

Proposed Series of Orbital Debris Remediation Activities

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ABSTRACT

Orbital debris is a growing hazard to reliable space operations and the sustainability of space-based systems that increasingly support national security and economic stability for many countries. Short-term attention has been focused on collision avoidance for operational payloads and enhanced debris mitigation guideline compliance while long-term attention has been focused on studying debris remediation via Active Debris Removal (ADR). These three activities must be continued and augmented by three new efforts that work together to provide improved debris remediation activities to enhance space flight safety. The three new efforts are (1) an international Spacecraft Anomalies and Failures Workshop; (2) Massive Collision Monitoring Activity (MCMA) operations; and (3) Just-in-time Collision Avoidance (JCA) development. This paper provides a plan that is focused on maintaining progress in three existing initiatives and starting three new ones.

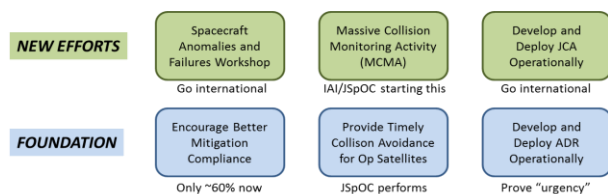


Figure 1. This paper proposes that there are three current efforts that need to be continued but three new efforts that should be started.

1 HYPOTHESIS

The spacefaring community is focused on environmental stability as the primary metric for responsible actions in space and for prioritizing investments related to debris remediation. While preventing a runaway cascading of collision events in Earth orbit (i.e., environmental instability) is a laudable and necessary goal, it is proposed that focusing more now on space flight safety as a relevant research, analysis, and development foundation is more cogent. It appears that for many, deploying active debris removal (ADR) is considered necessary only if it is proven that it will eventually prevent environmental instability. It is suggested that it is more relevant to determine if debris

remediation (such as ADR and just-in-time collision avoidance, JCA) is needed to insure spaceflight safety in the next decade¹ rather than inhibit the eventual cascading of collision breakups. Incidentally, *debris remediation that ensures spaceflight safety now will also prevent the onset of environmental instability later; however, the reverse is not necessarily true.* [1]

The erosion of space flight safety (i.e., degradation of reliable payload operations and reduction in operational lifetimes due to debris impacts) will occur well before the environment will manifest in outward signs of a runaway cascading effect. If we wait for a cascading of collisions before the global space community acts in earnest, it will be more difficult and costly to remediate the debris environment (i.e., “pay me now or pay me more later”). [2] In addition, the means to ensure spaceflight safety will require more proactiveness in debris remediation than is currently envisioned.² This is even more pronounced when considering clusters of massive derelicts that potentially have elevated³ risk levels.

2 SOLUTION COMPONENTS

Three related activities are proposed to proactively heighten awareness of space debris risks and dampen orbital debris evolution in the most responsible way by addressing current challenges detailed in the Hypothesis.

First, there is a significant benefit to measuring and quantifying the ensemble of spacecraft anomalies and failures that are tied to orbital debris impacts as this is a direct measure of the influence of the worsening debris environment on satellite operations. A paper was presented at the International Astronautical Congress (IAC) in Jerusalem in November 2015 and then published in Acta Astronautica in 2016 that provides a

¹ For ADR and/or JCA to be operational within a decade, development and testing needs to be ongoing now.

² For example, five derelict removals a year starting at some future indeterminate time is a typical sequence under consideration in Liou, J.-C., “An Active Debris Removal Parametric Study for LEO Environmental Remediation,” *Advances in Space Research*, 47 (2011) 1865-1876.

³ “Elevated” means higher probability than modeled by the traditional statistical probability of collision equation and much higher consequence due to mass involved.

detailed summary of related research over the last 25 years in this domain. [3]

Over the last four years, a **Spacecraft Anomalies and Failures (SCAF) Workshop** has been organized and hosted in Chantilly, VA, U.S. to advance the community's understanding of how orbital debris may be negatively affecting satellite operations. This workshop has been supported by and attended by NASA, NOAA, other USG organizations, U.S. industry, and U.S. academic institutions. The SCAF Workshop was catalyzed by the U.S. National Space Policy of 2010, Presidential Policy Directive-4, that states that the U.S. should *"improve, develop, and demonstrate, in cooperation with relevant departments and agencies and commercial and foreign entities, the ability to rapidly detect, warn, characterize, and attribute natural and man-made disturbances to space systems of U.S. interest."* This call to action is relevant to all spacefaring countries.

