# DEFORMATION OF SPACE DEBRIS AND SUBSEQUENT CHANGES IN ORBIT DURING ECLIPSE DUE TO SOLAR AND EARTH RADIATION FLUXES

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## ABSTRACT

Both solar radiation pressure (SRP) and Earth radiation pressure (ERP) can dramatically influence the orbit of space debris, particularly objects with a high area-tomass ratio (HAMR objects). Little work however has investigated how these objects deform when solar radiation pressure (SRP) and Earth radiation pressure (ERP) forces are applied.

In this work a flexible piece of debris is modeled as a lattice of interconnected particles. Internal and external forces are modeled on a particle by particle basis allowing for complex deformations to occur. The effect of these forces is shown on flat and curved objects and a coupled deformation/attitude model demonstrated.

Key words: Debris; HAMR; SRP; ERP; Flexible; Orbit Prediction; Eclipse.

### 1. INTRODUCTION

One class of HAMR object consists of a piece of multilayer insulation (MLI) that has broken free or spalled off from a spacecraft. MLI is typically extremely thin and has very little structural rigidity. This means that as well as being subject to orbital changes the non-conservative forces can also deform the object.

Previous studies have investigated how the SRP on a flexible object varies and how this is coupled with the orbit of the object. The object is typically modeled as a small number of flat elements connected by linkages which allow the composite object to mimic the material properties of a single piece of MLI type material. The use of a small number of connected flat plates allows for the analytical computation of self shadowing as well as the resulting normal and shear SRP forces.

In this work the structure of the piece of debris is modeled numerically using thousands of interconnected particles, allowing for the object to curl along any number of axes rather than simply fold around a single pre-defined axis. The incident radiation flux is computed on each individual element allowing for more accurate self-shadowing as well as variable flux across the debris object. The deformation of the object due to the uneven flux encountered when travelling through the penumbra is calculated along with the effect of both the SRP and ERP incident on the object.

This paper begins by outlining previous studies into HAMR debris and their findings. It will then go on to on to describe the orbital environment of these HAMR objects and the forces that effect them, particularly during eclipse.

The remaining part of the paper proceeds to describe a novel technique that allows the shape of the object to be modeled in a computationally efficient manner and describes a proof of concept simulation that has been carried out which demonstrates the coupled deformation-attitude model.

## 2. SYNOPSIS OF THE LITERATURE

Schildknechts study (2004) [SMP<sup>+</sup>04] discovered a new class of space debris objects in near geostationary orbits. The observations could only be fitted by assuming very high area-to-mass ratios (HAMR). One candidate for the origin of these HAMR objects is the multi-layer insulation (MLI) that covers many of the spacecraft surfaces. MLI is primarily used for thermal insulation purposes and is both abundant and has the observed area-to-mass ratio characteristics. MLI consists of many thin layers and the layers becoming detached is known as delamination. The layers typically are made of polyester or polyamide films such as Kapton or Mylar onto which a extremely thin metallic layer has been deposited. [FGM<sup>+</sup>11].

It has been established that the unknown space object materials and attitude dynamics tend to contribute to trajectory prediction errors over tens of days, whereas the combined solar-lunar gravitation and large non-conservative force effects contribute to the errors over longer timescales (weeks to years) [KJ11]. Data from several studies suggest that the AMR is not constant over time [FS12] and that due to the HAMR of the objects and the significant magnitude of the non-conservative effects the attitude and orbital motions are highly coupled. The effect

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of this coupling has been investigated in [FJ13] and found to be significant.

The existing body of research on the orbital behaviour of HAMR objects suggests that as well as the attitude the shape of the object plays a key role in the dynamics of the object. This is due to the effect shape has on the SRP due to changes in presented area and second order effects such as self-shadowing and reflection. HAMR objects are frequently modeled as flat plates but this model has been extended to model non-homogeneous surface properties [FKJ13].

Implementing a HAMR shape model consisting of a number of connected flat plates formed the central focus of a study by Früh [FJ14]. This allows the non-planar geometry of the object to be modeled and the author found significant variations in orbit when shape specific properties are taken into account.

### 3. ORBITAL ENVIRONMENT

The orbital environment and the associated forces acting on HAMR objects are of key importance when modelling their behaviour. As this work is primarily focused on HAMR objects at geostationary altitudes (~42164 km) a different set of forces must be taken into account when compared to objects in low Earth orbit.

Gravity is still the primary force acting on objects at this altitude and atmospheric drag can be neglected. Radiation pressure however, in relative terms, has much more significant effect. The primary source of radiation is from the sun and thus the dominant radiation pressure force is Solar Radiation Pressure (SRP). The other significant source of radiation is from the Earth itself in Earth Radiation Pressure (ERP). The Earth reflects a proportion of the suns incident radiation as short-wave radiation from the day side and also emits longer wave thermal radiation from both its day and night sides. At geostationary altitudes the magnitude of SRP is typically on the order of 100 times greater than ERP.

