IMPROVEMENTS ON MODELS AND ANALYSIS MEANS FOR FRAGMENTATION EVENTS

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ABSTRACT

This paper proposes the use of statistical means to study the space debris population. We have developed a deterministic space debris model using an orbit propagator and a breakup model to generate our populations. First, we introduce the concept of synthetic population. Second, we investigate how global statistical functions can be help us to characterize these populations.

Keywords: space debris; breakup; synthetic population; statistical analysis.

1. INTRODUCTION

Since the beginning of the space era more than 200 fragmentations events (explosions or collisions) in orbit around the Earth have been counted [9]. These fragmentation events constitute the main source of debris with a size greater than 1 cm. Moreover, only 12,000 debris with a size greater than 10 cm in Low Earth Orbit (LEO) region and 1 m in Geostationary Earth Orbit (GEO) region are observed, identified and tracked. There exists also a huge population, whose size is between 1 cm and 10 cm, which remains invisible because the pieces of space debris are too small but they have a high kinetic energy. This population is estimated into 800,000 objects, and the existence of these fragments increases the risk and cost of all space missions.

The fragmentations are modeled with the NASA breakup model in many space debris population simulations as SDM [14], MASTER [3] or LEGEND [11]. However, a large discrepancy between the model and the reality could exist because each fragmentation event depends on particular condition. Thus, we introduce new statistical means to overcome this limitations.

2. TOOLS TO MODEL A POPULATION OF SPACE DEBRIS

2.1. Orbit propagators

The University of Namur has developed two numerical orbit propagators to study the dynamical evolution of space debris. NIMASTEP was used to study the dynamics of the orbital motion around a terrestrial body [5]. It is based on a multistep method of Adams-Bashforth-Moulton of order 10, taking into accout the geopotential up to the 5th order and degree, the Sun, the Moon and the solar radiation pressure with shadowing effects. The atmospheric drag was added using different atmospheric density models like JB2008, DTM2013 or TD88 [13]. Thus, NIMASTEP has been adapted to compute all type of orbit, from the LEO to the GEO regions.

A second orbit propagator is SYMPLEC [6]. It is based on a symplectic integrator which has two main properties: it preserves the energy, and it allows a large integration step. Thus, it is very efficient to cumpute the effect of conservative forces on very long term and so, particularly adapted for the objects in the GEO region.

2.2. Fragmentation model

In order to simulate the population of space debris we use the NASA breakup model which gives the initial orbital elements and the $\frac{A}{M}$ ratio of the fragments produced by a collision or an explosion. It is based on a power law which gives the number of fragments with a size upper than a characteristic one, it also gives the distribution laws of the $\frac{A}{M}$ ratio and of the module of the increment velocity. We can use it for each historical fragmentation referenced by [9].

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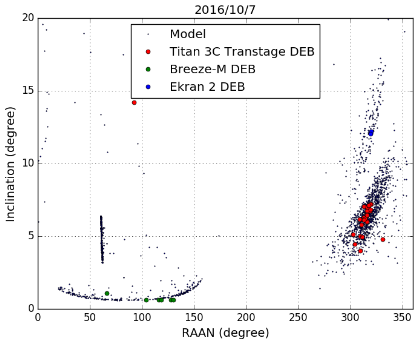

Figure 1. Simulation of the population in the GEO region

2.3. High computing

Using the libraiy MPI we are able to use several CPUs to calculate independent orbits at the same time. NIMASTEP or SYMPLEC are able to work on a personal computer but it is more efficient to use the cluster containing 112 nodes with two 6-cores Intel E5649 processors at 2.53 GHz and 48 GB of RAM (4 GB/core) of the 'Consortium des Équipements de Calcul Intensif' available at University of Namur.

3. CREATION OF A SYNTHETIC POPULATION BY STATISTICAL MEANS

3.1. Simulation of the GEO region

We propose to show how statistical informations can improve the classical simulations. We focus only on the GEO region where the atmospheric drag does not act and where the number of fragmentations is limited. Indeed, we count only 4 fragmentations. The first one occurs on June 25, 1978 and was caused by a malfunction of the battery of satellite Ekran 2. The second one occurs on February 21, 1992 and was caused by a failure of the upper stage Titan 3C Transtage [9]. In 2016, two additional fragmentatons happened with the satellites Breeze-M on January 16, and BeiDou G2 on June 26. Moreover, the observations have shown that more fragmentations should have happened but they have not been referenced [15] [8]. In figure 1 we plot the results of the simulation and the data coming from the TLE data. We find a good accordance.