Deliberations at SCAF Workshops have highlighted that (1) most space operators do not invest significant resources to resolve the cause of unknown non-recurring anomalies; (2) attributing anomaly cause is difficult and more of an art than a science largely due to the complex space environment and lack of anomaly diagnostics on spacecraft; and (3) most space operators will not share on-orbit anomaly and failure data due to concerns of proprietary technology, user/stockholder confidence, national security, and space insurance implications.

It would be prudent to have an international organization, such as the International Academy of Astronautics (IAA) and/or the International Association for the Advancement of Space Safety (IAASS), take on the charter to plan, organize, and conduct an annual *international* Spacecraft Anomalies and Failures Workshop or support the expansion of the existing SCAF Workshop. It is hoped that this would help to generate an impetus for spacefaring organizations to share information that will provide better understanding of how the manmade particulate environment has affected operational satellites in the past and support future operational assessments of anomaly and failure attribution. In addition, this workshop may help to develop designs and operational imperatives for efficient and effective impact diagnostics such as enhanced sharing of health maintenance data and installing accelerometers, flash detectors, acoustic sensors, instrumented witness plates, and/or cameras to provide insights about impacts. The utility of a camera was proven on 23 August 2016 when the European Space Agency quickly verified that an anomaly felt by the Copernicus Sentinel-1A was due to a particulate impact on a solar array. [4]

Indeed, the IAASS has decided to host the First International SCAF Workshop in Toulouse, France

on 16 and 17 October 2017.

Insight from SCAF workshops is necessary to validate the fidelity of debris environment models and to provide a measurable intermediate quantification of the evolution of the debris population. It will be the increased number of debris-induced anomalies and failures of operational spacecraft that will be the best indicator of changing orbital debris hazard severity from the debris population too small to be cataloged (i.e., smaller than 10cm).

More pointedly, while protecting operational satellites from the trackable population via collision warnings provides a quantifiable risk mitigation service and improves space flight safety, the primary debris threat to operational spacecraft comes from the lethal nontrackable (LNT) environment. LNT debris ranges from about 5mm to 10cm; these are fragments that are large enough to disrupt or terminate a satellite's mission upon impact but are too small to be cataloged. There is an estimated 500,000-700,000 LNT (between 1-10cm) in LEO currently.

Therefore, the cataloged population (~18,000 in LEO) that is evaded through active maneuvering is less than 5% of the lethal population. In the future, the LNT population will increase primarily by collisions between large objects in orbit as the number of LNT produced is proportional to the mass involved in a collision (or explosion).⁴ Cataloged debris produced from a catastrophic collision will be liberated at about 1.5-2.5 fragments per kilogram of mass "involved" while LNT production is around 15-25 fragments per kilogram of mass "involved."⁵

Our ability to model and estimate the rate of collisions is derived empirically from only one catastrophic accidental collision event and analytically on a statistical model based on the kinetic theory of gases (KTG).

However, clusters of massive objects that have the same inclinations with similar and overlapping apogees/perigees may indeed have a greater probability of collision than predicted by the KTG-based algorithms since the objects in the cluster are not randomly distributed and their orbital element evolution (e.g., change in right ascension of ascending node and argument of perigee) is also similar. [5]

It is hypothesized that these similarities will result in resonances of collision dynamics that will produce

⁴ Typically, catastrophic explosions create many fewer fragments per mass involved than catastrophic collisions.

⁵ Appendix A provides a more thorough discussion about debris produced as a result of collisions.

larger probability of collision and collision rate⁶ values than estimated by the Poisson distribution with a frequency based on the KTG analogy.⁷

For this reason, it is suggested that we focus on the collisions between the most massive derelicts. Three of the local concentrations of massive derelicts in LEO are depicted in the figure to the right. Each cluster is titled by the altitude center (e.g., Cluster 975, or C975, is the far right cluster in the figure). Cluster 775 (C775), while significant, will not be analyzed further at this time since the debris produced from its encounters will be shorter-lived than the other two clusters.

