SELF-INDUCED COLLISION RISK ANALYSIS FOR LARGE CONSTELLATIONS

H.G. Lewis[(1)], J. Radtke[(2)], J. Beck[(3)], B. Bastida Virgili[(4)], and H. Krag[(4)]

[(1)] *University of Southampton, United Kingdom, Email: hglewis@soton.ac.uk*
[(2)] *Technische Universitaet Braunschweig, Germany, Email: j.radtke@tu-bs.de*
[(3)] *Belstead Research Limited, United Kingdom, Email: james.beck@belstead.com*
[(4)] *ESA/ESOC Space Debris Office, Germany, Email: Benjamin.Bastida.Virgili@esa.int*
[(4)] *ESA/ESOC Space Debris Office, Germany, Email: Holger.Krag@esa.int*

ABSTRACT

Large constellations of communications satellites in low Earth orbit (LEO) can provide considerable benefits due to the global coverage and low latency. However, they also represent a possible risk to other space users and to the long-term health of the space environment, due to the increase in associated space traffic and if appropriate debris mitigation measures are not implemented. An assessment of the potential impact of large constellations on the space debris environment has been performed. This paper describes the analysis of long-term projections incorporating a large constellation made using evolutionary codes, with a particular focus on the detected "self-induced" conjunctions between constellation satellites and the reliability of the collision prediction method. The results show that with a sufficient number of Monte Carlo runs, the collision algorithm is able to compute reliable estimates of the constellation self-induced collision probability.

1 INTRODUCTION

Low Earth orbit (LEO) is experiencing a renaissance thanks to opportunities provided by small satellites and access to orbit. With plans by companies including Boeing, OneWeb and SpaceX to orbit large constellations of small, low-cost satellites to provide broadband internet services to the world, there is growing concern amongst existing space users about the long-term sustainability of the proposed small satellite activities in LEO.

Internet services are already offered by a number of companies with spacecraft in Geosynchronous Earth Orbit (GEO) but these suffer from high latency and, due to the complexity and resources required by the GEO satellites used, are relatively expensive. Communications satellites in LEO can provide considerable benefits due to the low latency but good coverage can only be achieved through the use of large numbers of satellites. For example, in filings with the Federal Communications Commission (FCC), SpaceX proposed a LEO constellation comprising 4,425 satellites, and OneWeb is planning for a constellation of 720 satellites. Both SpaceX and OneWeb have identified altitudes above 1000 km for their constellations, which provides a seemingly benign debris environment due to the relatively low debris spatial density there, as well as low latency.

There are encouraging signs that new constellation operators are seeking to comply with space debris mitigation guidelines, or in some cases to go beyond what is required to limit their impact on other space users. Yet, it is not clear that a reliance on existing debris mitigation measures will be sufficient to counter the effects of introducing large constellations of satellites, in spite of the commitment by entities to them. In the past, clarity on such space debris issues has been provided through the use of space debris evolutionary codes, which enable a range of scenarios to be investigated and effective debris mitigation measures to be identified.

Whilst the long-term effects arising from the introduction and maintenance of constellations have been investigated in the past (e.g. [1], [2], [3], [4], and [5]), few recent investigations of large constellations have been conducted using modern evolutionary codes. A recent initiative involving a number of European space agencies was reported in [6] and highlighted the importance of post-mission disposal (PMD) measures on the mitigation of debris resulting from a 1080-satellite constellation. Separately, [7] computed collision probabilities and the number of collision avoidance manoeuvres for the proposed OneWeb constellation, with different assumptions for the success of the PMD, mission altitude and lifetime. The results underlined the sensitivity to the mission altitude and the PMD success, with the need for very high PMD success rates for mission altitudes that experience little atmospheric drag.

In 2016 and 2017, a team comprising engineers from industry, academia and the European Space Agency performed an assessment of the potential impact of small satellites and large constellations on the space debris environment. This assessment included: (1) a review of historical and proposed future small satellite activities and associated technologies; (2) a large

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number of long-term projections using three evolutionary codes; and (3) detailed analysis of the results of the first two activities, to understand the sensitivity of the debris environment to key satellite and constellation parameters. Initial results from the projections were presented in [8] and, with respect to the large constellations considered, emphasised the importance of high PMD success rates for the mitigation of space debris. More detailed analysis of the simulation study results are presented here and also in [9].

As part of the assessment in this European study, the suitability of the collision risk method based on the "cube" approach described in [10] and implemented in the Debris Analysis and Monitoring Architecture to the Geosynchronous Environment (DAMAGE) and the Space Debris Mitigation long-term analysis program (SDM) was investigated. The focus remained on the DAMAGE implementation only and the investigation utilised a set of short-term, high temporal-resolution simulations of a large constellation in LEO. As well as aiming to measure the suitability of the cube method used by the evolutionary codes, the "high-definition" simulation study was also aimed at evaluating the role of self-induced collisions – events involving only satellites from the constellation – in the evolution of the space debris environment over the long-term. A particular objective was to determine the contribution to the collision activity made by active and inactive constellation satellites.

2 METHODOLOGY

2.1 DAMAGE model

The DAMAGE debris model is a three-dimensional computational model of the full LEO to GEO debris environment. It includes source models for objects down to 10 cm but is capable of evolving populations of objects down to 1 mm over short projection periods and for a limited set of target objects.

DAMAGE is supported by a fast, semi-analytical orbital propagator, a breakup model, several collision prediction algorithms, including the method based on the cube approach, and several satellite failure models. Most recently, a new constellation module has been added to DAMAGE to allow the investigation of a wide range of constellations and the variety of mitigation measures they may employ.

2.2 The cube method

The cube approach was designed to estimate the collision rate, $P_{i,j}$, between object i with a second object j and hence the number of collisions, in a fast and efficient manner. The method assumes that a collision is possible only when two objects are co-located within a small, cubic volume element. The collision probability of object i with object j in a small cubic volume element dU over a short time interval dt is expressed as [11]

$$dP_{i,j}(t) = s_i s_j v_{ij} \sigma \, dU \, dt \tag{1}$$

where s_i and $s_j$are the residential probabilities (also spatial densities) of objects i and j in the cube dU, v_{ij} is the velocity of object j relative to object i, and σ is the combined cross-sectional area of both objects measured in a plane normal to the relative velocity. The integration of (1) for all objects $j \neq i$ over a relatively long projection period (e.g. typically decades or centuries), and over the volume of near-Earth space provides an estimate of the cumulative collision probability, $P_{i,j}(t)$, for objects i and j. In practice, $dP_{i,j}(t)$ is calculated at discrete time intervals only for cases where two objects occupy the same cubic volume element. Thus, the computation time increases with N rather than N^2 for an environment containing N objects and sampling is performed over time so that new objects and changing orbital elements are incorporated [11]. A uniformly distributed random number is generated and compared with $P_{i,j}(t)$ to determine whether a collision between objects i and j at time t actually occurs. If so, DAMAGE makes use of the NASA Standard breakup model [12] to generate fragmentation debris. Some modification to this basic approach have been implemented in the DAMAGE model to account for occasions when two objects may be in close proximity but are resident in adjacent cubes.

