ANALYSIS OF CDM COVARIANCE CONSISTENCY IN OPERATIONAL COLLISION AVOIDANCE

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ABSTRACT

This paper investigates the covariance matrices used in conjunction data messages (CDM) provided by the Joint Space Operations Center (JSpOC) from a user perspective. The aim is to find the consistency of the covariance by comparing the given CDM state and covariance to operational mission orbits of ESA satellites. It was shown that the consistency percentage, giving the share of states within the CDM uncertainty, is nearly constant over the time to the closest approach. This suggests that the error growth is modelled realistically and matches the real error growth. Because the covariance was found to be slightly too small, it was attempted to find a scaling factor to match the covariance size, which is expected for a theoretical normal distribution. No common factor was found, because the scaling probably depends on individual satellite and orbit properties, which would suggest that at least a satellite-specific factor can be assumed. Finally, the planned upgrade of covariance tables for the ESA tool ARES in DRAMA based on CDM covariances is introduced.

Keywords: CDM; Covariance; ARES; DRAMA.

1. INTRODUCTION

Collision avoidance is not only reliant on a good estimate of the orbit, but also the uncertainty has to be considered in order to calculate a realistic probability of collision [2]. Since 2010, ESA uses conjunction summary messages (CSM), later replaced by conjunction data messages (CDM)[4], provided by the Joint Space Operations Center (JSpOC) for their routine collision avoidance, which is an improvement compared to the previously used low-precision Two-Line-Elements (TLE) [3]. The CDM contains, among other information, the Cartesian states (XYZ coordinates) and covariances (RTN coordinates) of the two objects at the time of the closest approach (TCA). The reliability and consistency of this covariance matrix shall be investigated in this paper by comparing the CDM states to operational mission orbits of ESA satellites.

The analysis of the covariance matrix in orbital mechanics is often discussed under the term *covariance realism*. Those studies, for example by Vallado [10], focus on the question whether the error of the orbit prediction can be represented by a covariance matrix, which assumes a normal (Gaussian) distribution of the errors. Possible deviations are a non-zero mean of the position errors or the effect of bending along the orbit for large covariances, which cannot be modelled with a matrix in Cartesian coordinates[8]. Instead of those rather theoretical approaches, this paper analyses the CDM covariances from a user perspective in collision avoidance by comparing the prediction errors of the CDMs with the given covariances.

2. METHODOLOGY

This analysis features satellites in different Low Earth Orbit (LEO) altitudes. Namely those are Cryosat-2 ($h \approx$ 720 km), Rapid Eye 1 ($h \approx 630$ km), Swarm B ($h \approx$ 500 km) and Swarm A/C ($h \approx 450$ km). All of those satellites are in near-circular, near-polar orbits. CDM data for conjunction events between October 2015 and October 2016 is used. The number of CDMs per satellite is given in Table 1. The operational mission orbits obtained from ESA Flight Dynamics are assumed as ground truth for the Swarm constellation and Cryosat-2. Ground truth for Rapid Eye 1 is given by an orbit determination from GPS measurements using the ESA Space Debris Office software tool ODIN (Orbit determination by improved normal equations) [1]. To avoid artificial errors, time spans around manoeuvres are erased from the data set.

The main collision avoidance approaches used by ESA aim to increase the separation to the chaser in along-track or radial direction [6]. Thus, these directions are studied for their representation in the covariance matrix. The standard satellite-centred RTN coordinate system is used because the CDM covariance is also given in this system. It shall be investigated how often the mission orbit $\vec{x}_{\mathrm{m}}$ is within the covariance, thus within the interval of CDM state $\vec{x}_{\mathrm{c}}$ plus/minus uncertainty σ_{C}. To focus on the R- and T-direction, only their values from the main diagonal

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Table 1. Number of CDMs per satellite for the analysis.

Satellite	Number of CDMs
Cryosat-2	65029
Rapid Eye 1	3332
Swarm A	10257
Swarm B	25373
Swarm C	10131

are used thus ignoring the overall shape of the covariance ellipsoid.

Disregarding the normal error also reduces the impact of a frame offset, which arises because the CDM states are given in an Earth-fixed coordinate system. To transform the ESA mission orbits, which are in an inertial frame, to an Earth-fixed system, the Earth orientation parameters (EOP) are necessary. If an Earth-fixed frame is used for a future state, like it is done for CDMs, either predicted EOP or the EOP from the day of the prediction could be used. It is not documented or known, what approach is used for CDMs, thus a small frame offset appears when comparing CDM orbits to mission orbits, which are transformed to Earth-fixed coordinates with the actual EOP, known in retrospective, from the day of the conjunction. The frame error is mainly due to the Earth rotation and thus artificially increases the normal error for near-polar orbits. Due to this offset, the results for the normal error cannot be analysed correctly.