However, it is noteworthy that the 775 cluster is centered near the altitude with the maximum cataloged spatial number density that currently exists at 790km (largely due to the Fengyun-1C breakup).

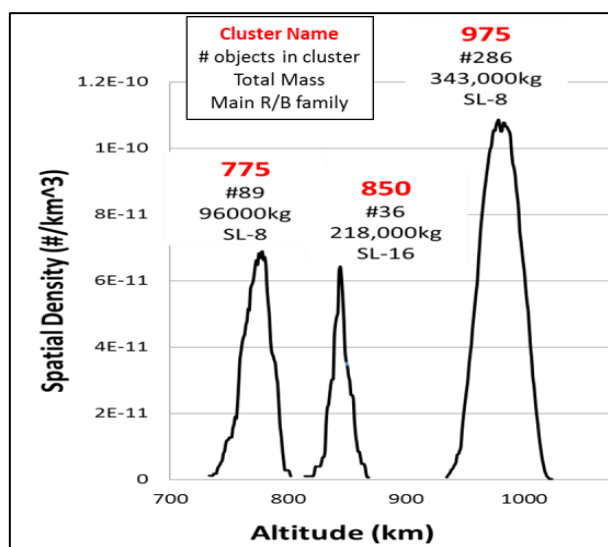


Figure 2. The three primary clusters comprise about a third of all derelict mass in LEO.

The **Massive Collision Monitoring Activity (MCMA)** is proposed whereby the encounters between members of these clusters are constantly monitored and close encounter information collected, plotted, analyzed, and shared. This would provide a rich research base for scientists and a predictive service for spacefaring countries.

⁶ The term “collision rate” is a bit of a misnomer. Collision rate is the total probability of collision between all of the objects within a cluster so it is actually still a probability. However, for small total PC the value is still a fair representation of the frequency of collisions between cluster members.

⁷ Appendix B contains a full explanation of this mathematical relationship.

The table below shows how ~340 of the largest derelict objects in LEO amounting to ~550,000kg (nearly 30% of LEO derelict mass) are in two mega-clusters we call 850 and 975 shown in the previous figure. Each cluster is comprised primarily of abandoned payloads and the expended rocket bodies that deployed them.

Table 1. The two primary clusters contain objects whose collisions would significantly affect the LEO

	Objects	Mass (kg)	#	L/D (m)	Altitude Extremes				Mass Span (kg/km)	Cumulative PC/Year (i.e., CR)	Produced From a Collision	
					High (km)	Low (km)	Span (km)	Inclination			Cataloged	Lethal, Nontrackable (LNT)
850	SL-16	8,300	18	11/3.9	860	814	46	71	3248	~1/3000	16,600	249,000
	SL-16 P/Ls	3,250	18	4/2	868	823	45	71	1300	~1/2500	13,000	162,500
975	SL-8	1,434	144	7/2.4	1020	935	85	83	2429	~1/250	2,868	43,020
	SL-8 P/Ls	800	142	3/2	1024	934	90	83	1262	~1/1000	3,200	40,000
	Other P/Ls	1500	18	4/2	997	905	94	64	293	~1/3500	6,000	75,000
TOTAL ->		~550,000	~340									

Despite the impressive amounts of massive derelicts in a limited altitude span, no one is specifically monitoring potential collisions between these objects.⁸ It is proposed that it would be a prudent risk management approach to ensure space flight safety by monitoring and characterizing this inter-cluster collision risk.

We are currently executing a subset of this proposed activity in conjunction with the Joint Space Operations Center (JSpOC). We have been monitoring the interaction dynamics between the SL-16 rocket body (R/B) population in the 814-860km altitude region (half of Cluster 850) since May 2015.