Inspection of (1) reveals that it is based on the kinetic theory of gas and assumes that objects i and j are equally likely to be found anywhere within the cube. Consequently, two objects will have a non-zero collision probability if they are resident in the same cube dU and have some relative velocity with respect to each other, regardless of the collision and orbital geometry. As such, it is possible for objects on orbits that do not intersect to contribute to the collision activity in any simulation. In general, this can be accepted because the exact states of objects in orbit will not be known with certainty. Indeed, the cube itself is a representation of the positional uncertainty for the objects. The along-track positional uncertainty tends to grow quickly for objects at relatively low altitudes and can reach several hundred kilometres in just a matter of days due to the effect of atmospheric drag [13]. As a result, a further assumption that is typically made for simulations of the debris population in LEO that use the cube method is that the distribution in right ascension is random. This is incorporated as a randomisation of the mean anomalies for inactive objects and debris in LEO such that the cube method effectively samples in time and in space. Consequently, it is important to make use of a sufficient number of Monte Carlo runs in order to build a reliable estimate of the collision probability between objects.

Whilst these assumptions are a recognised aspect of projections made by evolutionary codes that employ the cube method, they could be problematic when applied to scenarios involving a constellation, for two reasons. Firstly, the motion of active satellites in a constellation is synchronised within the same orbital plane and also across different orbital planes. In low-drag regions (e.g. above 1000 km), a satellite from the constellation that fails in the mission orbit will likely remain synchronous with the active satellites and may only become a collision risk after some time. Secondly, a failed constellation satellite that loses energy as a result of even a low level of atmospheric drag will separate from the active constellation satellites, assuming robust orbit control of the active constellation members is maintained. However, the cube method permits collisions between objects as long as their separation is smaller than the size of the cube, and this could lead to an over-estimation of the collision risk between active and failed constellation satellites. Similarly, the randomisation of the mean anomalies of the satellites could introduce collision risk where there might be none in reality (conversely, constellation satellites that are on eccentric disposal orbits with shorter orbital periods and perigees at low altitudes that are subject to atmospheric drag will quickly lose synchronicity with the constellation, and the randomisation of the mean anomalies for these satellites should provide an appropriate mechanism for estimating the collision probability).

To test the robustness of the assumptions inherent to the typical use of the cube method, the DAMAGE code was applied to a simulation study of a large constellation operating in LEO. The normal simulation outputs (e.g. number of objects, number of collisions, collision location, etc.) were enhanced by a simple metric that was used to characterise the orbital geometry of the colliding pair: one orbit was determined to be wholly within another orbit and, therefore, non-intersecting at the time of the collision (or close approach) if:

$$r_{p,i} < r_{p,j} \text{ and } r_{a,i} < r_{a,j}, \text{ or}$$
$$r_{p,j} < r_{p,i} \text{ and } r_{a,j} < r_{a,i}, \quad (2)$$

where $r_{p,i}$ is the perigee radius of object i, $r_{p,j}$ is the perigee radius of object j, $r_{a,i}$ is the apogee radius of object i, and $r_{a,j}$ is the apogee radius of object j at the time of the collision or close approach (Figure 1). This metric is not robust to some orbital geometry as (2) can be satisfied by orbits that do intersect. Hence, this metric was used to guide the analysis only.

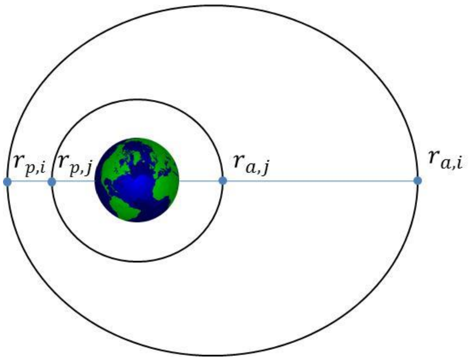


Figure 1. Schematic showing two non-intersecting orbits.

2.3 Simulation study

The scenario used for the simulations was based on a reference case comprising:

- Initial population: all objects ≥ 10 cm with perigee < 2000 km in orbit on 1 Jan 2013 (data from MASTER)
- Future launch traffic: repeat 2005-2012 launch cycle (data from MASTER)
- Projection period: 1 Jan 2013 to 1 Jan 2213
- Post-mission disposal (PMD) of 90% of spacecraft and rocket bodies to a 25-year orbit
- No explosions and no collision avoidance

The baseline constellation case (shown in Figure 2), which was the same as reported in [6] and [8], then included the following in addition to the reference:

- Walker-delta constellation comprising 1080 satellites in 20 orbital planes at 1100 km altitude and inclined at 85°
- Constellation satellite design lifetime of 5 years, 200 kg and 1 sq. metre
- Constellation build-up phase from 1 Jan 2018 to 1 Jan 2021 with 20 launches per year and 18 satellites per launch
- Constellation replenishment phase from 1 Jan 2021 to 1 Jan 2070 (50 years) with 12 launches per year and 18 satellites per launch. Note that the first replenishment launches commenced on 1 January 2023
- PMD of 90% of constellation spacecraft to a 400 × 1100 km disposal orbit
- Immediate de-orbit of rocket bodies

A “high-definition” version of the scenario was established by neglecting the reference, or background, population and performing the simulation with only the baseline constellation and at a high temporal resolution.

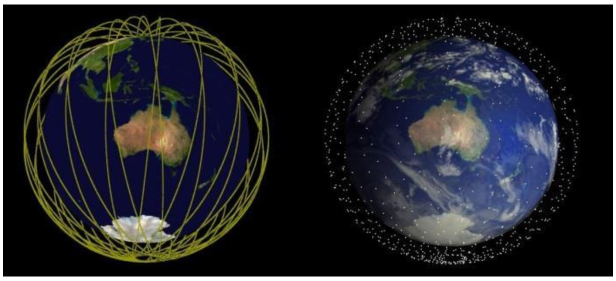


Figure 2. Baseline constellation orbits (left) and satellites (right) visualised using DAMAGE.

The high temporal resolution was obtained by switching from the default propagation time-step of five days to a new time-step of 10 seconds on 1 January 2025, by which point several thousand constellation satellites were on orbit, including those on eccentric disposal orbits and those that had failed (Figure 3). The projection then commenced from this time for one year. During this period, all close approaches < 5.5 km were identified based on a simple distance check for all possible object pairs. These data provided the "ground truth". At the same time, the cube method – operating without the randomisation of mean anomalies but at the 10 second time-step resolution – was used to identify close approaches at the same miss distance. The recorded conjunction events did not include close approaches that might have taken place between each time-step. Finally, the same cube method was used to record close approaches but at five-day intervals.