The position error ΔP at TCA is calculated by subtracting the reference state from the CDM state of the object and transforming this difference to the RTN system, in which the covariance is given, with the matrix B[9]:

$$\Delta P = B \cdot (\vec{x}_{\text{c}} - \vec{x}_{\text{m}}) \ . \qquad (1)$$

The main measure of this study is called *covariance consistency*, which gives the percentage of the number of states that are within 1-,2- or 3-σ of the CDM covariance. Thus the condition for n-σ in each direction is:

$$\Delta P_{R,T} < \text{n} \cdot \sigma_{R,T} \ , \qquad (2)$$

with the position difference ΔP from Equation 1 and the corresponding standard deviations obtained from the covariance matrix by using the square root of the variances on its main diagonal. A theoretical normal distribution would have 68.3% within 1-σ, 95.5% within 2-σ and 99.7% within 3-σ.

Before the actual analysis, it is checked how the position error behaves in R- and T-direction. If the mean error is close to zero, it can be assumed that the predicted positions spread evenly around the truth values and thus the approach to only use the uncertainties from the covariance matrix can be considered valid. Figure 1 and Figure 2 show the mean position error, calculated from

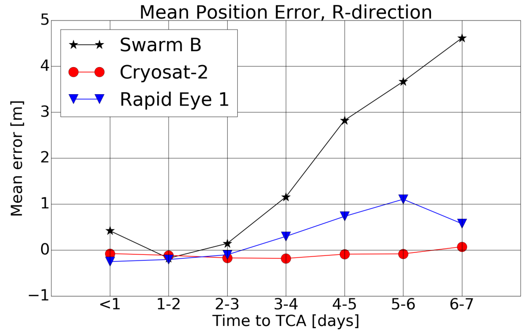

Figure 1. Mean error of operational vs. CDM position in the radial direction.

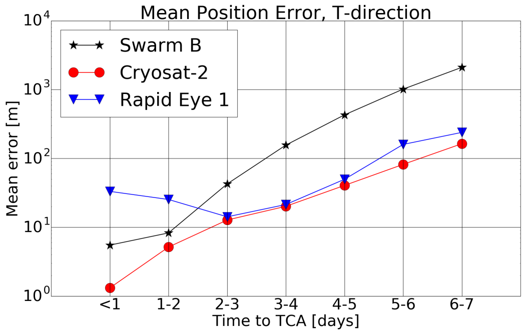

Figure 2. Mean error of operational vs. CDM position in the along-track direction.

Equation 1, of the satellites along the two directions over time to TCA, which is binned to one day steps. Only Swarm B is used because the deviations for Swarm A/C are similar. It can be seen that except for some outliers along the T-direction with more than 5 days to TCA, the mean is close to zero (compared to the size of the uncertainty). It is further assumed that the covariance is small enough to not bend along with the orbit and thus the assumption of a Cartesian covariance is considered valid.

3. COVARIANCE CONSISTENCY

Figure 3 shows the covariance consistency for Cryosat-2 in R- and T-direction for the levels of 1-, 2- and 3-σ. It can be seen that the graphs are relatively constant except for a drop for CDMs less than two day before TCA. The levels of the 1-,2- and 3-σ graphs are not equal for the two directions but in a similar order at approximately 60%-85%-95%, thus lower than it would be expected for a theoretical normal distribution, which implies a slight underestimation of the covariance.

The covariance consistency for Swarm B is shown in Figure 4. The general behaviour is similar to the previous one, but the percentages are lower. The levels for 1-,2- and 3-σ are at approximately 50%-80%-90%, thus also having an underestimation of the uncertainty. The lev-

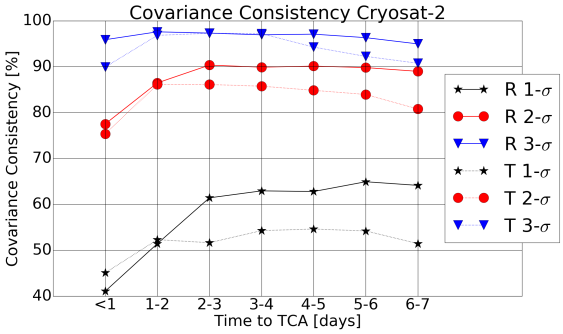

Figure 3. Covariance consistency for Cryosat-2.

