FUSION OF LASER RANGES AND ANGULAR MEASUREMENTS DATA FOR LEO AND GEO SPACE DEBRIS ORBIT DETERMINATION

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ABSTRACT

In order to deal with space debris objects, the precise and accurate knowledge of their positions is of fundamental importance. Past studies have shown that the accuracy of an orbit determination process depends on the length of the observed arc, the number of observations, the geometry of the observation (object-observer relative geometry), the observables used and their accuracy. The use of one observable, especially if the measurements stem from one single site, brings limits in the achievable accuracy when estimating an orbit. One possible way to improve the results of an orbit determination process is to utilize different kind of observables at the same time.
The main aim of this paper is to study the benefits of fusing laser ranges and angular measurements in an orbit determination process for space debris. We will treat the cases of the two highly populated orbital regions: LEO and GEO. In particular, with the use of only real measurements, we will show the improvements achievable using laser ranges in a typical space debris observation scenario. We will highlight the differences in the results obtained with merged measurements and short observation arcs, w.r.t. the classical angle-only orbit determination. Particular attention is dedicated to understand the main contributions of each observables, of the number of observations and their distribution, on the estimated parameters. The results shown are obtained using real angular/laser measurements provided by sensors of the Swiss Optical Ground Station and Geodynamics Observatory Zimmerwald owned by the Astronomical Institute of the University of Bern (AIUB) and in some cases using also real ranges from the International Laser Ranging Service (ILRS) stations.

Keywords: space debris, orbit determination, laser ranging, LEO, GEO.

1. INTRODUCTION

The increasing number of space debris objects, together with the risk associated for the active space missions, has as main consequence an increase of the working-load for the infrastructures dedicated to the space debris cataloging and orbit maintenance. Due to the high number of target objects and to the limitations of the observing systems, it is necessary to optimize the time available for observations. The scheduling of the observations acquisition for the maintenance of the catalog depends on the quality of the orbit previously determined.
The accuracy of an orbit determination depends on: the number of observations, the length of the observed arc [1, 2], the observation geometry [3] and the accuracy of the observations. Therefore to improve the orbit determination accuracy one can optimize either one or all of the just mentioned factors.
In this paper we want to investigate the effects given by the accuracies and the kind of information carried by different observables in the orbit determination process. Among the most commonly used observables in the orbit determination or improvement processes there are the range measurements and the angular ones. The first kind of measurement can be provided by radar and by laser facilities. Recent studies showed the possibility of successfully track space debris objects with 1 m level precision [4].
To prove the usefulness of the laser range in the space debris field we split our analysis in two parts. In the first, we will show how the two kind of observables affect the results of an orbit determination process performed on a short observation arc. The short arc is used to simulate a typical discovery and follow-up scenario with a limited number of measurements. In the second, we will show the effects of the data fusion on the space debris catalog maintenance.
The first analysis is focused essentially in the two regions mostly populated by space debris, respectively the LEO and the GEO region. The second is applied to a wider set of orbital regimes.
For the tests only real data are used: the angular and part of the laser measurements were provided by the sensors of the Swiss Optical Ground Station and Geodynamics Observatory Zimmerwald owned by the AIUB, while the other ranges were provided by ILRS stations.
The orbital elements of the objects used for the tests shown in this paper are summarized in Table 1. As one can see, the choice of the objects is made to cover a wide range of orbital regimes. For the LEO case we used TOPEX satellite (COSPAR ID 95052A), for the MEO case we used LAGEOS1 (76039A) and two GLONASS (10041B, 11009A), and for the GEO case we used IRNSS1A (13034A). All the selected objects have

Proc. 7th European Conference on Space Debris, Darmstadt, Germany, 18–21 April 2017, published by the ESA Space Debris Office
Ed. T. Flohrer & F. Schmitz, (http://spacedebris2017.sdo.esoc.esa.int, June 2017)

Table 1. Orbital elements of test objects.

Orbital Elements	92052A	76039A	10041B	11009A	13034A
a [km]	7721.073	12265.297	25509.465	25505.740	42164.705
e	0.00052	0.00446	0.00353	0.00062	0.00200
i [deg]	65.956	109.933	64.874	65.224	27.799
Ω [deg]	277.321	45.798	323.413	84.194	127.717
ω [deg]	152.058	341.833	169.149	210.319	190.810

an almost circular orbit, this choice was not made on purpose but it was mainly driven by the data availability since these objects are regularly tracked by ILRS.