3.2. IPF process

The creation of a synthetic population is based on the Iterative Proportional Fitting (IPF) process which will weigh a discretized cross-table of the initial population with statistical constraints (observational data, additional simulation, etc.).

The initial population is described by a contingency table $\Pi_{i,j}$ where i and j are related to a range of values for the variables like orbital elements or $\frac{A}{M}$ ratio. The table 1 gives a simplified example on two dimensions. Each cell counts the frequencies. The last line give the sum of the each column and the last column give the sum of each line.

	a_1	a_2	a_3	
$\frac{A}{M}1$	1	2	1	4
$\frac{A}{M}2$	4	1	1	6
$\frac{A}{M}3$	1	3	3	7
	6	6	5	17

Table 1. Expression of the contingency table restricted with two dimensions.

Let Π the contingency table and each cell noted $\Pi_{i,j}$. The marginal control for the i-th row and j-th column are noted m_{i+} and m_{+j} respectively. They are infered from statistical data. The IPF process is an iterative method, which is weighted with the marginal control until convergence. If we write Π^t the contingency table at the t-iteration, the row-fitting is implemented as,

$$\Pi^t_{i,j} = \Pi^{t-1}_{i,j,k,l}\frac{m_{i+}}{\Pi^{t-1}_{i+}}, \tag{1}$$

and the collumn-fitting is implemented as,

$$\Pi^t_{i,j} = \Pi^{t-1}_{i,j}\frac{m_{+j}}{\Pi^{t-1}_{+j}}. \tag{2}$$

The convergence is reached when the distance between the new contingency table and the contingency table at the last iteration is smaller than a parameter $\epsilon = 10^{-7}$. We compute the previous distance by using the following equation :

$$D(\Pi^t_{i,j}, \Pi^{t-1}_{i,j}) = \sum_{i,j} |\Pi^t_{i,j} - \Pi^{t-1}_{i,j}| \tag{3}$$

3.3. Application

In this application, we deal with four variables, the semi-major axis a, the inclination i, the right ascension of the ascending node Ω and the $\frac{A}{m}$ ratio, which are discretized to apply the IPF process. Our aim is to create new space debris with the same global statistical characteristics that the initial population. Thus, the controls are obtained fitting the distribution of the cloud to obtain the frequencies and to use them as contraints of the problem. The contingency table is then weighted by the IPF process and we convert the population obtained in the contingency table into new space debris population by a Monte-Carlo

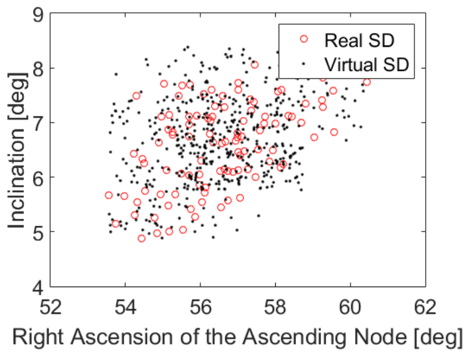

Figure 2. Comparison between the initial population and the synthetic population for a cloud of space debris coming from the fragmentation of the satellite BeiDou G2.

process. In figure 2 we compare both populations. This method helps us to overcome the computational limitation, but the final purpose is to use different constraints coming from observations to reduce discrepancy between the simulation and the real population.

4. STATISTICAL ANALYSIS OF THE DYNAMICS

4.1. Definition of spatial statistics

For this second point, we propose to investigate how the spatial distribution of a population of space debris could be related to global statistical functions. This way is based on two hypothesis: (i) the population of space debris is a realization of a point process (i.e. a configuration of points), (ii) the point process is considered as stationary.

In theory, the distribution of a point process is entirely known if all its moments are. In practice, in order to get some informations on the model that we could build, we perform an exploratory analysis. This analysis is based on summary characteristics which can give hints on the behaviour of the point process and the kind of interaction between the points (attraction/repulsion/no interaction).

