Phase Space Trajectory Development

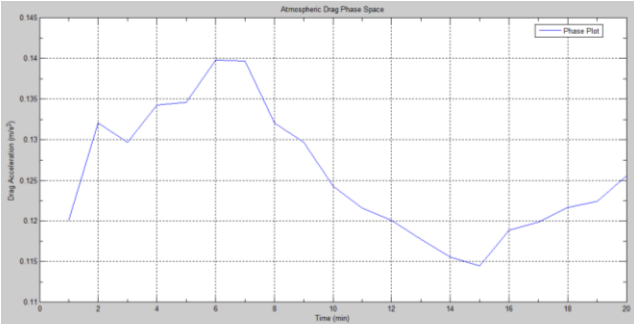

Figure 2: Atmospheric Drag Phase Space for Body #1

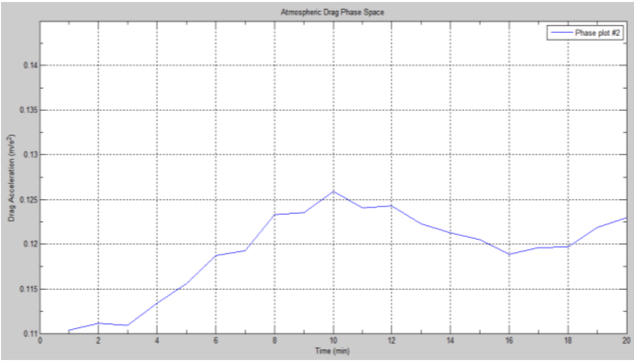

Figure 3: Atmospheric Drag Phase Space for Body #2

The data collected will be plotted in the phase space. Every point in Fig. 2 and Fig. 3 represent one state in which the system can exist or had existed. The trajectories in the phase spaces are the system's previous states.

5.3 Application of Dynamical Systems Theory

Calculating the LCE:

$$\lambda = \lim_{t\to\infty} \frac{1}{t} In \frac{|\delta x(t)|}{|\delta x(0)|} \quad (1)$$

[18]

where,

- λ is the LCE
- $\delta x(t)$ is the final separation between the two trajectories
- $\delta x(0)$ is the initial separation between the two trajectories

An arbitrary, linear phase line is the reference trajectory for the calculation of the Lyapunov Exponent of the body's phase space trajectory.

Larger timescales can also be considered for evaluating the LCE, creating different values and a range for the LCE. LCE also represents the chaoticity of trajectories in the phase spaces. We can also consider the motion of the trajectory's attractor to understand its chaoticity. Using attractors will supplement atmospheric drag and solar radiation pressure prediction models by reducing the range of possible values. This will necessitate a phase plot without the use of time.

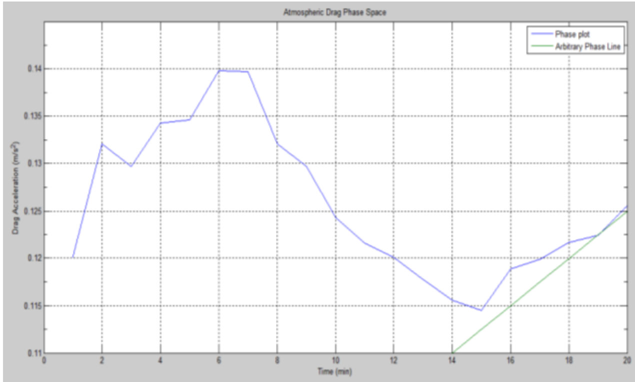

Figure 4: Atmospheric Drag LCE and LCE range calculation (Body #1)

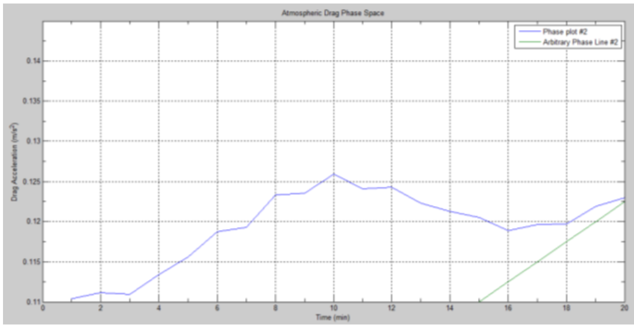

Figure 5: Atmospheric Drag LCE and LCE Drag Calculation (Body #2)

5.4 Predicting the Future Trajectory

Step #1: Non – Dynamical Orbit Calculations

(i) Atmospheric Drag Acceleration:

This equation is most widely used to calculate the drag force:

$$\overrightarrow{a_{drag}} = -\frac{1}{2}\rho \frac{C_D A}{m} v_{rel}^2 \frac{|\overrightarrow{v_{rel}}|}{\overrightarrow{v_{rel}}} \quad (2)$$