2. DISCOVERY AND FOLLOW-UP SCENARIO

With the following tests we want to answer the following questions: what are the benefits of the SLR measurements in terms of achievable orbit accuracy? And in the space debris field, what are the improvements, for example, for catalog maintenance applications? To illustrate these effects in the next examples we will compare the orbit determination/improvement (from now on simply called OD) results obtained using one or two nights of angular observations with those adding a very small number of ranges. The ephemerides generated after an OD are then compared with those obtained from a reference orbit. As reference ephemerides we used those provided by ILRS centers. Tables 2, 3 and 4 show the mean, over the propagation time, of the position differences w.r.t. the reference orbit in radial, along- and cross-track components (respectively R, S, and W). In addition also the total (over the 3 components) mean position difference is reported. For the GEO case also the mean differences of the osculating orbital elements at each ephemeris epoch are shown. From now on we refer to the mean position (or orbital elements) differences simply calling them errors.
Only a very small number of observations are taken into account in these tests so that we simulate the classical scenario of object discovery and first follow-ups (acquired in the same and in the following night). This way of operating is usually adopted at the AIUB and, especially for the GEO case, the obtained results could be easily compared with those shown in [1] to highlight the benefits given by the ranges. As just said, these tests were repeated for different orbital regimes (namely LEO and GEO).
It must be said that due to their different accuracies, we needed to make the system able to take the advantages of both observables without ignoring one or the other. In a Least Squares (LSQ) adjustment, the weight of a generic observable $p_i = \sigma_0^2/\sigma_i^2$. Putting $\sigma_0 = \sigma_\alpha$ we weighted relatively the two observables. σ_α is the standard deviation (STD) of the angular measurements, while σ_r is the STD for the ranges. Therefore, for the angular measurements the weight is equal to 1 while, for the ranges, the weight is equal to $p_r = \sigma_\alpha^2/\sigma_r^2$. Both STDs are determined experimentally. For the angles, the mean of the residuals obtained from the system time offset calibration [5] is used. For the ranges, the a posteriori root mean square (RMS) obtained for an optimal OD performed with only SLR measurements is used. While the resulting σ_α is constant to 0.5 arcsec for all the tests performed, for the ranges, a particular value was determined for each orbital regime.
The values of the used σ_r depend on: the dynamical model used for the OD, the number of available SLR measurements, the arc-length of the measurements, the portion of orbit covered by observations and the applied correction to the ranges. The determined σ_r values go from 0.55 m obtained for the LAGEOS1 satellite to 1.3 m for the GLONASS ones. The first value is essentially due to the physical model used and to the fact that we intentionally did not compensate the mismodelling using solve-for parameters like empirical accelerations. The second value is due to the fact that we did not apply the center-of-mass (CoM) corrections since their correct application depends on the object attitude which is unknown in our case. Since for both the LEO and the GEO case we did not have enough observations to estimate the correct σ_r values, we used in both cases the value of 1.3 m for the determination of the weight on the ranges.
For completeness, from now on we will indicate with 1D the number of laser measurements or equivalently normal points. We indicate with 2D the number of angular measurements which are provided in series of measurements called tracklet. Each tracklet is nominally constituted by 7 triplets of measurements, namely Right Ascension (RA), Declination (DE) and reference epoch.

2.1. LEO

One good candidate for this kind of analysis in the LEO regime is TOPEX POSEIDON (92052A). This satellite, which carries a retroreflectors array, was built to measure the surface topography and was decommissioned in 2006. This satellite, with its 2400 kg of mass and its fast spin period ($\sim$ 10 sec), is one of the case studied in the space debris field [6].
For the analysis of this case we used only the observations provided by the Zimmerwald observatory acquired during three consecutive passages of the satellite from one single night. In particular, we have an astrometric series for each passage and two ranges series belonging to the first two passages. The results of the tests and the precise number of used observations are summarized in Tables 2 and 3. We compared the ephemerides obtained from $\sim$ 7 days of propagation starting from the first observation epoch, which is also the epoch were we

Table 2. Results of the OD tests for the LEO satellite in the 1 passage case.