It is commonly assumed that these forces act on the objects from different directions but do not differ in magnitude across the object itself. This assumption however breaks when the object enters and exits regions where the sun is in eclipse.

#### 3.1. Eclipse Season

When an object is being eclipsed there are two possible states. In the first the sun is entirely obscured by the Earth, the Umbra, or the sun can be partially obscured by the Earth, the Penumbra. The object must pass through a penumbral region on it's way into and out of the umbra.

The transition from full solar illumination to no solar illumination through the penumbra takes up to 2 minutes



Figure 1. Variation in normalised illumination as an object passes through the penumbra into the umbra during an eclipse transition.



Figure 2. Superimposed normalised illumination levels for a single objects eclipse season. Each line is one days orbit.

as shown in Figure 1. For this work the eclipse was computed assuming Earth was an oblate spheroid as described in [ASZC04].

An object in a geostationary orbit will enter eclipse during the period around the vernal and autumnal equinoxes. These sets of eclipses are known as eclipse seasons and by superimposing the illumination during 24 hour long orbits on an object (Figure 2) we can see all of the eclipses occurring during one season.

#### 3.2. Variation in Radiation Pressure over Body

During a typical eclipse roughly two minutes is spent travelling through the penumbra at the beginning and end of the eclipse. A total of 60 to 70 minutes are spent in the umbra. As the object moves through the penumbra the illumination is either decreasing or increasing as it moves.



Figure 3. Variation of normalised illumination across a 1m by 1m flat body while transiting the penumbra.

For an extended body this also means that there will also be a small variation in illumination across the body as shown in Figure 3.

### 3.3. The effect of shape on radiation pressure forces

The variation in SRP and ERP will effect the objects orbit, attitude and shape. The orbit, attitude and shape of the object are all highly coupled and previous studies have shown that the introduction of flexibility to the object significantly changes the objects orbital behaviour [Cha16], even with the flexibility modelled by a simple model consisting of two plates connected by a hinge.

Much of the previous work in this field has been based on the assumption of a flat plate object with uniform material properties and evenly distributed mass, no torque is induced. This work demonstrates a more complete model of the shape and behaviour of the debris that can capture the more complex shape dynamics of a HAMR object.

## 4. LATTICE MODEL FOR FLEXIBLE OBJECTS

In this section a new model for both the shape of these HAMR debris and how it evolves over time is developed. This work extends the previous approach of the object being modeled by a small number of connected rigid bodies that have limited degrees of freedom to an approach that breaks the object into many thousands of individual objects that are interconnected in a far more general manner.

The debris object is treated as an interconnected set of particles. Each particle experiences individual forces and is subject to constraints. The set of accelerations that are acting on each particle from the environment and from other particles are numerically integrated and the set of geometrical constraints are iteratively satisfied.



Figure 4. Example of the lattice model capturing the change of shape of a flexible HAMR object when subjected to a force

In order to meet the dual aims of maintaining numerical stability and maximising computational efficiency the symplectic Newton–Störmer–Verlet–leapfrog integration method is used. Throughout this paper, the term NSVL will refer to the Newton–Störmer–Verlet–leapfrog integration method.

The NSVL method offers a number of advantages over more traditional explicit numerical integrators. The primary advantages are that the total energy of the system is stable (although it may oscillate around this value it is not liable to 'explode') and that the method is highly computationally efficient.

### 4.1. Introduction to Newton-Störmer-Verletleapfrog integration

A more complete treatment of NSVL integration is given in [HLW03] but a basic overview of the method and description of how its key properties apply to the problem of flexible objects is given below.

Our flexible object can be considered a large set of particles that are interconnected through springs, dampers and constraints. An acceleration on a particle creates a change in position of that particle. The springs and dampers connecting this particle to others then transmits an acceleration to its adjacent particles. At any given time step the particles can all be experiencing different accelerations and these must all be resolved. This can be done sequentially or in unconnected groups.

In order to find the new position of a particle based on its resultant acceleration its acceleration must be numerically integrated. In this case the position is not dependent on velocity, only acceleration so it has the following approximate form.

$$\ddot{q} = f(q),\tag{1}$$

This formulation is commonly used in molecular dynamics but is also very well suited to our set of constrained particles.

### 4.2. Two-step formulation

If for the integration we choose a step size of h and grid points  $t_n = t_0 + nh$ , the discretisation of equation (1) is:

$$q_{n+1} - 2q_n + q_{n-1} = h^2 f(q_n), \tag{2}$$

which allows us to calculate  $q_{n+1}$  whenever we know  $q_n$  and  $q_{n-1}$ . This leads to one of the peculiarities of this method in that it cannot 'cold start' and needs two time steps to initialise. Geometrically this amounts to determining an interpolating parabola which at its mid point assumes the second derivative from equation (1).