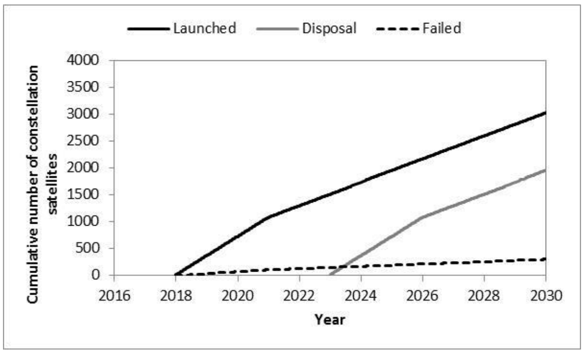


Figure 3. Cumulative number of constellation satellites launched, on disposal orbits or failed.

In summary, the high-definition study sought to provide a robust test of the cube method via the analysis of three cases: (i) a simple distance test to generate the "ground truth", which was performed at 10 second intervals but did not permit the computation of collision probabilities, (ii) the cube method, without randomisation of the mean anomaly, applied at 10 second intervals, and (iii) the cube method, without randomisation of the mean anomaly, applied at 5 day intervals. The cube method with 10 second time-step was used to capture the same events as the "ground truth" but with an estimate of the collision probability for every event. Then, the cube method with 5 day time-step provided a test of the typical implementation of the cube method – with a sampling interval of several days – to determine the sufficiency of the sampling in time and to evaluate the characterisation of the events and the accuracy of the collision probability estimates.

200 Monte Carlo runs were performed for the reference and baseline constellation cases. However, due to the computational effort required for the high-definition study, the number of Monte Carlo runs for this was limited to 20 (which, nevertheless, required several months on an 8-core PC node to complete). Given the low number of Monte Carlo runs, the short projection period and the use of a relatively small miss distance for the high-definition study, the cube method employing a 5 day time-step was not expected to record a significant number of conjunction events. Therefore, further analysis was performed using the close approaches < 17 km recorded by this cube method.

3 RESULTS

To provide some context for the high-definition study, the results from the baseline constellation case are presented first.

3.1 Baseline constellation case

The evolution of the effective number of objects in LEO for the baseline constellation case and the reference case is shown in Figure 4.

These results are consistent with those presented in [6] and [8]. They show that the impact of the constellation on the orbital object population can be separated into three components: a quick population rise during the constellation build-up and replenishment; a period of population decay as PMD measures reduce the number of constellation satellites; and a long-term, gradual increase in the population due to collisions involving long-lived, failed constellation satellites.

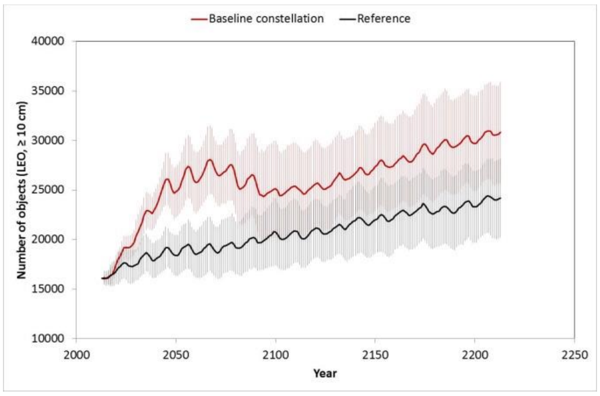


Figure 4. Effective number of objects in LEO from the reference and baseline constellation cases. Average and standard deviation shown for each case.

The cumulative number of catastrophic collisions for the baseline constellation case is shown as a function of time in Figure 5.

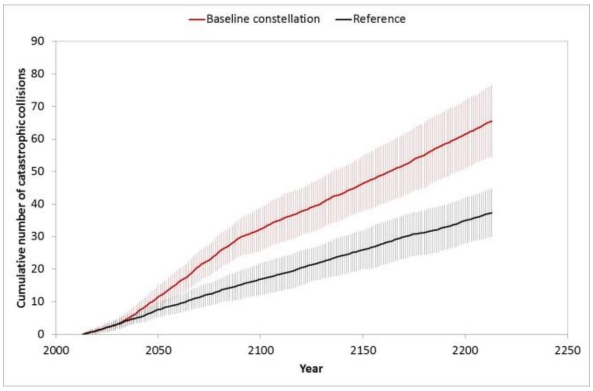


Figure 5. Cumulative number of catastrophic collisions from the reference and baseline constellation cases. Average and standard deviation shown for each case.

The increase in the number of catastrophic collisions, with respect to the reference case, arises because of the additional events involving inactive constellation satellites and fragments of constellation satellites. Of these additional events, approximately half were caused by collisions involving only constellation-related objects and the remaining half were caused by collisions involving a constellation-related object and an object from the background population (Table 1).

Table 1. Average number of catastrophic collisions from the baseline constellation case.

All	Constellation versus background	Constellation versus constellation ("self-induced")
65.6	15.1	15.0

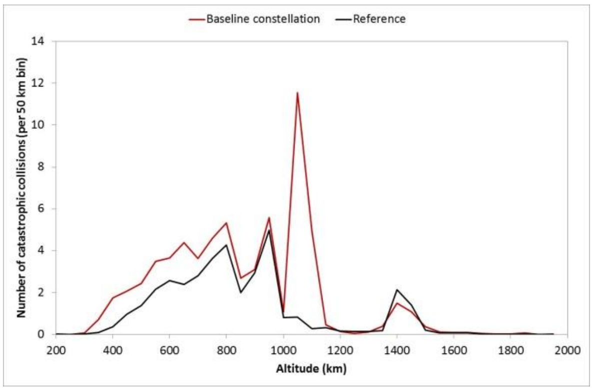


Figure 6. Altitude distribution for catastrophic collisions from the reference and baseline constellation cases.

The effect of the constellation on the altitude distribution of catastrophic collisions (from the entire projection period) is shown in Figure 6 as a function of the altitude of the event. The constellation resulted in a significant increase in the number of catastrophic collisions at the mission altitude and an equally important increase at altitudes below the mission altitude.

Figure 7 provides a view of the number of collisions per cubic kilometre (i.e. the collision density) for events that involved only intact, inactive constellation satellites. Active satellites were assumed to avoid all collisions during their lifetime. The figure highlights the distribution in terms of altitude (similar to the distribution shown in Figure 6) and in declination, or latitude. Unsurprisingly, it is apparent that collision events involving intact constellation satellites were common at the mission altitude (1100 km) and at high latitudes (85°), due to the orbital elements of constellation satellites. However, self-induced collisions were also important at lower altitudes, especially around the perigees of the disposal orbits used by the constellation (400 km).