In August 2016, we added the associated payloads of the 18 SL-16⁹ rocket bodies (which completed Cluster 850). We started monitoring Cluster 975 in August 2016 at IAI.¹⁰ This overall MCMA effort amounts to monitoring daily the mutual conjunctions between all

⁸ Current JSpOC procedures are to screen possible collisions between all operational payloads and the cataloged population but there is no current requirement to screen for collisions between massive derelicts. However, the SOCRATES site run by AGI does look at the top ten possible conjunctions (by probability and miss distance) daily for the entire catalog.

⁹ The SL-16 launch vehicle is also known as the Zenit and the SL-8 is also known as the Kosmos launch family.

¹⁰ IAI is the acronym for Integrity Applications Inc. where all of the authors are employed.

340 objects in the two clusters detailed in the previous table.

During the 11May2015 – 11May2016 timeframe, the 18 SL-16 R/Bs had 232 conjunctions less than 5km and ten less than 1km. This amounts to approximately 20 sub-5km encounters per month and one sub-1km pass; this is a 20:1 ratio.¹¹ The average closing velocity of these encounters was ~11km/s.

The closest approach was 425m on 1AUG2015 with a relative velocity of ~8km/s. The second closest approach was 447m on 8JAN2016 with a relative velocity of ~14km/s.

As discussed earlier, the model used to estimate the collision rate of massive derelicts is based on the kinetic theory of gases (KTG) and the Poisson probability distribution function.

However, because these derelicts are in clusters (groups of objects with similar orbital parameters) we have hypothesized that this may lead to an under-prediction of the collision rate within a cluster as compared to what the Poisson/KTG distribution would estimate.

Note that for a Poisson distribution the mean is equal to the variance so significant variability is expected. By backing out the frequency that would have represented the measured encounter data, the annual collision rate for the full SL-16 R/B cluster is ~ 1/2500 (vice the 1/3045 estimated from the Poisson/KTG model).

As we approach the collision cross-section for an SL-16 collision we are close to the positional uncertainty of the orbital elements of these objects. As a result, the actual probability of collision will still be statistical in nature based largely upon the position covariances of the objects for a particular encounter. [6]

The purpose of the MCMA is not just to better characterize the dynamics of clusters of massive derelicts but also to examine the possibility of using the characterization to better predict future encounters.

From previous conjunction analyses, we identified the likelihood that objects with similar inclinations would exhibit a repeatable sequence of encounters getting continually closer in regular intervals; we call these “walk-ins.”

The figure below depicts a small portion of a “walk-in” between two SL-16 R/Bs whose closest approach occurred on 19 March 2015.¹² Eventually, they came within 447m of each other.¹³ Each point represents the next closest pass of the derelicts, exactly one orbital period apart (i.e., ~101min).

The consistency with which the objects approach each other hints at the possibility that we could use this sequence to predict where and when the “turning point” (i.e., time of closest approach, TCA) will occur days before it occurs.¹⁴

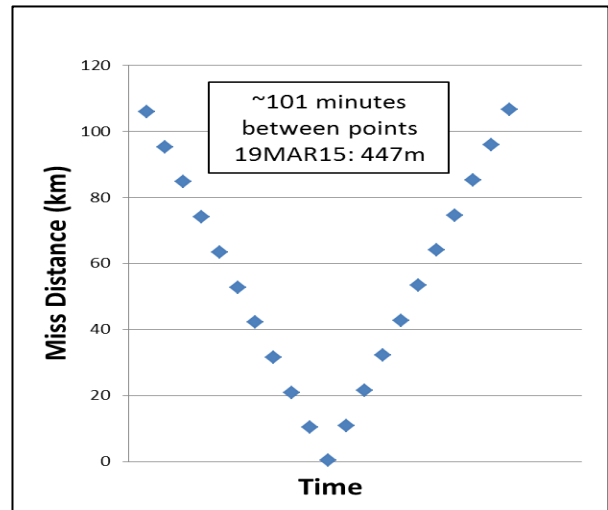


Figure 3. Walk-in sequences may be used to more accurately predict future conjunctions.

