Covariance Matrix Uncertainty Analysis and Correction

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INTRODUCTION

Since the first orbital launch in 1957, the number of artificial objects in Earth orbit has been increasing [1]. This has led to a corresponding increase in the threat to active satellites from hypervelocity collisions, putting in jeopardy crucial services that benefit human society. To mitigate such risks, one of the approaches commonly adopted consists in the planning and the execution of collision avoidance maneuvers for maneuverable operational satellites. This approach relies on the analysis of several geometric and probabilistic criteria, computed from the state vectors and covariance matrices of the two objects involved in the conjunction at the TCA (Time of Closest Approach). Thanks to a major international collaboration, a standardization of the data format needed to assess a collision risk between two orbiting objects has been possible [2].

In order to determine the proper action to mitigate a possible collision risk, it is necessary to have access to accurate inputs representing the orbital states of the two orbiting objects at the TCA and most of all their uncertainties. The covariance matrices representing the uncertainties of the orbital states at a given date are however obtained via an optimization process (orbit determination process). As such, they may not represent the actual uncertainties of the state vector if un-modelled errors have not been taken into account during the orbit determination process, or if the expected uncertainties of the observations used do not correspond to the reality.

Lack of representativeness on the covariance matrix, will lead to a wrong estimation of the collision probability (cf. Eq. 1) and therefore may lead to take the wrong decision concerning the mitigation action.

$$P_c = \frac{1}{\sqrt{(2\pi)^3|C|}} \iiint_V exp\left(-\frac{1}{2}\vec{X}^T C^{-1}\vec{X}\right) dxdydz \tag{1}$$

Several situations are regularly observed, when the updates on an orbit determination (OD) process are propagated up to the date of closest approach (TCA), in order to monitor the evolution of a given risk (cf. Fig. 1).

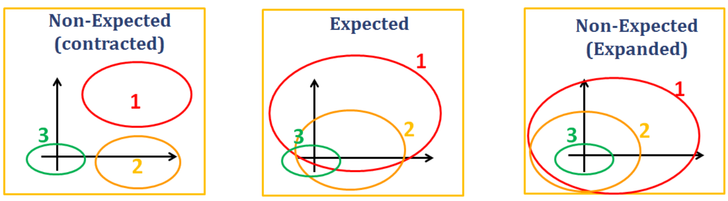

Figure 1.- Projections on a 2-D hyperplane of OD's local orbital frame.
The numbers (1, 2, 3) represents the OD updates done a three different times (t1< t2< t3<TCA).

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METHOD

In our work, we study the possibility to increase the realism of the covariance matrices via the application of scaling factors. Prior to the estimation of the scaling factors to apply in order to enhance the covariance matrix representativeness, it is of key importance to validate that such enhancement can be reached via the application of these factors.

If we consider a series of OD updates, whose state vector and orbital errors ($X_i, \sum_i$) are propagated at the same date (e.g. TCA), and we consider that at that date we dispose of a reference state vector($X_N, \sum_N$). We are able to compute the Mahalanobis distance (${k_\sigma}^2$) between each of the propagated state vectors and the reference one (Eq. 2)

$$ {k_\sigma}^2 = (X_i - X_N)^T (\Sigma_i + \Sigma_N)(X_i - X_N) \tag{2} $$

If the covariance of each of the state vectors is representative of reality, ${k_\sigma}^2$ is the result of the sum of three standard normal distributions. Therefore and only if the covariances are realistic, ${k_\sigma}^2$ follows a χ2 distribution of 3 degrees of freedom.

In coherence what has been said previously, and considering a temporal series of collision data messages or in-house orbit determination processes, we have developed two approaches in order to enhance the realisms of the covariance matrices. Such approaches are based on the application of a scaling factor (${k_*}^2$) to the covariance matrices (Eq. 3).

$$ {d_M}^2 = \frac{{k_\sigma}^2}{{k_*}^2} = (X_i - X_N)^T \left({k_*}^2 C\right)^{-1} (X_i - X_N) \tag{3} $$

If the application of the scaling factor (${k_*}^2$) allows for the enhancement of the covariance realism, by definition ${d_M}^2$ will follow a χ2 distribution of 3 degrees of freedom (dof). Two methods have been developed to search for the scaling factors that allow the data to be distributed following such distribution:

- The scaling factors, and their associated weights, are estimated by a fitting process considering a mixture of N 3 dof χ2 laws (Eq. 4).

$$ f(x) = \sum_{l=1}^{N} f_{{k_l^2}_* \chi_2 (x) w_l =} \sum_{i=l}^{N} \frac{1}{{k_l^2}_*} f_{x_2} \left(\frac{x}{{k_l^2}_*}\right) w_l \tag{4} $$

- The scaling factors are computed by a point-to-point comparison with the percentile value of the theoretical χ2 distribution and the empirical data.