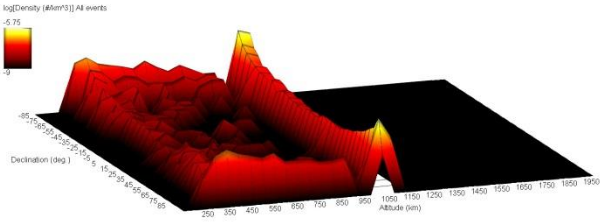


Figure 7. Average number of collisions per km^3, or collision density, involving only intact, inactive constellation satellites as a function of altitude and declination. Data from the baseline constellation case.

The results shown in Figure 8 indicated that failed constellation satellites were responsible for the more numerous self-induced collisions at the mission altitude, whilst constellation satellites that had been transferred successfully to disposal orbits at the end of their lifetimes were responsible for a relatively few self-induced collisions below the mission altitude (Table 2). In general, impacts speeds were high (> 14 km/s) at the mission altitude, and at altitudes corresponding with the disposal orbit perigees (400 km) for events involving constellation satellites, but were relatively low when they occurred elsewhere in LEO (Figure 9).

The proportion of all self-induced and other catastrophic collisions involving objects on intersecting orbits determined using (2) was 56.0% for the reference case and 63.3% for all catastrophic collisions in the baseline constellation. Table 3 details the proportion of intersecting orbits for all self-induced, catastrophic collisions delineated by the collision partners.

Table 2. Average number of self-induced catastrophic collisions from the baseline constellation case.

	Number of self-induced collisions	Collision rate (#/year)
All events	13.39	0.072
Both failed	10.69	0.057
Both disposal	2.66	0.052
Including failed	10.73	0.057
Including disposal	2.70	0.052

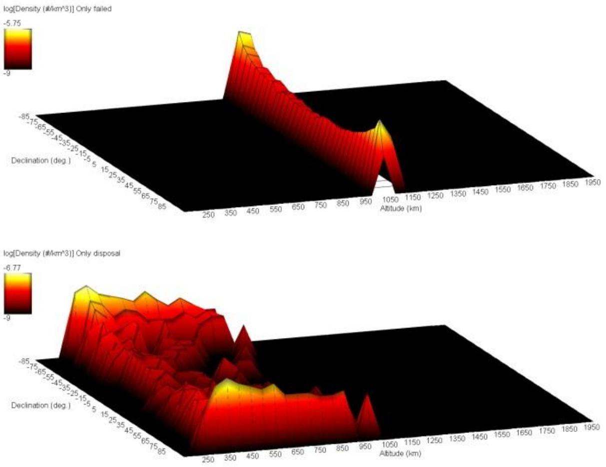


Figure 8. Average number of collisions per km^3 involving only intact, failed constellation satellites (top) or constellation satellites on disposal orbits (bottom) as a function of altitude and declination. Data from the baseline constellation case.

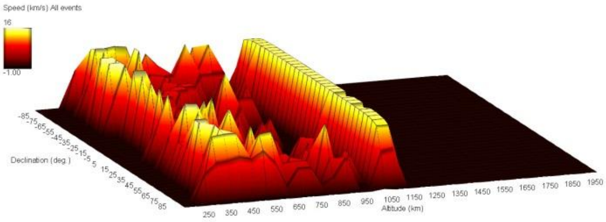


Figure 9. Average impact speeds for intact, inactive constellation satellites as a function of altitude and declination. Data from the baseline constellation case.

The results in Table 3 indicate that catastrophic collisions involving failed constellation satellites – the suggested cause of the long-term impact of the constellation on the space environment in [6] and [8] – were generally due to objects on intersecting orbits according to (2). This result should add confidence to the conclusions that have been drawn in [6] and [8], as it suggests that the cube method is identifying collisions involving failed constellation satellites appropriately. Conversely, catastrophic collisions involving constellation satellites on disposal orbits provided the lowest proportion of intersecting orbits (27.1%), although these were generally few in number (20% of all catastrophic collisions involving satellites from the constellation),.

Table 3. Average number of self-induced collisions and proportion of collisions involving objects on intersecting orbits from the baseline constellation case delineated by collision partner.

	Number of self-induced collisions in 200 year projection	Proportion of those self-induced collisions involving intersecting orbits, based on (2)
All events	13.32	77.1%
Only failed	10.64	89.8%
Only disposal	2.64	27.1%
Including failed	10.68	89.4%
Including disposal	2.68	26.7%

3.2 High-definition study

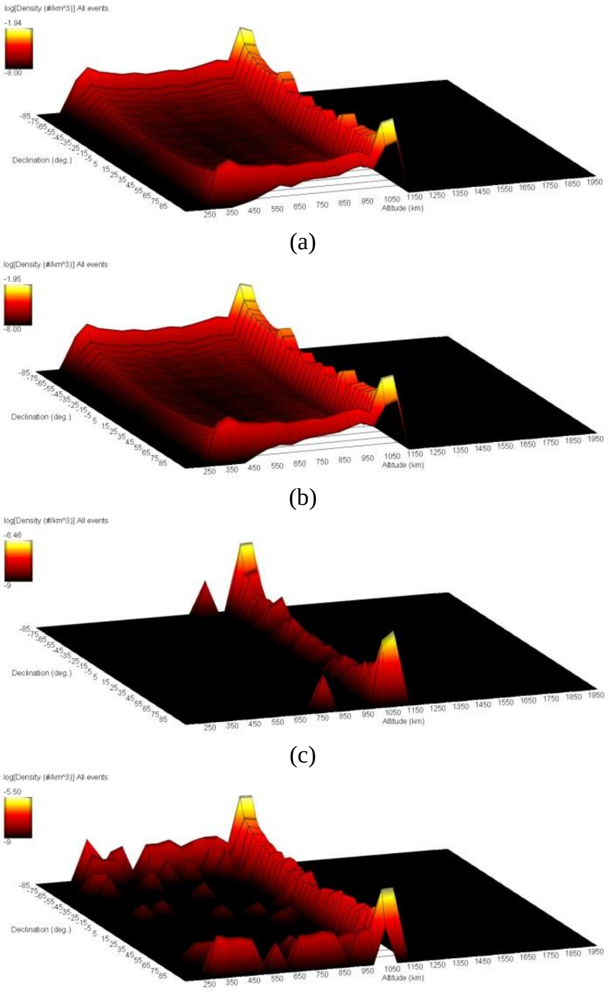
The high-definition study recorded an average of 577,885 instances when two constellation satellites were within 5.5 km and where the energy-to-mass ratio of the satellite pair exceeded 40 J/g. The cube method with a 10 second time-step recorded 585,412 high-energy events, on average, and the cube method with a 5-day time-step resulted in the detection of only 12.85 high-energy conjunctions, on average, for the year (Table 4).