92052A Mean Position Error [m]		Angle-Only Circular	Angle-Only 6 ele.	All angles, 2 ranges	1 angle, all ranges	16% of ranges	Merged All Meas.
1 Passage	Obs*	$-/3$	$-/3$	$2/3$	$21/1$	$4/3$	$21/3$
	R	$2.655 \cdot 10^4$	$8.397 \cdot 10^6$	$3.989 \cdot 10^5$	5561	113.7	1258
	S	$5.391 \cdot 10^5$	$4.929 \cdot 10^6$	$2.122 \cdot 10^6$	$2.388 \cdot 10^5$	$2.308 \cdot 10^4$	$1.059 \cdot 10^5$
	W	246.2	$1.248 \cdot 10^5$	794.5	1197	418.8	861.9
	Tot.	$5.399 \cdot 10^5$	$1.030 \cdot 10^7$	$2.164 \cdot 10^6$	$2.389 \cdot 10^5$	$2.308 \cdot 10^4$	$1.059 \cdot 10^5$

* Number of observations respectively 1D/2D

estimated the orbit. For completeness we report that the time interval between two successive ephemerides positions is 1 minute.

We want to compare the results obtained by the classical angles-only solution with those using merged measurements varying the number of ranges used in the OD. Furthermore we want to report, for completeness, the results obtained by the ranges-only OD. For the satellite considered, this kind of experiments was not always possible especially for the 1-passage case. We decided to split the results in two tables: in the first (Table 2), the results obtained for the 1-passage case are shown, and in the second (Table 3), the ones obtained for the 2- and 3-passage cases are reported. Since the observed part of the orbit, in the 1-passage case, is too short to perform a LSQ to determine the orbit with homogeneous measurements we needed to change the method to calculate the orbit. For the "Angle-Only" case we used the Gauss-method as described in [7], in particular we reported the case where only a circular orbit was estimated and the case with the estimation of the entire set of orbital parameters (namely "Angle-Only Circular" and "Angle-Only 6 ele."). These results are obtained processing an extremely short arc of observation made of only 1 tracklets constituted by 3 measurement epochs. The results in the "Angle-Only Circular" column were obtained estimating only the semi-major axis, the inclination, the Right Ascension of the Ascending Node (RAAN) and the argument of latitude (respectively a, i, Ω and u_0) assuming $e = 0$ and $\omega = 0$.

For the "Ranges-only" case, we were obliged to use a minimum number of merged observations to be able to estimate an orbit. In particular, for the 1-passage case, we decided to report the results obtained with all ranges available plus the minimum number of angular observation to ensure the LSQ convergence (the "1 Angle, All Ranges" case). We also reported the results obtained using all angular measurements available plus the minimum number of ranges (the "All Angles, 2 ranges" case). As one can see from the Table 3 the workarounds used in the "Angle-Only Circular", "All angles, 2 ranges" and "1 angle, all ranges" cases were not necessary for the $2/3$-passages scenarios, so they were not replicated. The last two columns of the tables show the results obtained using only $1/6$ of the ranges and using all measurements available. As said before no empirical parameters are used during the OD process. On the other hand, since the object is still influenced by the effects of the atmosphere, we used the MSISe90 model to determine the atmosphere density and to model the atmospheric drag [7]. Finally, due to its fast attitude dynamics (tumbling period $\sim$ 10 sec [6]) we could not use the nominal value of AMR. We estimated the AMR fitting all available observations (both ranges and angular) coming from 1 week of observations from different laser stations. The observation arc used to determine the AMR was chosen in a way that the observations used for our tests are in the center of this arc. Once estimated the AMR the ODs for the next tests were performed without estimating any empirical forces/parameters.

Starting now from the angles-only examples in the 1-passages case shown in Table 2, we report a comparison between the results obtained by the application of the Gauss-method to estimate the complete and the reduced set of orbital elements. As one can see, the total error obtained estimating the circular orbit is 2 orders of magnitude better than those obtained estimating the full set of parameters. This is due, obviously, by the fact that the observed arc is extremely small and the target object is in an almost circular orbit (the true eccentricity is $\simeq 4 \cdot 10^{-4}$).