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RESULTS AND CONCLUSIONS

From the two methods listed above, the one having the most computational complexity is of course the first one. Nevertheless, it allows for a deeper analysis of the data as it allows splitting the overall amount of data on N 3 dof χ2 laws, with their associated weights. Therefore, it allows for an easy identification of clusters of data similar characteristics (e.g. time delay, ballistic coefficient…). This clustering of data allows of course for a reduction of the domain of scaling factor to apply for the enhancement of the covariance realism.

Fig. 2 shows the result of the first method on real data. The color code corresponds to different families of OD updates depending on the propagation time of each of the OD updates.

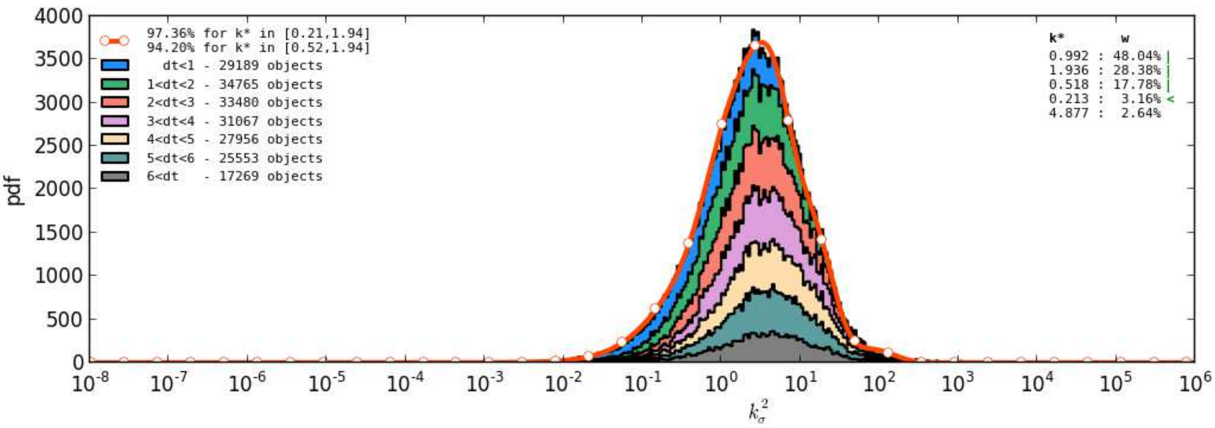

Fig. 2.- Fitting of the real data, considered distributed according as a 3 dof χ2 law mixture, to estimate the scaling factors as a function of the propagation time of the OD result.

Nevertheless the advantage of the second method is that it is straightforward to apply, as just by a point-to-point comparison with the percentile value of the theoretical χ2 distribution and the empirical data is needed to estimate the scaling factor domain to be applied.

Both methods have been validated on known datasets in order to evaluate their capacity to estimate the scaling factors to apply to the data in order to recover known factors applied to a priori known probability density functions.

Quantile	Expected Value	Mixture of laws		Single law
		Value	Weight	
2.5%	0.467	0.463	9.93%	0.910
50%	1.125	1.129	72.23%	1.147
97.5%	2.227	2.097	17.84%	1.606

Table 1.- Expected and computed values for a known dataset. The scaling factors follow a mixture of three log normal law, with a standard deviation of 0.1 and mean values of 0.5 (10%), 1.1 (70%) and 2 (20%).

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Table 1 present the results of one of the validation test cases. On this test case the scaling factors follows a mixture of three log normal laws with a standard deviation of 0.1 and mean values of 0.5 (10%), 1.1 (70%) and 2 (20%).

As can be deduced from Tab. 1, the mixture law approach offers the best results, and allows for the enhancement of covariance realism. As far as the single law approach is not always able to recover the proper scaling factors and may under-estimate the value of the scaling factors.

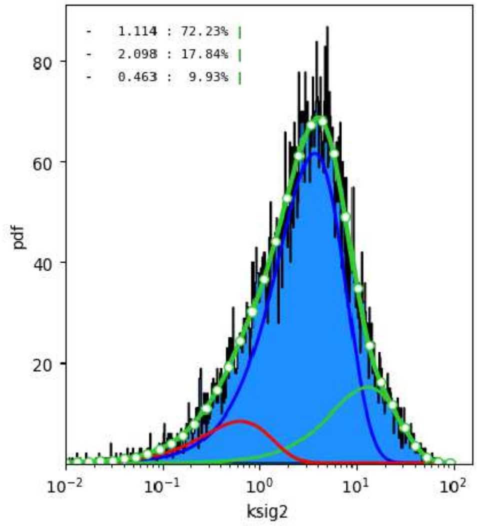

Fig. 3.- Fitting of the test data set presented at Tab. 1 with the mixture law approach.

References

1. Space Debris Program Office, Orbital Debris Quarterly News, Volume 19, Issue 1, page 9, January 2015.
2. Conjunction Data Message (CDM) CCSDS Recommended Standard 508.0-B-1

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