[19]

where,

- ρ is the atmospheric density
- C_D is the drag coefficient of the body, which describes its resistance to air flow
- A is the area of perpendicular cross section
- $\overrightarrow{v_{rel}} = \overrightarrow{v_{sat}} + \overrightarrow{v_{atm}}$ (Vector Addition)
- $\overrightarrow{a_{drag}}$ is the acceleration caused due to drag on the body

To calculate the drag force, the geometry and characteristics of the body must be known. The drag coefficient of a body is the representation of its geometry and surface structure. To calculate the C_D there must be complete understanding of debris structure. Our debris surface descriptions too are insufficient. Simpler calculation of the C_D:

$$C_D = \frac{2m\overrightarrow{a_{drag}}|\overrightarrow{v_{rel}}|}{\rho A {v_{rel}}^2 \overrightarrow{v_{rel}}} \quad (3)$$

[19]

Since the drag coefficient does not change on an infinitesimal time scale, the same coefficient can be applied to both the equations, but with different vector values. Since accuracy of a_{drag} is low, a_{drag}'s value is used as a boundary for the area of possible trajectories, *A*.

(ii) Solar Radiation Pressure:

Solar cycle predictions are necessary to determine the orbit of space debris with high accuracy [20]. Solar radiation pressure is the pressure caused due to events like the Solar Wind, Coronal Mass Ejection among others. It influences the conditions of the atmosphere. This must also be taken into consideration.

Solar cycles and the magnitude of the solar wind cannot be predicted with high certainty [19]. Most equations are dependent on empirical pattern observations. Predicting R_Z (maximum sunspot number) is very important to determine solar cycle patterns. Aggregated parameters like $F_{10.7}$ are required in computing the value of the solar radiation acceleration [19]. Empirical equations used in solar radiation acceleration prediction are:

$$F_{10.7} = 63.7 + 0.728\,R + 0.000\,89R_2 \quad (4)$$

$$F_{10.7} = 145 + 75\cos(0.001696t + 0.35\mathrm{SIN}(0.00001695)) \quad (5)$$

[19]

where 't' is the time from January 1, 1981.

In the LEO, the effect of solar radiation is smaller than the effect of atmospheric drag. Nevertheless, it is a major influencing factor of the properties of the atmosphere [19].

Step #2: Identifying Lyapunov time:

Lyapunov time determines the time till when accurate prediction can be performed.

For any finite accuracy of the initial data,

$$\delta(x) = \delta x(0) \quad (6)$$

The dynamics of the system and the system trajectory is predictable only up to a finite Lyapunov time (T_{Lyap}):

$$T_{Lyap} = \frac{-1}{\lambda} In|\delta x/L| \quad (7)$$

[18]

where L = $|\delta x(t)|$

The Lyapunov time can be considered for all the values of the LCE within the range of LCEs. This paper will be considering the least of these Lyapunov times for the model, usually the most chaotic system.

Step #3: Identifying the Separation in Trajectory

The separation of two infinitesimally close trajectories in phase space in time *t* is quantified as:

$$|\delta x(t)| \approx e^{t\lambda}|\delta x(0)| \quad (8)$$

[18]

Equation 4 dictates the mean separation between the two trajectories at time *t*, and with the range of exponents, this trajectory will have a range of values in the phase space encompassing area *A*.

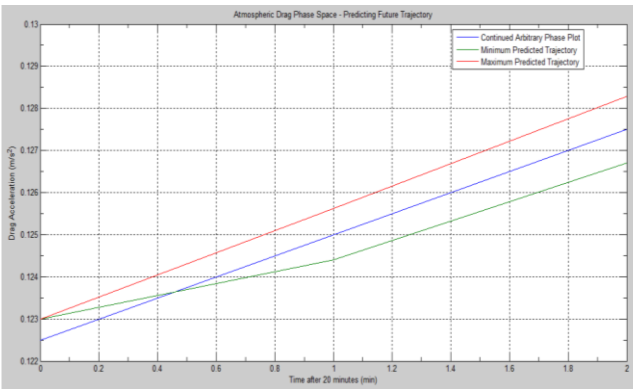

Figure 6: Predicting Future Trajectory for Body #1

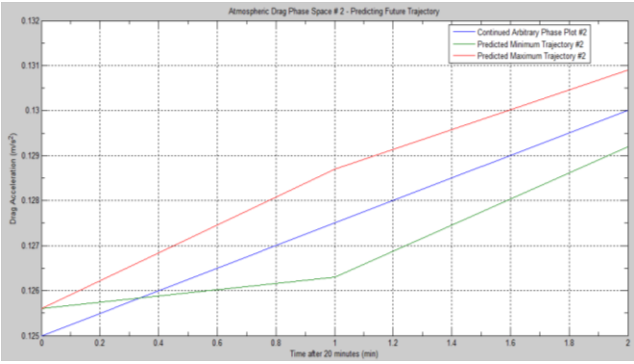

Figure 7: Predicting Future Trajectory for Body #2

5.5 Range of LCE and Area of Phase Space

Let the range of LCE's be expressed as R_λ. The area of phase space encompassed by the probable future trajectories in radius - time phase space is *A*. We need to express R_λ in terms of *A*. This is done with equation (8). The value $|\delta x(t)|$ takes different values with differing expressions for LCE (and Lyapunov time too). This results in varying values of acceleration from non-gravitational forces. This acceleration determines the range and thus influences *A*.