The case "All angles, 2 ranges" was reported since it represents the minimum number of observations (with a majority of the angular ones) for which the LSQ converges. The total observation time interval is about 6 minutes which does not allow for a good OD but one can already see the improvements given by the ranges. The radial error decreased by 1 order of magnitude w.r.t. the "Angle-only 6 ele." case. Looking at the "1 angle, all ranges" case we can see how the accuracy of the ranges produces an improvement of 2 orders of magnitude w.r.t. the angles-only case. In particular, the main improvements are visible for the radial component (3 order of magnitudes better) and secondarily for the along-track one of the error. At the same time this example shows the importance of the angular observations.

The obtained error in cross-track component is roughly 1.5 times bigger than in the “All angles, 2 ranges” case where a significantly smaller number of observations is used, the observed arc is shorter and 2 more angular measurements were used. We decided also to evaluate the influence of the number of observations w.r.t. the length of the observed arc keeping constant the number of angular observations considered (namely the cases: “All angles, 2 ranges”, “16% of Ranges” and “Merged All Meas.”). In the “All angles, 2 ranges” case we just used the first two ranges measurements available, in the “16% of Ranges” one we used a subset of ranges homogeneously distributed over the entire passage and finally, the “Merged All Meas.” case considers simply all the measurements available for that satellite passage. These three examples show the importance of the distribution of the observations. In fact, as one can see by comparison of the “All angles, 2 ranges” case against the “16% of Ranges” one, the addition of only 2 ranges, with a different distribution over the arc, improves the solution by 2 orders of magnitude. Adding further measurements leads to a degradation of the solution. This degradation is due to the fact that the OD fits nicely the observations over the observed arc which is too short to describe correctly the shape of the real orbit.
The same considerations can be deduced looking at Figure 1. In particular, it is interesting to notice that when estimating the 6 orbital elements from only 3 angular measurements, the LSQ converges on a “local minimum” solution whose estimated eccentricity value is far from the truth, producing the sinusoidal behavior of the error visible in Figure 1. At the same time it is possible to see the importance of the observation distribution w.r.t. the number of measurements since the best solution is obtained with only the 16% of the range measurements.

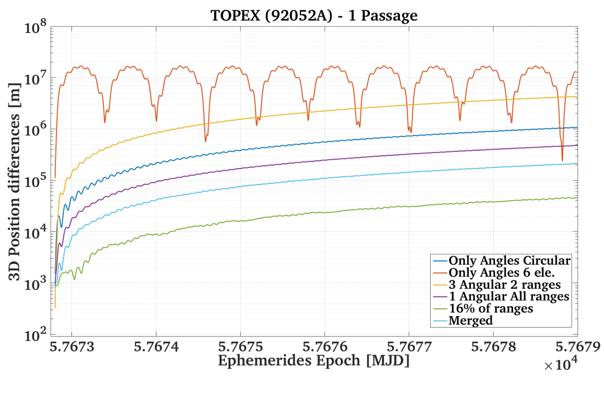

Figure 1. Behaviour of the total position errors resulting from the OD tests for the LEO case with only the observations collected during the first passage.