From the analysis of 30 more prediction exercises, examining eGP final results versus predictions by eGp up to 7 days out, it was found that using SGP4 on the walk-in algorithm can reliably get within 10-15% of the miss distance five days before the event. One might ask with this new information, what if an impending collision between two massive derelict objects is predicted, what can be done? Do you want to be sitting at the console watching the miss distance between two SL-16s closing in orbit after orbit knowing that this one

¹¹ While the encounters for Cluster 975 will not be discussed in this paper it is interesting to note the results from the first day of Cluster 975 calculations for 11 August 2016; there were 60 sub-5km encounters and three less than 1km. This is the same 20:1 ratio between 5km and 1km encounters as we found in examining a year worth of SL-16 R/B encounters.

¹² This sequence was part of IAI’s expansion of the SL-16 encounter dynamics that went back to 2012 and was not part of the JSpOC exercise.

¹³ It should be noted that all of the walk-in sequences include both a northern hemisphere walk-in and southern hemisphere walk-in. The results in the figure above are for the northern hemisphere crossings; if the passes for both northern and southern are combined then the sequence is not as consistent, and thus, is not as useful.

¹⁴ The initial approach assumed a 2nd order polynomial to the shape of the absolute miss (i.e., total range) profile.

collision has at least a $1/3045^{15}$ chance of occurring each year and that it would likely double the debris population in a single event, cannot be prevented? This leads us to the third and final operational initiative being proposed – **Just-in-Time Collision Avoidance (JCA)**.

The concept of JCA is to release a cloud of gas or very small particles in the path of one of the two potentially colliding derelicts to deflect its path to prevent the imminent collision (or widen the predicted miss distance to an acceptable level). This nudger cloud may be created by releasing a cloud of low density particles at the apex of a ballistic trajectory [7] or by using a rocket motor's plume as the cloud that deflects a derelict object. [8] JCA provides a timely means to prevent a significant imminent debris-generating event between two massive uncontrolled objects in space. JCA would cost about \$1-3M per launch and, even with a few false alarms a year, would be 1000x less expensive than ADR operations [9] as measured by cost per collision prevented (~\$300M-\$3B).¹⁶

Analysis has shown that at least two widely used sounding rockets, Black Brant XII and Oriole IV, can be employed from 2-4 existing launch sites to provide sufficient launch responsiveness, conjunction coverage, and mass-to-orbit capability for the JCA mission. The last engineering issue is to place the nudger cloud accurately enough (within 50m) to insure sufficient interaction with the derelict (50-200gm nudger mass impinging on the derelict); the sounding rockets do not currently have the terminal guidance to meet the 100m positional placement uncertainty (PPU) requirement. [10] This means that we need the nudger cloud's center to be placed in inertial space within 100m of a desired location to assure the derelict sweeps through the cloud sufficiently to encounter the desired nudger mass. To do this, it is likely that we may have to use small space launch vehicles (vice sounding rockets) that have more accurate payload placement capabilities.

A successful JCA mission will require accurate predictive abilities days before a potential event in order to reduce the number of false positives when determining targets for JCA and to improve the effectiveness of a nudger cloud interaction. The recent prediction experiment hints that JSpOC might be able to predict out more than a week before TCA with high accuracy for these types of conjunctions (i.e., massive objects in high-LEO) and we can predict out five days

with 10-15% error using “over the counter” (i.e., commercially available) tools. The further in advance an encounter can be predicted before a JCA system is deployed, the less trajectory correction is needed. This will reduce the cost of the operation and increase probability of success.

Even if a JCA system can be built, JCA is not proposed to replace ADR. Rather, a debris remediation strategy would be to determine the best combination of ADR and JCA missions that would minimize the chances of massive derelicts from colliding. It is expected that JCA and ADR operations could eventually be performed in tandem, reinforcing each other with ADR operations removing “frequent JCA offenders”; JCA creating risk statistics to make more relevant decisions about removal of the derelicts; and JCA preventing collisions until an ADR mission is mobilized to remove a specific derelict object.

However, with the recent conjunction experiment results, we might have a completely new concept of operations. If we can predict a conjunction between massive derelicts in high-LEO to within 1-10% a week or two in advance, this might make a compromise between ADR and JCA a possibility. If ADR can be responsive enough to remove an object within 1-2 weeks then possibly we could eliminate the need for JCA and just impose a requirement of a “responsive ADR” to enhance the overall efficiency of this remediation option. This would obviate the calculus of needing 35-50 removals to prevent one collision with this new “Just-in-time ADR (JADR)”.