The number of conjunctions detected by the cube methods is highly correlated with the number of events in the “ground truth”, regardless of the time-step or miss distance ($R^2 > 0.9850$).

Figure 10 shows the number of high-energy conjunction events per cubic kilometre as a function of altitude and declination for the detected events. Unsurprisingly, the distribution of “real” close approaches follows closely the spatial density distribution of the constellation satellites (Figure 11; showing the spatial density at the end of the projection period), with most events occurring close to the constellation mission altitude (1100 km). These events involved the active constellation satellites and the failed constellation satellites. Fewer, but still important, events took place at all altitudes below the constellation and involved constellation satellites that were placed successfully onto eccentric disposal orbits.

Table 4.Average number of high-energy (> 40 J/g) conjunction events from the high-definition constellation case. The proportion of events involving satellites on orbits that were classified as intersecting is shown in parentheses.

	"ground truth", < 5.5 km	**cube method, 10 sec time-step, < 5.5 km**	**cube method, 5 day time-step, < 5.5 km**	**cube method, 5 day time-step, < 17 km**
All events	577,885.55 (95.8%)	585,412.60 (95.8%)	12.85 (95.7%)	184.25 (96.3%)
Both failed	142,654.95 (97.0%)	143,010.60 (97.0%)	3.40 (97.1%)	51.70 (98.3%)
both disposal	7,508.25 (31.0%)	7,107.55 (30.9%)	0.10 (50.0%)	4.80 (35.4%)
Both active	165,375.95 (100%)	171,209.20 (100%)	2.60 (100%)	41.60 (100%)
Including active	427,697.10 (96.5%)	435,270.05 (96.5%)	9.35 (95.7%)	127.65 (97.8%)
Including failed	404,994.85 (95.2%)	407,089.65 (95.2%)	10.15 (95.1%)	137.75 (97.3%)
Including disposal	7,540.00 (30.8%)	7,138.15 (30.8%)	0.10 (50.0%)	5.00 (34.0%)
Not active	150,188.45 (93.7%)	150,142.55 (93.9%)	3.50 (95.7%)	56.60 (92.8%)



(a)

(b)

(c)

(d)

Figure 10. Average number of conjunctions > 40 J/g per km^3 involving intact constellation satellites as a function of altitude and declination. High-definition case for (a) "ground truth", (b) cube algorithm with 10 sec time-step, (c) cube algorithm with 5 day time-step and miss distance < 5.5 km, and (d) cube algorithm with 5 day time-step and miss distance < 17 km.

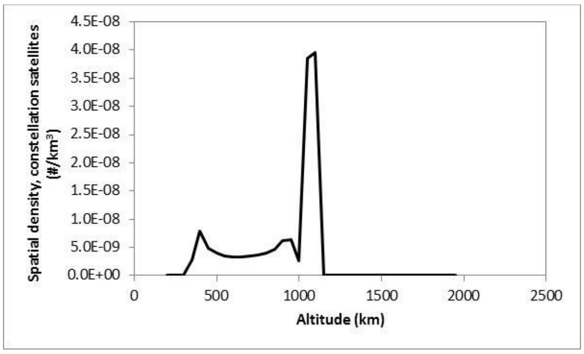


Figure 11. Average spatial density of constellation satellites at the end of the high-definition projection period as a function of altitude.

It can be readily argued that changing the spatial density would result in a different distribution of conjunction events. Such a change could be achieved by, for example, an increase or decrease of the PMD success rate: a decrease in the PMD success rate would result in fewer constellation satellites being transferred to the disposal orbits, which would limit the number of satellites traversing the altitude regime below the mission altitude an, so, limit the number of conjunctions taking place there. Conversely, an increase of the PMD success rate would produce a corresponding increase in the number of satellites on disposal orbits and, hence, the number of conjunctions below the mission altitude (although there is an upper limit which would be achieved when the PMD success rate reaches 100%).

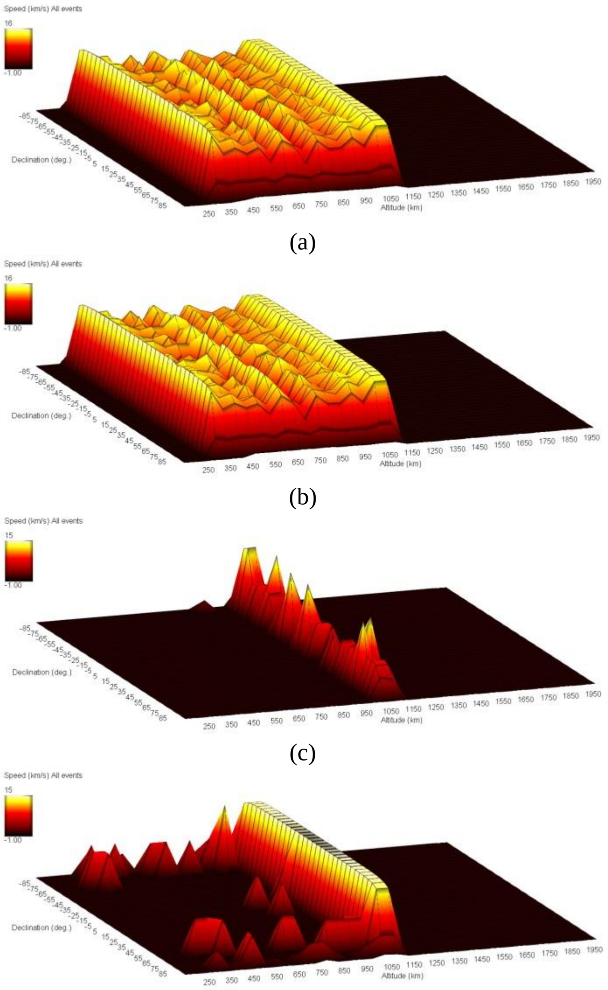
Figure 10 also shows that the altitude-declination distribution of the "true" close approach event density is almost exactly reproduced by the cube method if the time-step is 10 seconds (Figure 10 b; R^2 = 0.9990) and is reasonably approximated if the time-step is 5 days (Figure 10 c; R^2 = 0.7076). In particular, the preference for events at the mission altitude and at high latitudes is captured effectively.

Of the 577,855 conjunction events recorded in the year from 1 January 2025, 95.8% involved satellites on orbits that were classified as intersecting, according to (2). This proportion is significantly higher than the proportion calculated for all catastrophic collision in the reference case (56.0%), and the proportion determined for the self-induced catastrophic collisions in the baseline case (77.1%). This suggests that the conjunctions involving constellation satellites feature more intersecting orbits than non-constellation conjunctions. A similarly high proportion of intersecting orbits (95.8%) was identified using the cube method with a 10 second time-step, and a nearly identical proportion when a 5 day time-step was used (95.7%).