Looking now at the 2-passages examples shown in Table 3 we can see how, due to the higher number of observations available, we were able to carry out an OD using a LSQ adjustment even with homogeneous measurements. It must be said that the convergence of the algorithm using only ranges is probably due to the fact that we used an initial orbit to initialize the OD tool and we improved an initial solution. The same considerations can be done for the 3-passages cases. The highest improvement in the accuracy of the solution can be seen observing the angles-only case where the error now is 4 order of magnitude smaller w.r.t. its analogous in the 1-passage case. As expected the main component of the error is the along-track one which depends on the estimation of the semi-major axis, the eccentricity, the argument of perigee and the perigee passing time. These parameters, excluding the semi-major axis, are difficult to estimate if the observations are taken with a distance of one orbital period. In the ranges-only case the along-track component is 10 times smaller than the angles-only case. This is due to the nature of the observables which helps in the estimation of the just mentioned parameters. The comparison of the merged cases is another evidence of the importance of the distribution of the observations: as one can see, the results obtained with one sixth of the available observations are comparable to those obtained using all of them.
Looking at the results of the 3-passages cases shown in Table 3, it must be said that the ranges-only results were just copied from the 2-passages case, for an easier comparison, since we did not have any ranges from the third passage. Considering now the angles-only case it is possible to see how the further addition of observations still improves the results. Performing the same comparison with the merged cases, a much smaller improvement is visible. There are two probably main reasons: the first is due to the fact that no other ranges are added w.r.t. the 2-passages cases, the second is caused by the object-observer relative geometry. The information gained from the geometry is limited by the fact that only one station provides measurements and the considered arc is relatively short, in fact the total observation arc is only 4 hours.
The same plateau and the reduction of the improvements, in the results accuracy, given by the introduction of the range measurements with longer observation arcs can be seen also in Figure 2. The figure shows the behavior of the total position error of the propagated orbit w.r.t. the reference ephemerides. Each plot shows the comparison between the results obtained by an OD using homogeneous and merged measurements together with the observation epochs. The epochs of each measurement are identified with the green vertical lines for the ranges, and with the red ones for the angles. As one can see the first two plots are in a logarithmic scale while the latter in a linear one.

Table 3. Results of the OD tests for the LEO satellite in the 2 and 3 passages case.

92052A Mean Position Error [m]		Angle-Only	Ranges-Only	16% of ranges	Merged All Meas.
2 Passages	Obs*	−/20	59/−	10/20	59/20
	R	180.0	184.2	147.2	159.4
	S	8023	644.0	528.0	509.7
	W	263.7	291.1	228.7	234.6
	Tot.	8034	789.1	641.8	634.9
3 Passages	Obs*	−/25	59/−	10/25	59/25
	R	110.5	184.2	148.5	159.2
	S	1336	644.0	472.1	474.7
	W	346.8	291.1	235.0	239.7
	Tot.	1412	789.1	598.1	607.9

* Number of observations respectively 1D/2D

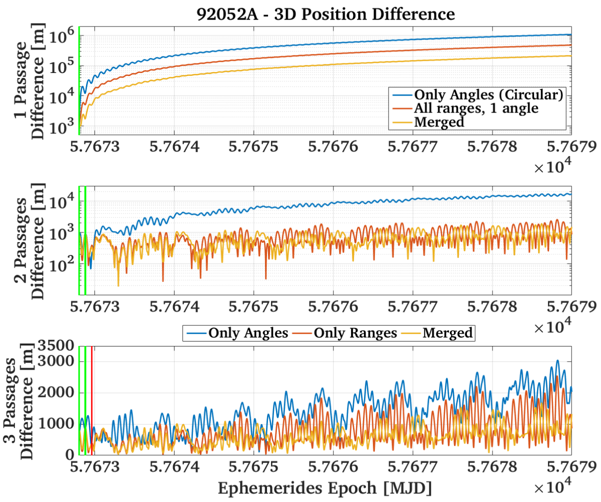

Figure 2. Behaviour of the total position errors and distribution of the used observations resulting from the OD tests for the LEO case.

2.2. GEO

The other orbital regime that we wanted to investigate is the geostationary one. As everybody knows, being this region one of the most exploited, it presents also a high density of space debris. The satellite used for these tests is the IRNSS1A (13034A). We will compare the position and orbital elements, generated after an OD performed over a maximum of two nights of observations, with those coming from a reference orbit. The angular data are provided by the Zimmerwald observatory, while the ranges by Hertsmonceux (United Kingdom). We would like to highlight that, in this case, the usual comparison is made using SLR measurements which were available only for the first night of observation. This was probably due to the rising difficulties of the SLR stations in tracking high altitude satellites. A total of 3 tracklets (even if not complete) spread over 3 hours were available for the first night and other 2 consecutive ones for the second night. Looking at Table 4, adding the 2 ranges produces a jump of 3 orders of magnitude in the mean error that will ensure the recovery of the object even after 6 days (final error around 1 arcmin for the considered object). In the angles-only case, as previously shown by Musci et al. in [1], a follow-up in the successive nights becomes mandatory. Comparing quickly the results shown by Musci et al. in [1] for the same orbital regimes we can say that 1 night of observations made by 3 tracklets and 2 ranges provides the same accuracy achievable with 4 (angular-only) follow-ups spread over 3 observation nights.