5.6 Range of Net Acceleration of the Bodies

$$\vec{a}_{net} = -\frac{GM}{r^3}\vec{r} + \vec{a}_g + \vec{a}_{ng} \qquad (9)$$

[10]

The net acceleration is the vector sum of all contributing accelerations, where:

- $\vec{a}_g$ is the vector sum of all gravitational forces including perturbations from other planets
- $\vec{a}_{ng}$ is the vector sum of all non-gravitational accelerations (atmospheric drag, Solar and Earth radiation pressure and Albedo effect)
- $\vec{r}$ *is the* radius vector.

The range of net accelerations for two bodies at time *t* (within their respective Lyapunov times) is:

$$-\frac{GM}{r^3}\vec{r} + \vec{a}_g + \vec{a}_{ng} + \vec{a}_{miu} < \vec{a}_{net} < -\frac{GM}{r^3}\vec{r} + \vec{a}_g + \vec{a}_{ng} + \vec{a}_{mau} \qquad (10)$$

where *mau* is maximum uncertainty and *miu* is minimum uncertainty.

5.7 Evaluation of Range$_{\vec{r},\vec{v}}$

Conversion of acceleration to radius and velocity terms: The r and v vectors are the Cartesian orbital elements. The Cartesian state vectors incorporate perturbations better than Keplerian state vectors.

$$\dot{v} = a = \frac{dv}{dt}$$

$$\dot{r} = v = \frac{dr}{dt}$$

$$\ddot{r} = a = \frac{d^2r}{dt^2}$$

The range predicted values of radius and velocity will form the future plot in phase space covering an area *A*.

5.8 Collision Probability

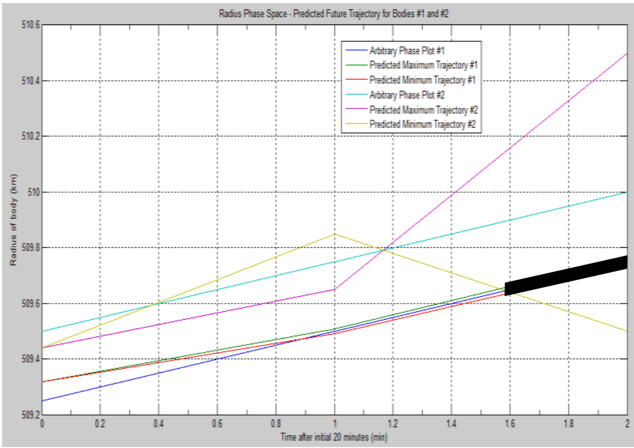

Figure 8: Identifying the collision probability (area shaded black is the convergence of trajectories in the radius-time space)

In this paper, collision probability (P_C) at a certain point or interval of time is:

$$P_C = \frac{Common\ Area}{Total\ Area\ Covered\ by\ Future\ Trajectory} \qquad (11)$$

where the numerator (Common Area) is the area of overlap of the areas encompassed by the trajectories of both bodies and the denominator (Total Area Covered By Future Trajectory) is the area of all possible radius - time states of one of the bodies for which the P_c is calculated. The probability that satellite or debris # 1 may collide with satellite or debris # 2 need not be equal to the inverse. The common area (A_2) is a different fraction of the areas of the individual states.

$$P_{C2(avg.)} = \frac{Common\ Area}{Total\ Area\ of\ \#2} \approx \frac{0.0168}{2.415} = 0.69\%$$

$$P_{C2(avg.)} = \frac{Common\ Area}{Total\ Area\ of\ \#1} \approx \frac{0.0168}{0.08} = 21\%$$

These are the values of the simulation results. This is the expression of ratio of convergence of trajectories and the total possible states.

6 SUMMARY

Chaos theory has been put to use especially in weather prediction and stock markets for event prediction. Application of deterministic Chaos (Lyapunov exponents in particular) for space debris collision prediction and probability evaluation is a novel approach by this paper.

This paper has attempted to address the problem of inaccurate space debris orbit determination. Inaccuracies arise due to two reasons - the uncertainties in the initial position of the space debris / satellite and the uncertainties in the modeling structures. For orbit prediction, dynamical systems theory has necessitated the use of relevant real time space debris data, empirical relation derivation and accuracy analysis of the model. This paper attempts higher accuracy in orbit prediction and collision probability evaluation using the tools of chaos theory and dynamical systems theory. New method for identifying collision probability has also been discussed. The results of the simulation support the hypothesis.

7 FUTURE SCOPE

The model can be strengthened using relevant and real time data. This model can utilize other facets of chaos theory and dynamics (for example, attractors among others) and involve Equinoctial orbital elements instead of Cartesian state vectors for making the collision prediction results more accurate.

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