### 4.3. One-step formulation

The two-step model is not ideal due to its startup time and its computational complexity. We can improve the method however by introducing v and turning equation (1) into a first order system:

$$\dot{q} = v, \qquad \dot{v} = f(q)$$
 (3)

Using this we can introduce discrete approximations of v and q as follows:

$$v_n = \frac{q_{n+1} - q_{n-1}}{2h}$$
$$v_{n-\frac{1}{2}} = \frac{q_n - q_{n-1}}{h}$$
$$q_{n-\frac{1}{2}} = \frac{q_n + q_{n-1}}{2}$$

You will notice the use of  $t_{n-\frac{1}{2}}, t_{n+\frac{1}{2}}, \ldots$ ; this 'staggered grid' preserves the second order and symmetry. Inserting these into equation (2) it can be seen that we now have a one-step method  $\Phi_h : (q_n, v_n) \mapsto (q_{n+1}, v_{n+1})$ , given by

$$v_{n+\frac{1}{2}} = v_n + \frac{h}{2}f(q_n)$$

$$q_{n+1} = q_n + hv_{n+\frac{1}{2}}$$

$$(4)$$

$$v_{n+1} = v_{n+\frac{1}{2}} + \frac{h}{2}f(q_{n+1})$$

There is also a dual variant of the method that uses the staggered grid  $(v_{n-\frac{1}{2}}, q_{n-\frac{1}{2}}) \mapsto (v_{n+\frac{1}{2}}, q_{n+\frac{1}{2}})$  as follows:

$$q_{n} = q_{n-\frac{1}{2}} + \frac{h}{2}v_{n-\frac{1}{2}},$$

$$v_{n+\frac{1}{2}} = v_{n-\frac{1}{2}} + hf(q_{n}),$$

$$q_{n+\frac{1}{2}} = q_{n} + \frac{h}{2}v_{n+\frac{1}{2}}$$
(5)

In practice when applying the step-by-step procedure of the numerical integrator we can concatenate the last line of the current step with the first line of the next step. With this simplification both schemes become the same with the q-values evaluated on the original grid and the v-values evaluated on the half grid:

$$v_{n+\frac{1}{2}} = v_{n-\frac{1}{2}} + hf(q_n),$$

$$q_{n+1} = q_n + hv_{n+\frac{1}{2}}$$
(6)

This formulation is the most computationally economic and is more numerically stable than equation (2) as outlined in [HNW93].

#### 4.4. Geometric Properties

The NSVL method has many interesting geometric properties but two are useful in our modeling of a particle lattice. These are its symmetric (and thus reversible) and symplectic nature.

The NSVL is symmetric with respect to changing the direction of time. In the one step formulation (equation (6)) reflecting time at the centre  $(t_{n+\frac{1}{2}})$  gives the same method again. This time-symmetric property implies reversibility. This mean that by inverting the direction of the initial velocity we do not change the solution trajectory, it just inverts the direction of motion.

The fact that the NVSL is a second order symplectic integrator means that there is no build up in total system -energy through error. At worst the energy of the system will oscillate around the starting value. This solves one of the common problems encountered when integrating constrained particle lattices using non-symplectic integrators such as Euler or Runge-Kutta methods, in that the total energy of the system increases until the whole system 'explodes' and particles move to infinity.

#### 4.5. Integration Error

As a demonstration of the reduction in global error for a problem similar to the particle lattice we are investigating we can look at the Toda-lattice [Tod70]. Figure 5 shows the difference in global error when using a Runge-Kutta integrator vs a Störmer–Verlet method for integrating the forces across a lattice.



Figure 5. Global error of the Störmer–Verlet and the second-order Runge method on the Toda ([Tod70]) lattice.



Figure 6. An example of the interconnection of a 5x5 lattice (left) and the different types of connections needed to model the physical properties of the HAMR object (right).

## 5. MODELLING A HAMR OBJECT AS A LAT-TICE

The NSVL method offers an efficient method of integrating the forces acting between a large number of elements in a lattice. In order for these forces to propagate through the lattice the particles within the lattice must be connected. These connections act as both conduits for forces and as constraints. In order to successfully model a flexible HAMR object the particles must be connected in such a way that they model how the object reacts and resists forces acting upon it.

Figure 6 shows how the particles in the lattice are connected. The most fundamental connection is the connection between adjacent particles in lattice (red connections in Figure 6). This allows forces to be transmitted across the lattice but offers no resistance to shearing (deformation due to in plane forces) and bending (deformation due to out of plane forces). In order to resist shear forces diagonal connections are made (blue connections in Figure 6). The direct and shear links allow for forces in the lattice plane to modelled effectively.