For any conjunction involving two intact constellation satellites and detected using the cube method, the impact speed provides the only varying parameter in the computation of collision probability in (1) and also in the calculation of the energy-to-mass ratio, which determines whether a subsequent collision is catastrophic (> 40 J/g) or non-catastrophic (< 40 J/g). Consequently, a comparison of the distribution of impact speeds with the “ground truth” is important to the evaluation of the cube method. Figure 12 shows the average impact speed (close approach speed in the context of the high-definition study) as a function of the altitude and declination. The average speeds were generally high for the catastrophic collisions (as expected) across all altitudes below the constellation and revealed no significant influence of declination. This distribution was also found using the cube method, but only for the 10 second time-step; when the 5-day time-step was used, the cube method was unable to capture the same distribution, even if the miss distance was increased to 17 km, because of under-sampling of conjunction events at altitudes below the constellation.

Importantly, the results in Figure 12 also suggest that under-sampling occurred within the same altitude region in the 200-year projection of the baseline constellation (Figure 9) because those results differ from the “ground truth” in more-or-less the same way. An exception to this is at 400 km altitude, which shows high average impact speeds in Figure 9. The consequence of any under-estimation of the impact speeds could be the corresponding under-estimation of catastrophic collisions (and potentially over-estimation of non-catastrophic collisions) above 400 km and below 1100 km. As these events are driven by intact constellation satellites that have been transferred successfully to disposal orbits, analyses focused on that object type as well as the failed and active satellites were conducted.

However, the collision rate for each event type (i.e. total collision probability over the projection period divided by the time) predicted by the cube methods with 10 second and 5 day time-steps, and for a miss distance of 5.5 km, were highly correlated: $R^2 = 1.0$. In addition, the absolute error in the collision rate prediction computed over all conjunctions was 15.1%, meaning that reasonable estimates of the total collision probability were obtained even with the 5 day time-step. In general, therefore, in can be argued that the cube method with a 5 day time-step was able to broadly capture the distribution of the key high-energy events – those involving active and failed constellation satellites – and estimate the characteristics (speed and energy) of the events. In addition, the orbits of the satellites involved in those conjunction events appeared to be predominantly intersecting, according to (2), making the cube method an appropriate choice. Nevertheless, some issues were identified with particular conjunctions, especially those involving satellites on disposal orbits. Consequently, an analysis by object type was conducted and the results are reported below.



(a)

(b)

(c)

(d)

Figure 12. Average impact speed for conjunctions > 40 J/g per km^3 and involving intact constellation satellites as a function of altitude and declination. High-definition case for (a) “ground truth”, (b) cube algorithm with 10 sec time-step, (c) cube algorithm with 5 day time-step and miss distance < 5.5 km, and (d) cube algorithm with 5 day time-step and miss distance < 17 km.

3.2.1 Events involving active satellites

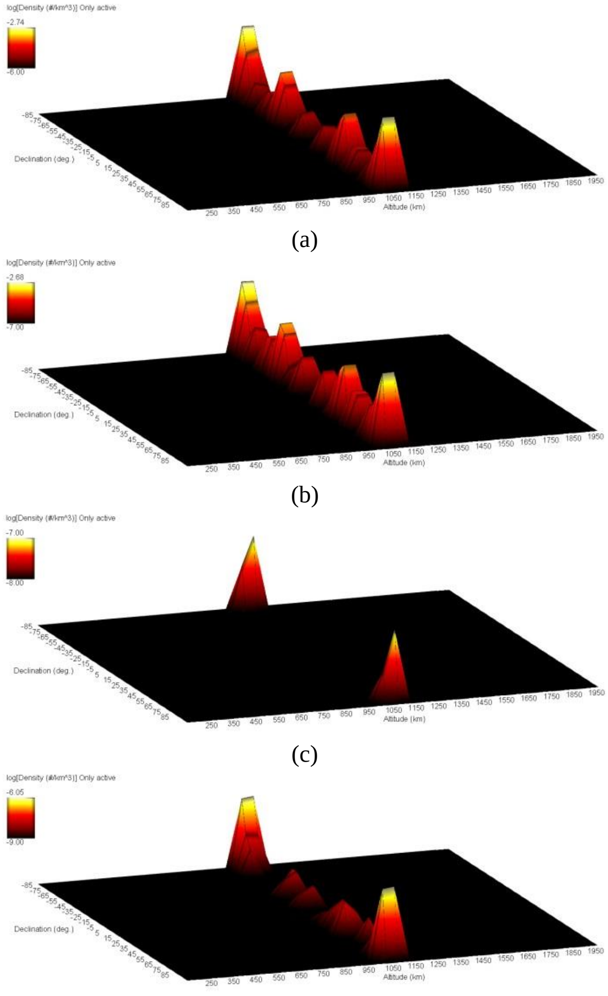
Figure 13 shows the number of high-energy conjunction events per cubic kilometre involving only active constellation satellites as a function of altitude and declination for the detected events. As expected, the conjunction events only took place at the mission altitude, and most events occurred at high- and mid-latitudes where the orbital planes of the Walker-delta constellation crossed. As the time-step increased from 10 seconds to 5 days, the cube method struggled to capture the latitude distribution, even when the miss distance was increased (although in that latter case, events occurring at mid-latitudes were detected).

The average close approach speeds as a function of the altitude and declination and computed by the cube method with a 10 second time-step for events only involving active satellites were perfectly correlated with the “ground truth” (R^2 = 1.0) but were quite poorly estimated by the cube method with a 5 day time-step (R^2 = 0.2049). Here (and below), these correlations were computed only for cells in the control volume where both methods had recorded conjunction events. Similarly, the altitude-declination distribution of the average collision probabilities was moderately correlated: (R^2 = 0.5392). The collision rate predictions made by the cube methods (< 5.5 km miss distance) for the active satellites are shown in Figure 14.

3.2.2 Events involving failed satellites

Conjunctions involving failed constellation satellites were the second-most numerous events in the high-definition study (Table 4) so it is not surprising that, even with a time-step of 5 days, there were an adequate number of samples by the cube method to generate a good reproduction of the altitude-declination distribution of high-energy conjunction event density. Figure 15 shows the average number of events > 40 J/g per cubic kilometre involving only the failed satellites for the detected events.

For conjunction events involving only failed constellation satellites, the correlation in the altitude-declination event density distribution with respect to the “ground truth” was calculated to be R^2 = 0.9978 for the cube method with a 10 second time-step and R^2 = 0.9382 for a 5 day time-step. Predictions by the cube method with a 5 day time-step of the distributions for the average close approach speeds (R^2 = 0.3840) and collision probabilities (R^2 = 0.5907) were slightly improved compared with the corresponding predictions for the active satellites. In absolute terms, the collision rate for failed satellites predicted by the cube method with a 5 day time-step (Figure 14) was a good estimate of the rate predicted using a 10 second time-step.