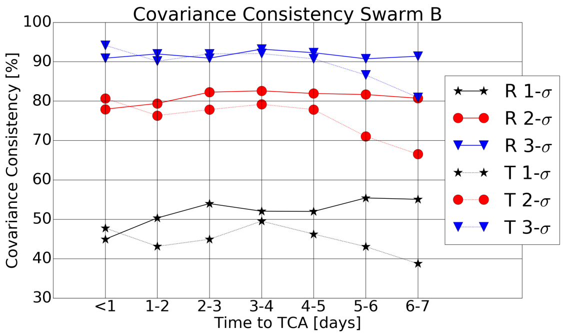

Figure 4. Covariance consistency for Swarm B.

els for Swarm A in Figure 5 are similar but show larger differences between the R- and T-directions. The consistency of Swarm C is nearly equal to that of Swarm A.

For Rapid Eye 1 (Figure 6), the distribution is clearly different. The percentages of the consistency are less stable than previously, but it is still visible that there is a constant trend. A possible explanation for this more unsteady data set is that there are significantly less CDMs for Rapid Eye than for the other satellites.

If the covariance consistency is nearly constant over the time to TCA, it can be inferred that the covariance propagation is modelling the error growth realistically. A decreasing (or increasing) consistency percentage with a

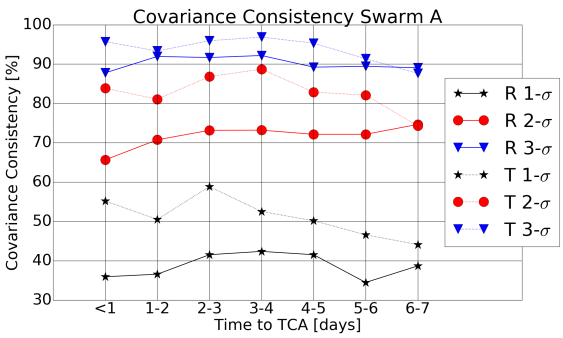

Figure 5. Covariance consistency for Swarm A.

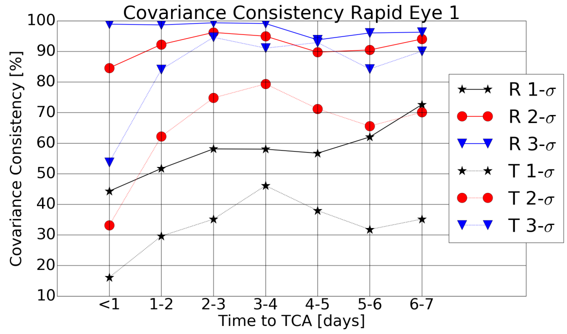

Figure 6. Covariance consistency for Rapid Eye 1.

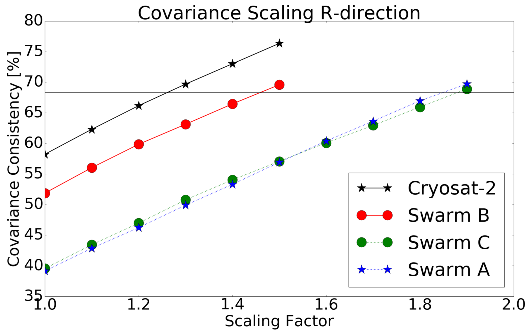

Figure 7. Covariance scaling for the radial direction and testing 1-σ consistency.

decreasing time to TCA would imply a too fast (or too slow) reduction of the covariance. Although it appears to be slightly underestimated, the consistency shows that the covariance for a single satellite has a predictable and stable behaviour, which is necessary for a reliable use in collision avoidance.

4. SCALING THE COVARIANCE

As shown in the previous section, the covariance is slightly underestimated compared to a theoretical normal distribution. Thus it shall be investigated whether it is possible to scale the uncertainty with a common factor to obtain a consistency that matches the normal distribution. This is attempted for Cryosat-2 and the Swarm constellation. Rapid Eye 1 is left out, because its behaviour showed a non-constant consistency.

Instead of a function of time as it was in the previous section, the covariance consistency is calculated for all CDMs within 0-7 days to TCA and thus is given as an overall value over the scaling factor. The consistency is calculated via Equation 2, but in this case n represents the scaling factor, which is used as the independent parameter.

Figure 7 shows the covariance consistency percentage for n-σ in the radial direction over the applied scaling factor

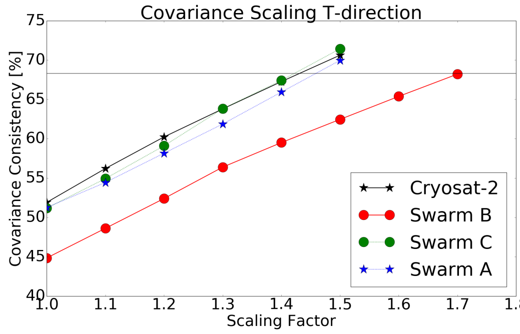

Figure 8. Covariance scaling for the along-track direction and testing 1-σ consistency.