The power of using the lattice approach is in its ability to model how the shape of the flexible object changes in 3 dimensions. The direct and shear links offer no resistance at all to bending due to out of plane forces. Connections that skip a particle's nearest neighbour and connect further out into the lattice (green connections in Figure 6) model this property.

#### 5.1. Lattice Integration Performance

For this approach to be useful in practice it must be as computationally inexpensive as possible as the shape/attitude of the object must be calculated in full at each orbital integration time-step.

The speed of the lattice update computation is dependent on its size. Small lattices can be computed quickly but do not accurately approximate the curved shapes required for HAMR debris. The initial performance target was 100 full lattice integrations per second on a single CPU thread. The lattice approach lends itself to parallelisation but this has been left for future work.

A fully interconnected lattice (see Figure 6) made up of 64 rows of 64 particles has been used for all analyses in this work and can be integrated within the 10ms time window.

One key optimisation is to reduce as much as possible the interdependence of subsequent connection calculations. When two connections share any nodes then they are dependent on each other and both must access the same memory location for the shared particle and be calculated before the lattice computation as a whole can proceed.

This slowdown has been avoided by reordering the computation of the links between particles to make sure that wherever possible subsequent links have no shared nodes.

## 6. ANALYSIS

An initial proof of concept analysis using the lattice model was undertaken. The simulation included both SRP and ERP acting on a 1 metre by 1 metre flexible HAMR sheet (Along with gravitational forces modelled using the GRACE-GGM03C gravity model ([TRB<sup>+</sup>07]) up to degree and order 20. Physical properties of the object were based on those of 0.25mm thick coated PET sheet. An orbit close to the vernal equinox was chosen as a basis for this investigation.

#### 6.1. Flat object

The analysis focused on the deformation of the HAMR due to incident forces as it this stage the lattice model has not been fully coupled to the orbital model. In the first analysis a flat plate on our vernal equinox geostationary orbit was modeled. The SRP and ERP were precomputed and the SRP and ERP forces applied to the lattice model. With zero initial rotation or deformation the lattice stayed completely planar throughout its orbit. This is the expected result given the homogeneity of the HAMR object in question. The variation in solar flux during the penumbra was also modelled and this induced a small rotation as expected.



Figure 7. The deformation of a curved flexible object due to SRP and ERP forces over one geostationary orbit during vernal equinox modeled using the lattice method.

This simulation shows that the lattice model can capture the same attitude behaviour as simpler models and no significant errors are introduced by computing the SRP and ERP forces on each individual particle rather than the object as whole.

### 6.2. Non-planar objects

A far more interesting simulation is to try and model the change in shape of the flexible object due to these same forces. The only change vs the previous flat plate simulation is that the initial condition of the HAMR object is curved. This seemingly small change brings into play phenomena that are typically very difficult or impossible to model using analytical methods. These include the self shadowing (from two radiation sources), the centre of mass not being in the object plane and most importantly the ability of the flexible structure to deform.

Figure 7 show the deformation of the simulated flexible object over the course of one orbit. The self-shadowing and variation of the direction of the radiation sources in the body frame result in both a complex deformation and a rotation of the object, coupled behaviours that cannot be modeled to this fidelity using existing methods.

## 7. CONCLUSION AND FUTURE WORK

This study set out to develop a model for the deformation of flexible HAMR debris objects due to SRP and ERP forces. Prior to this study it was difficult to make predictions about how the shape of flexible debris objects changed over time. Previous work propagated the attitude along with the orbit but at best modeled the flexible object as 2 flat plates connected with a single degree of freedom joint.

This work treats the flexible debris as a collection of 4096 independent particles connected via linkages that model



Figure 8. Rendered lattice showing both the micro and macro scale shape effects that can be captured using a lattice model.

the material's physical properties. This lattice of particles can be arranged in any configuration and a flat plate was chosen only to make comparison with the effects shown using analytical models easier.

Complex geometries at the micro and the macro scale can be modelled (Figure 8) and the deformation and attitude evolution modelled. As forces are applied on a particle by particle basis complex effects like self shadowing have already been integrated.

The major limitation of this study is the fact that the attitude/deformation model was not directly coupled with the orbital model. This will be the first area of work tackled in in the future.

A further study could assess the long-term effects of initial conditions when coupled deformation and attitude are taken into account. Further research should also focus on determining the effect of gravitational torques on the objects as the gravitational force also varies across the body and can be computed on a particle by particle basis.

A natural progression of this work is to analyse the process of HAMR object creation by modelling flexible sheets that are still attached to objects and modify the linkages to allow for tearing to model how these objects might break apart and form.

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