Looking now at the bottom graph of Figure 3, we can see a sinusoidal behavior of the errors. In this case, the error sinusoidal component is more pronounced than the drifting one. Furthermore, from the vertical green and red lines, it is easy to see how the smallest error occurs close to the observed part of the orbit. This effect is due to the distribution of the angular measurements which are precisely one day apart and the orbital period of the object that, being a GEO satellite, is coincident with the sidereal day. As main consequence this observation distribution improves strongly the estimation of the semi-major axis but at the same time, as shown in [3], does not give enough information to estimate correctly the eccentricity of the orbit. This effect, even if less pronounced, is also visible for the two nights case with merged measurements. In this case the ranges together with the distance information increase also the geometry changes. Since they are acquired two hours later than the last angular observation, they help constrain both, the semi-major axis and the eccentricity reducing the amplitude of the error oscillations (see elements error in Table 4). Consequently, improving the estimation of a and e, the ranges have strong effects even in the estimation of the argument of perigee and the mean anomaly, as can be seen looking at the error for ω and M.

Another consequence of the distribution of the observations can be seen in the parameters which describe the orientation of the orbital plane. Looking at the distributions of the angular measurements they are spread over 3 hours during the 1st night of observations, and the time distance between the 1st and the last tracklet is about 24 hours. As shown in [3] for GEO objects, series of angular observations in the same part of the orbit, since the arc covered by the tracklet is relatively small, do not provide enough information to determine correctly i and Ω. This effect can be easily seen from the error value reported in Table 4: the errors for i and Ω are relatively high and stay on the same order of magnitude (especially i) for the angles-only cases. In this case the ranges help also the estimation of these parameters (i and Ω) enlarging the observed portion of the orbit. The arc observation increases in fact from 3 to 5 hours. This last test shows the strength of the SLR measurements: looking at the last

Table 4. Results of the OD tests for the GEO case.

13034A		1 Night		2 Nights	
		Angles-Only	Merged	Angles-Only	Merged
# of Obs.	1D	–	2	–	2
	2D	19	19	33	33
Pos. Error [m]	R	$7.193 \cdot 10^5$	486.8	5436	86.07
	S	$5.219 \cdot 10^6$	9017	$1.698 \cdot 10^4$	199.6
	W	$8.992 \cdot 10^3$	50.99	703.9	20.87
	Tot.	$5.282 \cdot 10^6$	9056	$1.850 \cdot 10^4$	234.3
Orbital Elements Error	a [m]	$2.793 \cdot 10^5$	497.2	61.71	6.729
	e	$4.997 \cdot 10^{-3}$	$1.822 \cdot 10^{-6}$	$8.270 \cdot 10^{-5}$	$3.089 \cdot 10^{-6}$
	i [deg]	$4.921 \cdot 10^{-3}$	$3.190 \cdot 10^{-5}$	$1.387 \cdot 10^{-3}$	$4.500 \cdot 10^{-5}$
	Ω [deg]	$3.997 \cdot 10^{-2}$	$2.207 \cdot 10^{-4}$	$1.407 \cdot 10^{-3}$	$9.800 \cdot 10^{-6}$
	ω [deg]	22.93	$2.995 \cdot 10^{-1}$	5.032	$1.760 \cdot 10^{-2}$
	M [deg]	16.75	$2.865 \cdot 10^{-1}$	5.055	$1.771 \cdot 10^{-2}$

column of Table 4, even a small number of ranges (only 2) produces an improvement of the average error from roughly 2 arcmin to 2 arcsec.

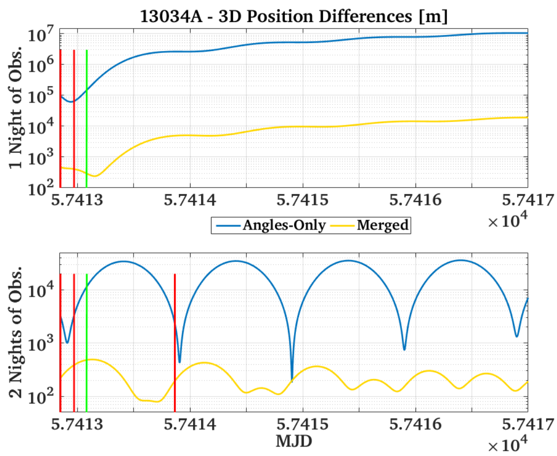

Figure 3. Behavior of the 3D position errors and distribution of the used observations resulting from the OD tests for the GEO case.