It is suggested that a new metric for all ADR system concepts be introduced: an ADR mission must be able to be mobilized and executed within 5-7days. This approach might transform the concept of operations for ADR (into JADR) and make it much more viable technically, fiscally, and politically.

3 SUMMARY

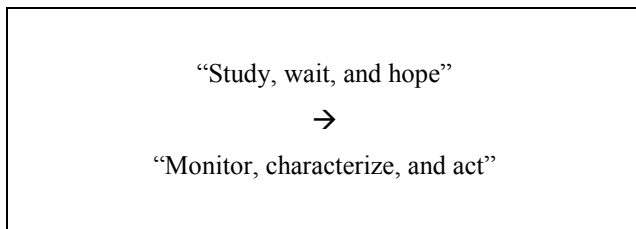
The combination of these three activities (i.e., SCAF Workshop, MCMA, and JCA) will provide (1) an advanced space situational awareness perspective for the international community and (2) a more defensible, proactive debris remediation stance as part of the emerging domain of space traffic management. This proposal assumes that continued pressure and resources will be applied to (1) increase the worldwide compliance to existing debris mitigation guidelines, (2) move ADR (and possibly JDR) concepts to an operational state, and (3) continue conjunction warnings for operational satellites. These three core activities are essential complementary work that must be continued along with the three new proposed activities.

The strategic motivation for this paper is to advance the space community's position on orbital debris response

¹⁵ As mentioned earlier, by using empirical observations the annual collision rate is estimated to be 1/2500.

¹⁶ The current approach to ADR is that of a long-term statistical cleanup effort that prevents one collision for every 35-50 derelict removals which produces a cost of approximately \$300M-3B per collision prevented. ADR solutions are currently not operational and even as envisioned are not responsive enough to react to MCMA Alerts.

well past a philosophy of “study, wait, and hope” to “monitor, characterize, and act.”



It is hoped that this paper will energize the immediate application of resources (diplomatic, operational, engineering, and fiscal) to increase efforts in the three proposed areas without taking from the existing three initiatives. There is some question how to best “divide and conquer” globally in the execution of these efforts.

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APPENDIX A: Technical Discussion About “Mass Involved”

The Iridium and Cosmos payloads that collided had a total mass of about 2,000kg¹⁷ and produced over 3,000 trackable fragments (i.e., 1.5x mass of colliding objects) and likely 30,000¹⁸ LNT debris. The purposeful collisional breakup of the Fengyun-1C spacecraft yielded around 2,200 trackable fragments (i.e., over 2.5x “mass involved”) and likely over 25,000 LNT from only ~850kg of mass involved. While it is important to prevent any collision from occurring in the future, the consequence of a collision (based on number of LNT produced) will be proportional to the mass involved in the collision. This “rule of thumb” model is consistent with detailed breakup models used by NASA.¹⁹

The figure below summarizes the mass involvement scenarios which highlight why the massive-on-massive collisions are the focus of our analyses. The type of objects involved in a hypervelocity collision will drive how much mass will be “involved” in debris generation and thus drive the amount of cataloged and lethal, nontrackable debris produced.

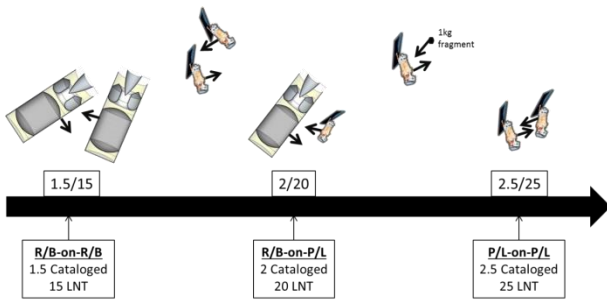


Figure A1. “Mass involvement” is a function of both type of objects colliding and the geometry of impact.