These summary characteristics that we used are 2 functions [7]:

- Empty space function *F(r)*:
  Let us draw a ball *b(u,r)* of radius $r > 0$ centred in *u* an arbitrary location in the observation window and let $\rho(u, X)$ be the minimum Euclidean distance between *u* and *X*. The *F*-function describes the probability of finding a point of the point process *X* in that ball,
  $$F(r) = \mathbb{P}(\rho(u, X) \leq r) \tag{4}$$
- Nearest-neighbour distance distribution function *G(r)*:
  Let *b(x,r)* be a ball or radius $r > 0$ centred in *x* a point of the point process *X* and again, let $\rho(x, X)$ be the minimum Euclidean distance between *x* and another point of *X*. The *G*-function describes the probability of finding a point of *X* in that ball without counting *x* itself,
  $$G(r) = \mathbb{P}_x(\rho(x, X\backslash\{x\}) \leq r) \tag{5}$$
  This function is defined as a Palm distribution, for more information about these distributions see [4] or [7].

4.2. Estimation and interpretation

Knowing the definition of the summary statistics, they now have to be estimated by using a *Border Method estimator* (see [16], [12], [1]) as follows:

$$\hat{F}(r) = \frac{\sum_{u\in I} \mathbb{1}\{\rho(u, X) \leq r\}.\mathbb{1}\{\rho(u, \partial W) > r\}}{\sum_{u\in I} \mathbb{1}\{\rho(u, \partial W) > r\}} \tag{6}$$

$$\hat{G}(r) = \frac{\sum_{x\in X} \mathbb{1}\{\rho(x, X\backslash\{x\}) \leq r\}.\mathbb{1}\{\rho(x, \partial W) > r\}}{\sum_{x\in X} \mathbb{1}\{\rho(x, \partial W) > r\}} \tag{7}$$

with $\mathbb{1}(.)$ the indicator function. The estimates are then compared with known analitycal formulas of $F(r)$ and $G(r)$ existing for a Poisson point process. The Poisson point process has been chosen because it is considered as the reference model when there is no interactions between the points of a point process (Complete Spatial Randomness (CSR)). The deviations from the Poisson expressions can be interpreted as follows [7], [17]:

- For the *F*-function:
  - if $F(r) = F_{Poisson}(r)$, the process studied may be described as a Poisson process
  - if $F(r) < F_{Poisson}(r)$, there is a tendency to the formation of aggregates (sort of attraction between the points) because more empty spaces are detected than in the Poisson case
  - if $F(r) > F_{Poisson}(r)$, there is a tendency to repulsion between the points because there are less empty spaces than in the Poisson case
- For the *G*-function:
  - if $G(r) = G_{Poisson}(r)$, the process studied may be described as a Poisson process
  - if $G(r) < G_{Poisson}(r)$, there is a tendency to repulsion between the points because there are less close-neighbours around a point than in the Poisson case

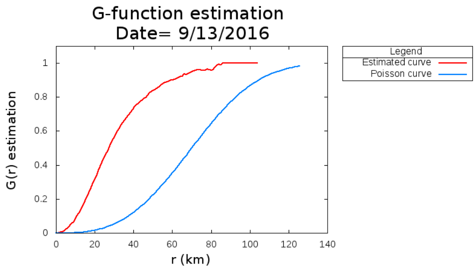

Figure 3. Estimation of the $G(r)$ function for a population evolving with the effect of J_2, $C_{2,2}$, and $S_{2,2}$.

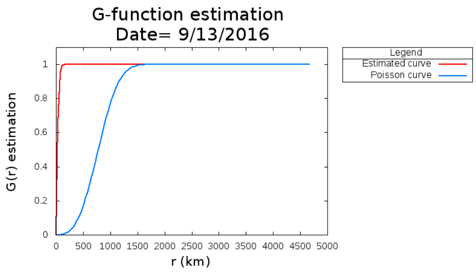

Figure 4. Estimation of the $G(r)$ function for a population evolving with the effect of J_2, $C_{2,2}$, $S_{2,2}$,and the SRP.