3. ERROR EVOLUTION WITH TIME

With the last tests we want to compare the influence of few SLR measurements in a short observation arc on the time needed by an observed object to go outside of the field of view (FoV) of a telescope. This time interval is important for an observer since if it does not want to lose an object, it has to acquire new measurements to improve its orbit before it leaves the FoV of the telescope. The maximum time interval available before losing the object will be called, from now on, recovery period. In the following tests we will highlight the benefits of the use of SLR measurements in terms of length of the recovery period for an observed object. We want to compare the time needed by an observed object to go outside of the FoV of a telescope. For these tests we used the FoV of the Zimmerwald telescopes, namely Zimlat and Zimsmart. The first has a FoV of $\simeq$ 26 arcmin while the second $\simeq$ 3.5 deg; consequently, we will consider an object as lost when the vectorial sum of the errors in the along- and cross-track directions, w.r.t. the reference orbit, is greater than the half of the FoV. This comparison will be made between results obtained from an OD with different kinds of observables, number of observations, lengths of the observed arc and for different orbital regimes. Particular emphasis is given to the comparison between the angles-only case and the merged one in order to highlight the improvements of the SLR measurements w.r.t. the results achievable with the actually used angular observations method. This test is of fundamental importance for catalog maintenance. The increase of the recovery time allows a more relaxed observation schedule with a smaller number of required observations permitting the tracking of a bigger number of objects. The test will be performed mainly on short observation arcs using a very small number of ranges to better simulate the space debris observation case. Table 5 shows the time interval needed by the propagated orbit to go outside the FoV for the first time for the different cases analyzed. We report the smallest time interval needed to go outside the FoV since it can happen that, due to the observation geometry, certain OD produce results like those shown in the bottom graph of Figure 3. In this case the object, before getting completely lost, appears again inside the FoV after a certain period. This effect can be reproduced easily if we compare the OD results with the true orbit.

Table 5 shows also the arc-length and the number of observations used in each case. As mentioned just above, only short observation arcs are used. In particular, for the Lageos 1 and TOPEX cases (namely 76039A and 92052A), the results obtained by 1, 2 and 3 passages within the same night are reported, while in the other cases the comparison between 1 and 2 nights of observation are shown. Looking now at the case of 10041B, a further comparison is done changing the number of observations used in the case of 2 nights of observation arc. As one can see the first 3 tests were carried out using 3 tracklets of angular observations (14 observations in the table) varying the number of used range from 2 to 4. A second test was performed comparing the results given by the 4 tracklets (18 angular observations) case with the 4 tracklets and 4 ranges one.

Looking at the Table 5 we can see how in the angles-only 1 passage cases for 76039A and 92052A and the one-night cases for 13034A and 10041B, the recovery time is roughly 4 hours or even smaller (if one look at the results using the FoV of Zimlat). For 11009A even in the worst case scenario the recovery time is at least one day. The reason for it is the length of the observed arc. In fact, the observations are distributed over $\simeq 4.5$ hours, which roughly coincide with one third of the orbital period. Already adding a couple of ranges the solution improves by at least one order of magnitude. As one can see from the results obtained with the smallest observation arc, the recovery time using few SLR measurements increases from few hours to almost one week (look at 10041B) and even more, depending of course on the number of angular observations and their distribution. As shown in [1], extending the observation arc adding follow-ups, will produce an increase of the recovery time also using only angular measurements. Also in this case the improvement given by the ranges is noticeable but generally less pronounced. It is also interesting to highlight two cases where the angles-only solution of two nights of observations is compared with the merged one using only the range measurements acquired during the first night (namely the 2 nights cases for 10041B and 13034A marked with a *). These two examples highlight the importance of merging different kinds of observables since each one is acting differently on the estimated parameters. The combination of angular follow-up and a constant number of ranges produces a results 9 times better than the angles-only solution (see 13034A case). One last thing that can be noticed from the table is a general, quite obvious, trend in the improvement of the solution given by an increase of the observation time. There is only one case which goes against this trend: the 76039A case. For this satellite the comparison between the one passage merged solution and the two passages one, shows that the one passage case has a longer recovery time. We do not deduce any general conclusion from this case, as we think that the distribution of the observations makes the system converge on a slightly worse solution. One last remark can be seen from the recovery times obtained for the TOPEX case which are considerably smaller w.r.t. those obtained for the other satellites, this is probably due to the more complex orbital dynamics and the increasing difficulty in modeling all the forces acting on the LEO regime.