(a)

(b)

(c)

(d)

Figure 13. Average number of conjunctions > 40 J/g per km^3 involving only active constellation satellites as a function of altitude and declination. High-definition case for (a) “ground truth”, (b) cube algorithm with 10 sec time-step, (c) cube algorithm with 5 day time-step and miss distance < 5.5 km, and (d) cube algorithm with 5 day time-step and miss distance < 17 km.

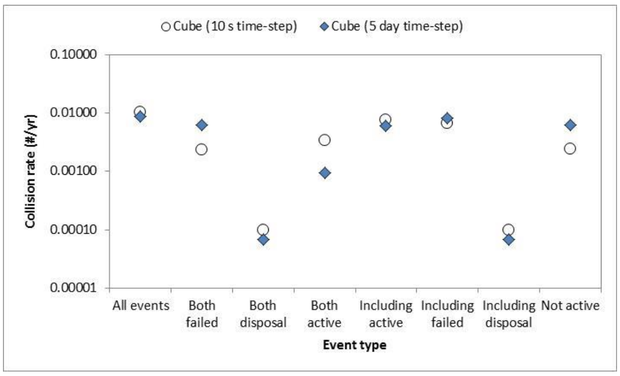
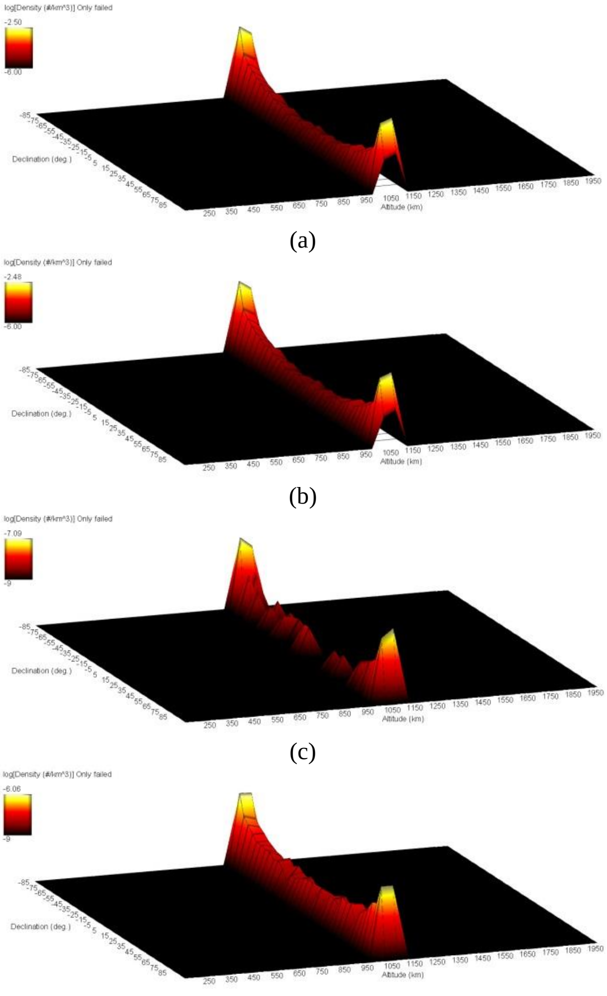


Figure 14. Catastrophic collision rates (#/year) computed by the cube method with 10 second and 5 day time-steps, and based on events < 5.5 km.

3.2.3 Events involving satellites on disposal orbits

The analysis reported above identified conjunctions involving constellation satellites that had been transferred successfully to disposal orbits as of particular concern, with respect to the robustness of the cube method. These events were the least numerous in the high-definition study (Table 4) and the evidence from section 3.2 suggested that the 5 day time-step resulted in an under-sampling, even over long projection periods. As a consequence, the expectation was that high-energy events involving satellites on disposal orbits would not be characterised well, in terms of their altitude-declination distribution, close approach speed and collision probability.



(a)

(b)

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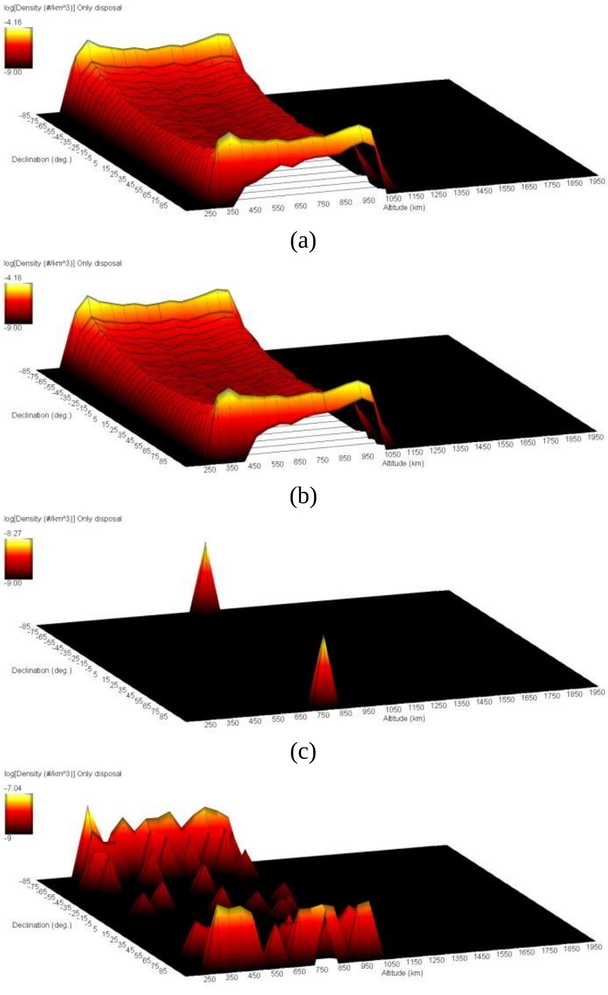
(d)

Figure 15. Average number of conjunctions > 40 J/g per km^3 involving only failed constellation satellites as a function of altitude and declination. High-definition case for (a) "ground truth", (b) cube algorithm with 10 sec time-step, (c) cube algorithm with 5 day time-step and miss distance < 5.5 km, and (d) cube algorithm with 5 day time-step and miss distance < 17 km.

Figure 16 shows the average number of events > 40 J/g per cubic kilometre involving only constellation satellites on disposal orbits as a function of altitude and declination for the detected events. As expected, the figure clearly shows that a 5 day time-step was not sufficient to capture the full altitude-declination distribution of the events. In particular, no events were detected at 400 km altitude, where the perigees of the disposal orbits resulted in a relatively high density of events in the "ground truth". The distribution in latitude is somewhat captured by the cube method when the maximum miss distance was increased to 17 km, and some events at mid-latitudes were also seen. Nevertheless there were clear indications that an insufficient number of samples were made.