- if $G(r) > G_{Poisson}(r)$, there is a tendency to attraction (clustering) between the points because there are more close-neighbours around a point than in the Poisson case

4.3. Application

We want to use these functions to characterize the distribution of a population of space debris. For this purpose, we compute with NIMASTEP the orbit of a set of initial conditions near the geosynchronous resonance. The mean longitudes are spanned between 0 and 360° and the semi-major axis over a 42,164 $\pm$ 35 km range. The other variables are fixed to e = 0.002, i = 0, Ω = 0, ω = 0, and $\frac{A}{M}$ = 5.

We compare two cases. The first one takes into account the J_2, $C_{2,2}$, and $S_{2,2}$ effects. In the second one we add the solar radiation pressure (SRP) which includes dissipation and thus resonance crossing. We perform the computation over 15 years and we apply the statistical functions.

On figures 3 and 3, the blue line is associated to the value of $G(r)$ for a distribution generated by a Poisson process. The red line is associated to the value of $G(r)$ for the distribution of a GEO random population which has evolved taking into account J_2, C_{22} and S_{22} for the figure 3, and adding the solar radiation pressure for the figure 4.

For the two cases, the estimation of the G-function shows a tendency to attraction between the points, as the red curve is above the blue one. This is even clearer in the case with SRP but some nuances can be found in this interpretation. Indeed, if some points are isolated from the rest of the set (that could be considered as outliers), they may induce a detection of attraction that does not represent very well the reality. However, especially in the first case, a kind of collision probability can be deduced from the plot. For example, the probability of finding a space debris in a radius of 20 km from another one can be read on the y-axis: it is equal to 0.2.

5. CONCLUSION

The statistical means could be useful to characterize a population when we have a serious lack of information. Thus, we have started to investigate two approaches to link populations of space debris and statistical tools. The first one uses the IPF process to infer a synthetic population whose the global properties should be the same that the real one. The second one uses global statistical functions with the purpose to compare the population of space debris with Poisson process. These two statistical analysis should complete the informations given by the classical way.

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REFERENCES

1. Baddeley, A., & Rubak, E., & Turner, R. 2016, Spatial Point Patterns Methodology and Applications with R, Taylor and Francis Group
2. Baddeley, A. 1999, Spatial Sampling and Censoring, Stochastic Geometry : Likelihood and Computation, Chapmand and Hall
3. Bendisch, J., Bunte, K., Klinkrad, H., Krag, H., Martin, C., Sdunnus, H., Walker, R., Wegener, P., & Wiedemann, C.: The MASTER-2001 model, *Advances in Space Research*, Vol. 34(5), pp. 959-968 (2004).
4. Chiu, S.N., & Stoyan, D., & Kendall, W.S., & Mecke, J. 2013, Stochastic Geometry and its Applications, John Wiley and Sons, d. 3
5. Delsate, N., and Compère, A. 2012, A&A, 540, A120
6. Hubaux, C., Lemaître, A., Delsate, N., & Carletti, T. 2012, Advances in Space Research, 49, 1472

7. Illian, J., & Penttinen, A., & Stoyan, H., & Stoyan, D. 2008, Statistical Analysis and Modelling of Spatial Point Patterns,John Wiley and Sons

8. Jehn, R., Ariafar, S., Schildknecht, T., Musci, R., & Oswald, M. 2006, Acta Astronautica, 59, 84

9. Johnson, N. L., 2008, Orbital Debris Program Office

10. Lewis, H. G., Swinerd, G., Williams, N., & Gittins, G. 2001, Space Debris, 473, 373

11. Liou, J.-C., Hall, D. T., Krisko, P. H., & Opiela, J. N. 2004, Advances in Space Research, 34, 981

12. Moller, J., & Waagepetersen, R.P. 2004, Statistical Inference and Simulation for Spatial Point Processes, Chapman and Hall/CRC

13. Petit, A., & Lemaitre, A. 2016, Advances in Space Research, 57, 2245

14. Rossi, A., Anselmo, L., Pardini, C., Jehn, R., & Valsecchi, G. B. 2009, Fifth European Conference on Space Debris, 672, 90

15. Schildknecht, T., Musci, R., Ploner, M., et al. 2004, Advances in Space Research, 34, 901

16. Stoyan, D. 2006, On estimators of the nearest neighbour distance distribution function for stationary point processes, Metrika, 64, 139-150

17. Van Lieshout, M.N.M., & Baddeley, A., 1996, A Nonparametric Measure of Spatial Interaction in Point Patterns, Statistica Neerlandica, 50, 344-361