Table 5. Results of the recovery time tests.

		# of Obs.		Recovery Time [days]	
		1D	2D	Zimlat	Zimsmart
92052A	1 Pass.*	–	3	0.18	0.35
	1 Pass.	4	3	0.80	5.78
	2 Pass.	–	20	1.92	16.30
	2 Pass.	10	20	47.64	403.2
	3 Pass.	–	25	17.30	153.3
	3 Pass.	10	25	60.13	511.4
76039A	1 Pass.	–	41	< 0.1667	0.5417
	1 Pass.	3	41	917.2	7400
	2 Pass.	–	59	248.7	2006
	2 Pass.	10	59	674.1	5443
10041B	1 Night	–	9	< 0.1667	< 0.1667
	1 Night	2	9	5.83	45.83
	2 Nights	–	14	60	493.7
	2 Nights	2^{*1}	14	435.6	3515
	2 Nights	4	14	499.3	4032
	2 Nights	–	18	1177	9470
	2 Nights	4	18	1319	10650
11009A	1 Night	–	10	1.167	8
	1 Night	3	10	340.6	2749
	2 Nights	–	14	415.3	3350
	2 Nights	4	14	886.2	7153
13034A	1 Night	–	19	< 0.1667	0.5
	1 Night	2	19	29.83	310.2
	2 Nights	–	33	314.7	3039
	2 Nights	2^{*1}	33	2920	23660

* OD using the Gauss-method estimating a circular orbit (estimated parameters: a, i, Ω and u_0)

^{*1} ranges belonging to the first observation night

4. CONCLUSIONS

We investigated the improvements that the high precision ranges provided by an SLR station, could give to the OD process based on the classical angular measurements.

After the weights definition, some studies were performed to highlight the consequences of the use of the laser ranges in the OD process. We simulated a classical discovery and follow-up scenario and by means of these analyses we evaluated how each single observable is acting on the estimated parameters, the influence of the number of SLR observations used in the OD, and also the influence of the relative object-observer geometry. Furthermore, we focused on the achievable improvements given by a very small number of ranges on a relatively short observation arc. Then we showed the benefits of processing merged observables in terms of catalog maintenance activities. All these tests were performed using exclusively real angular and SLR measurements provided respectively by the Zimmerwald observatory and the ILRS. The tests showed that, using merged measurements we can improve by orders of magnitude the orbit determina-

tion results. This leads to a huge increase of the recovery time especially for short observation arc. For the GEO case for example using only 2 ranges added to the 3 angular tracklets we increase the recovery time from less than 4 hours to $\sim$ 30 days. On the other hand, since the SLR measurements are much more precise than the angular ones, a fine tuning of the measurements weights is needed so that the system will not ignore the angles. The improvements given by the SLR data on long observation arcs are less pronounced w.r.t. those obtained for short arcs.
Of course this problem needs further investigation, but it is already proved the benefits of the use of SLR measurements in the OD for space debris. However, to have more general outcomes one should analyze the results coming from the application to a wider set of observations concerning different orbital regimes. Actually, it would be very interesting to analyze the effects of the ranges on the OD of objects on eccentric orbits. Further improvements can be given by the investigation of the geometry influence in a more theoretical way. Further studies can be carried out using simulation in order to not have constraints given by the availability of the measurements.

ACKNOWLEDGMENTS

The first author would like to thank the Swiss National Science Foundation for providing the funding that supported this work (SNF Grant 200020 157062).

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