(a)

(b)

(c)

(d)

Figure 16. Average number of conjunctions > 40 J/g per km^3 involving only constellation satellites on disposal orbits as a function of altitude and declination. High-definition case for (a) "ground truth", (b) cube algorithm with 10 sec time-step, (c) cube algorithm with 5 day time-step and miss distance < 5.5 km, and (d) cube algorithm with 5 day time-step and miss distance < 17 km..

For conjunction events involving only constellation satellites on disposal orbits, the correlation in the altitude-declination event density distribution with respect to the “ground truth” was calculated to be R^2 = 0.9928 for the cube method with a 10 second time-step and R^2 = 1.0 for a 5 day time-step. The caveat, here, is that the correlations were based on the comparison of cells from the control volume where both methods had detected conjunction events. Given the very low number of events detected by the cube method with a 5 day time-step, the correlation statistic is not useful. Similar issues existed with correlations based on the altitude-declination distributions of the average close approach speeds (R^2 = 1.0) and collision probabilities (R^2 = 1.0). However, the collision rate predictions (Figure 14) were of the right order of magnitude, in absolute terms.

4 DISCUSSION

The high-definition study, reported above, was motivated by the need to build confidence in the cube collision prediction method employed in evolutionary models that have been used to study the impact of large constellations on the space environment. In particular, the assumptions based on the kinetic theory of gas and sampling through time, as described by [11], were thought to be potentially problematic for constellations where the orbital and collision geometry cannot be considered in the same manner as for debris.

At the same time, conducting the high-definition study allowed a detailed look at one of the possible key drivers of the space debris population: self-induced collisions involving satellites from a large constellation. Results from the baseline constellation study suggested that nearly 25% of all catastrophic collisions predicted over the 200 year projection period were self-induced.

Through the work conducted and reported in [6] and [8], evolutionary codes have identified collisions involving failed constellation satellites as the leading source of long-term debris growth following the operation of a constellation in LEO. It is likely that the number of failing satellites will be some proportion of the total number of satellites launched, with the PMD success rate determined by the number of satellites that remain functional at the end of life. The self-induced collision rate between failed satellites will therefore increase over time while the constellation is operational because the failed satellites accumulate at the mission altitude without the influence of atmospheric drag, then the rate remains constant after the end of the mission, once all constellation replenishment launches have ceased.

In contrast, the number of satellites on disposal orbits will always be constrained by the satellite replenishment launch rate (i.e. the number of satellites on disposal orbits will be a function of launch rate and the PMD success rate). It becomes, effectively (and at best), a “one in, one out” system and can never increase beyond this. Indeed, shortly after the end of the constellation mission, the number of satellites on disposal orbits will reach zero. As a result, the contribution to the self-induced collisions will be limited in number, and over time, compared to the contribution coming from the failed satellites. The results from the baseline and high-definition studies confirm this hypothesis. Nevertheless, collisions below the mission altitude involving constellation satellites and objects from the background population represented approximately 25% of all the catastrophic collisions in the baseline constellation case, predicted over the 200 year projection period. Consequently, this population of constellation satellites is still as important as the population of failed constellation satellites. Making sure that the prediction of these collisions is being performed correctly is, therefore, of considerable importance.

The high-definition study also highlighted the large proportion of high-energy conjunction events involving active constellation satellites. In fact, 74% of all of the conjunctions identified in the “ground truth” data involved at least one active satellite, and 28.6% of the conjunctions involved two active constellation satellites. Given the equally high proportion of conjunction events involving at least one failed satellite (95.2%) it is simple to conclude that close approaches to active satellites by failed satellites represented the predominant conjunction type in the high-definition study. The number of these conjunctions can be reduced through a high PMD success rate; although it is worth noting that an optimistic value of 90% was assumed for this study. Maintaining good situational awareness and manoeuvring active constellation satellites when close approaches involving failed satellites are predicted are, therefore, crucial requirements for the operation of large constellations.

With predictions made by evolutionary models likely to contribute to future discussions on the regulation of large constellations and the mitigation of harmful impacts, having confidence in the ability of the cube method is vital. This requirement is made more important because of the significant influence that large constellations can have on the space environment, as predicted by these models. The results from the comparison of the cube method with the “ground truth” in the high-definition study have shown that this approach can provide good estimates of the overall self-induced collision rate even for relatively long intervals between samples, but the altitude, declination, and speed may not be adequately characterised without additional sampling for some event types. In particular, this issue affected conjunctions involving constellation satellites on disposal orbits (only 1.3% of all close approaches < 5.5 km in the “ground truth”). Improved sampling can be achieved by decreasing the time-step, increasing the length of the projection period, or increasing the number of Monte Carlo runs. If the latter option is adopted, the cube method will only provide additional sampling if the time-step is also varied between each Monte Carlo run (e.g. by at most the orbit period of a constellation satellite orbit) or the mean anomalies are randomised. This is not trivial, however, and can introduce additional problems. Ultimately, the solution used for the ESA study reported in [9] used an approach that randomised the mean anomalies of inactive constellation satellites, including those on disposal orbits.

Promisingly, the cube method was able to adequately characterise conjunction events involving failed constellation satellites, which play an important role in the long-term evolution of the debris environment following the operation of a large constellation, as discussed. Simultaneously, the application of the intersection test (2) for the orbits of collision and close approach satellite pairs, underlined the suitability of the cube method for the evaluation of events involving failed satellites, for which uncertainties in the orbits and positions are likely. Events satisfying (2) represented more than 95% of the conjunctions involving at least one failed constellation satellite.

5 CONCLUSIONS

A simulation study, motivated by the need to build confidence in the cube collision prediction, has been performed using the DAMAGE evolutionary code. The study focused on self-induced collisions involving the satellites of a large constellation, similar to those proposed by SpaceX, OneWeb and Boeing, for example. A key feature of the study was its very high temporal resolution, achieved through the use of a 10 second time-step applied over a relatively short (year-long) projection period. Analysis of the results of 20 Monte Carlo runs, which generated approximately 11.6 million close approaches in total (0.6 million per Monte Carlo run, on average) indicated that the cube method was able to provide good estimates of the expected collision rates for active and inactive constellation satellites at intervals of 5 days. However, better estimates of the collision rates and better characterisation of the close approaches – especially those involving satellites on disposal orbits – can be obtained if the number of samples is increase, e.g. by decreasing the simulation time-step, increasing the length of the projection period, or increasing the number of Monte Carlo runs (in conjunction with a modification to the sampling in space performed by the mean anomaly randomisation).

In general, the results support the use of the cube method in evolutionary codes, to assess the impact of large constellations on the space environment.

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