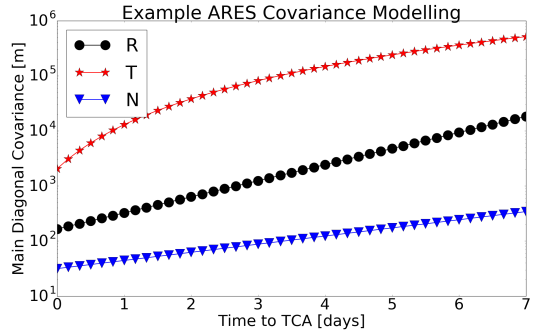

Figure 9. Example of modelled covariance functions for ARES.

n. The factor is approximately 1.25 for Cryosat-2, 1.5 for Swarm B and 1.9 for Swarm A/C, which would suggest a dependency of the scaling factor on the altitude. In contrast to that, Swarm A/C and Cryosat-2 share the same factor of approx. 1.45 in the along-track direction compared to 1.7 for Swarm B, as shown in Figure 8. Even the R- and T-direction do not have the same factor for the same satellite, which shows that a pure scaling is not enough to match the covariance of a theoretical normal distribution.

From these plots, it can also be noted that the consistency percentage is linear over the scaling factor. Because the second derivative of the normal distribution is zero at 1-σ, this indicates that the given covariance is close to the 1-σ region.

5. USE OF CDMS FOR COVARIANCE TABLES IN ARES

The ESA software DRAMA (Debris Risk Assessment and Mitigation Analysis) contains the tool ARES (Assessment of Risk Event Statistics), which is capable of e.g. estimating the number of manoeuvres a satellite has to perform based on the accepted risk level and orbital regime [7]. Therefore an artificial covariance has to be assigned to each object solely based on its size and orbit. The current version of ARES includes covariance tables for the RTN-directions based on old CSM data and TLE estimation [5]. After showing that the CDM covariances are a reliable source, those tables shall be upgraded by analysing a large group of CDMs for different objects. Of course, the covariance is highly dependent on e.g. observation techniques, which cannot be represented by a fixed model, but it shall be attempted to find at least a good approximation of the covariance matrix.

The original implementation featured an exponential function of the type 10^{ax} for all directions. Analysis of the CDM data showed that an exponential polynomial of the type $e^{a\log(x)} = x^a$ is more appropriate for the along-track direction, because this led to smaller residuals in 70% of the cases. In contrast to that, the other two directions only showed lower residuals in approx. 40% of the cases, thus they keep the original model. This leads to the following fit functions for the standard deviations:

$$\sigma_{R,N} = C_{R,N} \cdot 10^{\alpha_{R,N} \cdot t} , \quad (3)$$

$$\sigma_T = C_T \cdot (t+1)^{\alpha_T} , \quad (4)$$

with the time to TCA t in days and the parameters C and α, which are fitted to the CDM covariance data with a robust regression method.

To reflect the different uncertainties for different types of objects and orbits, the CDM data is binned according to size (small, medium, large), perigee altitude, eccentricity and inclination. The bins are chosen to have a good resolution, but still enough data to have statistical significant results for as many bins as possible. Bins with no or insufficient data are filled by either copying from similar bins or scaling from other bins. For example. the uncertainty is relatively stable over different inclinations, whereas it is increasing with a decreasing perigee altitude due to the increase of drag forces, which are more difficult to predict. Figure 9 shows an example for such a modelled covariance function. Typically, the largest uncertainties are in along-track direction and the smallest in cross-track direction.

6. CONCLUSION

The results of the consistency analysis showed that the CDM covariance is a good indication for the uncertainty of the orbit, although it seems to be slightly underestimated, which would confirm earlier suggestions by Alfriend [2] from 1999. The percentage of states within the given uncertainty was shown to be nearly constant, which suggests that the covariance propagation is well fitted to the real error growth. A common scaling factor could not be found, which could mean that this factor is dependent on the orbit and the observations. Swarm A and C are very close to each other on similar orbits and also had nearly equal scaling factors, which would support this idea. In this case, the scaling factor could be assumed as a constant for each satellite and thus used operationally including regular checks for consistency.

After it was shown that the CDM covariance is a realistic estimate, it will be used as a reference value for the upgrade of the covariance tables in ARES to model the changes in the covariance over time for different orbits.

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