The term “mass involved” implies a good coupling of the impactor mass with the target mass. For a large fragment (e.g., ~1kg) striking a typical payload (that is densely built) in its main satellite body (vice striking a solar array or other appendage) at hypervelocity speeds (i.e., above 6km/s) will result in all the mass being

¹⁷ However, it is clear that the Iridium spacecraft was left largely intact so (from a physics perspective) not all of its mass was “involved” in the collision. That is why the number of cataloged fragments was only 1.5x the mass of the two objects.

¹⁸ The LNT production is probably lower than average due to incomplete fragmentation of objects involved.

¹⁹Johnson, N.L., et al, “NASA’s New Breakup Model of Evolve 4.0,” Advances in Space Research. Issue 9, 2001, pp. 1377-1384.

“involved” in the debris. However, a large fragment striking a derelict rocket body, due to the way that the mass is concentrated at the ends of a rocket body, will likely not result in all of the mass being “involved” in the liberated debris. However, it is likely that when two large derelicts, either rocket bodies or payloads, collide with each other, then most or all of the mass will be involved due to the likely direct physical interaction between the mass.

APPENDIX B: Technical Description of the Poisson Distribution Applied to Orbital Debris Encounters

In order to test the hypothesis that the Poisson probability is an underestimation, empirical encounter rates (ER) were calculated at various miss distances (from 500m-5km in 500m intervals) and compared to a Poisson distribution. The empirical ERs were calculated from JSpOC data gathered from May 2015-May 2016 and encounter statistics created by Integrity Applications Incorporated (IAI) for this same timeframe. These were then compared to the ER found using equations (1-4) where λ is the frequency within the Poisson probability density function (i.e., $P(k)$) taken from the kinetic theory of gases analogy.

$$\lambda = AC * VR * SPD \quad (1)$$

where $SPD = \frac{N}{Vol} = \text{spatial density, \#/km}^3$

$N = \text{number of derelicts,}$

$Vol = \text{volume swept out by cluster, km}^3$

$AC = \text{collision cross section, km}^2$

$VR = \text{relative velocity, km/s}$

$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!} \quad (2)$$

where $\lambda = \text{expected \# of occurrences over time, } t$

$k = \text{number of occurrences } (k = 0, 1, \dots)$

When it is assumed that there will be very few events, the probability of that rare event can be determined by 1 (i.e., the total all possible occurrences) minus the probability of no events. The result is represented by the well-known expression in equation (3).

$$P(1) = 1 - e^{-\lambda t} \quad (3)$$

The PC is the collision hazard to one satellite from N objects in the population. When we are looking at PC we are only concerned about the target, e.g., operational satellite getting hit by cataloged debris. Conversely,

when we have a cluster of massive derelicts we are concerned about collisions between any two of the N objects in the cluster. This is called the collision rate (CR) and is the cumulative PC for N objects on each other. CR is represented by:

$$CR = \sum_1^N PC = \left(\frac{1}{2}\right) N (AC * VR * SPD * T) \quad (4)^{20}$$

$$= (N^2/2) * (AC * VR * T) / (Vol)$$

When the encounter dimension is considered to be half of the miss distance then the collision rate is equivalent to the encounter rate (ER).

The next logical question is “if we accept the probability found with a Poisson distribution, when might the first collision occur?” Using a gamma distribution this can be evaluated for a given confidence level in equation (5).

$$\Gamma = -\ln(1 - C) * \left(\frac{1}{CR}\right) \quad (5)$$

where Γ = # of years until the first event

C = confidence interval

CR is Poisson-derived encounter rate

The table below shows the number of years for the first Poisson event predicted by the gamma distribution at different confidence levels for a CR of 1/3045. Please note that we have already shown that the Poisson distribution may underestimate the actual physical encounter rate so these may overestimate the time until the first collision event. Using the empirically-derived collision rate of 1/2500, the first Poisson event would occur within 25yrs with a 1% confidence. Note that the SL-16 cluster has been intact since 2007, so the “clock started ticking ten years ago.”

Table B2. The probability of the first collision within the SL-16 cluster is 1% in the next 20 years since the cluster was fully formed in 2007.

Confidence	Years Before First Event
1%	31
5%	156
10%	321
25%	876
50%	2110
75%	4221
90%	7011

²⁰ Note that the ½ term appears to insure that we do not double count possible encounters within the cluster.