BAYESIAN GROUP TRACKING METHOD FOR SPACE DEBRIS

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ABSTRACT

With the intense increase in space debris, it is necessary to efficiently track and catalog the extensive dense clusters of space debris. As the main instrument for LEO space surveillance, ground-based radar system is usually limited by its resolution while tracking small space debris with high density. Thus, the obtained measurement information could have been seriously missed, which makes the traditional tracking method inefficient. To address this issue, we conceived the concept of group tracking. For group tracking, the overall motional tendency of the group objects is expected to be revealed, and the trajectories of individual objects are simultaneously reconstructed explicitly. According to model the interaction between the group center and individual trajectories using the MRF within Bayesian framework, the objects' number and individual trajectory can be estimated more accurately. The MCMC-Particle algorithm was utilized for solving the Bayesian integral problem. Finally, simulation was carried out to validate the efficiency of the proposed method.

1 INTRUDUCTION

With the increasing amount of low earth orbit (LEO) space objects, especially space debris, space surveillance has become the foundation for utilizing space resources and avoiding the threats of space debris. Most of the current space surveillance networks can only track and catalog individual space object larger than 10 cm. However, objects larger than 1 cm can seriously damage or disable an operational spacecraft. Space debris of small size usually emerges in groups forming high-dense debris cloud [1]. Ground-based radar system is the main instrument for LEO space surveillance [2]. Unfortunately, this radar system cannot always meet the requirement for resolving the space debris cloud, which makes it difficult to track and catalog the objects individually. Instead of the traditional individual object tracking, tracking multiple space debris in group is becoming a potential demand and tendency. In addition, as an important application of space surveillance, collision avoidance is commonly based on calculating the collision probability using the "bulk" of the predicted orbital covariance [3]. Group tracking describes the "bulk" evolution of multiple closed orbital objects, which just satisfies the need. Group space debris can be defined as the hardly distinguishable objects that have similar orbit parameters during the observed period. The undispersed space debris clouds created by orbital collision possess the typical group character. Tracking or cataloging space debris in group can not only describe the overall evolution, but also potentially improve the accuracy of individual tracks using prior information regarding the group, which has profound significance in space situational awareness and collision avoidance.

Group tracking has some differences with respect to the traditional multiple targets tracking. MHT [4] and JPDA filtering [5] are the two classical and effective methods for tracking multi-targets. These methods implement multi-targets tracking based on data association. To alleviate the computational intractability and track the unknown number of objects, Mahler and Vo et al. [6][7][8][9] proposed a series of recursive Bayesian filters including PHD, CPHD, GMPHD, GMCPHD, which are the low-order statistical moments of the multi-targets posterior density based on the finite-set statistics (FISST). The PHD filter operates on the single state space and avoids the combinatorial problem that arises from data association. However, the PHD filter does not consider target identity. Furthermore, these methods suffer from performance degradation when the environment is characterized by higher clutter rate and low target detection probability [10]. Sometimes, tracking the multi-targets as independent individuals hardly improves the tracking performance. Consequently, Khan [11] incorporated the Markov random field (MRF) to model the interactions between multiple targets. The proposed approach was implemented using MCMC, and the efficiency was verified by vision-based ant tracking. However, the progress of group tracking was largely hindered by the problems resulting from splitting and merging of groups. Pang et al. [12] developed a group structure transition model that can describe the splitting and merging of groups smartly, as well as the interaction models for closely spaced targets. They simultaneously tackled the problem of group structure inference and joint detection and tracking for group targets within a Bayesian framework.

In this present study, we have focused on tracking the overall group evolution as well as individual objects' trajectories. By analyzing the orbital mechanics of

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space debris, we have constructed the characteristics parameters for describing the movement and structure of groups. The group objects are tracked within the Bayesian framework. By establishing the interaction model between the nominal group center and individuals, we can not only obtain a more robust estimation of object number and improve the accuracy of the estimated individual trajectory, but also depict the evolution of the groups in the case of low object detection probability.

The paper is organized as follows. Section 2 describes the kinetic model of space debris, the characteristics parameters of groups, group structure model of the existence state, and the Bayesian group tracking model. Section 3 breaks down the Bayesian tracking procedure into some detailed modules: the state transition model of space debris, the state transition model of group center, the interaction model between group center and individual trajectories, and the likelihood probability model of observation. In Section 4, MCMC-Particle algorithm has been utilized to calculate the Bayesian integral and fulfill group tracking. Section 5 presents the simulation of a single group tracking and analyzes its performance, and the conclusions are presented in Section 6.

2 BAYESIAN TRACKING MODEL OF GROUP SPACE DEBRIS

2.1 Kinetic model and observation of group space debris

Based on the two-body problem, the individual orbit can be fixed according to the three-dimensional position vector and velocity vector at a certain time.
First, let us define some parameters as follows: for the i th object, the motion state is $X_{t,i} = \left[\boldsymbol{r}_{t,i}^T, \boldsymbol{v}_{t,i}^T\right]^T = \left[x_{t,i}, y_{t,i}, z_{t,i}, \dot{x}_{t,i}, \dot{y}_{t,i}, \dot{z}_{t,i}\right]^T$ at time t, where $\boldsymbol{r}_{t,i}$ and $\boldsymbol{v}_{t,i}$ represent the position vector and velocity vector, respectively. $X_t = \left[X_{t,i}, \ldots, X_{t,N_{\max}}\right]^T$ denotes the state of $N_{\max}$ objects at time t. During a short time interval, the Keplerian trajectory and the propagation perturbation of the object's orbit can be approximately calculated using an elegant transition matrix presented in Eq. 1. The transition procedure is nonlinear and has no closed-form solution, and requires solving the Kepler's equation by iterations [13]:

$$X_{t,i} = FX_{t-1,i} + Q_u \tag{1}$$

where Q_u is the perturbation of the motion model. The details about F can be found in [13].

Furthermore, we can describe the group evolution as follows. Let $\begin{bmatrix} 300\boldsymbol{I}_{3\times3} & \boldsymbol{0} \\ \boldsymbol{0} & 100\boldsymbol{I}_{3\times3} \end{bmatrix}$ represent the group characteristics at time Q_L, then:

$$L_t = \Theta\left(X_t\right),\ L_t = F_l\left(L_{t-1}\right) + Q_L \tag{2}$$

where Θ is the function that extracts the group characteristics from the multiple objects' states, F_l is the prediction function of the group characteristics, and Q_L is the corresponding prediction error. The group characteristics have many different expressions, such as the average heading of flocks, and the "bulk" of multiple objects. Although there is no real orbital center for multiple closed space debris, it still has nominal characteristics parameter for describing group evolution.

The observation model can be expressed as:

$$Z_t = H\left(X_t\right) + Q_\omega \tag{3}$$

where Q_ω is the observation noise. Here, we simplify the scenario by only considering the miss and false alarm. Simultaneously, for displaying the tracking procedure intuitively and analyzing the performance easily, we first transfer the measurements of radars, such as range, azimuth angle, and elevation angle within the radar coordinate frame into the position vector within the Earth centered inertial (ECI) coordinate. The noise distribution is also simplified into the Gaussian white noise. Thus, the function H can be written as the matrix $\left[\mathbf{1}_{1\times3} \quad \mathbf{0}_{1\times3}\right]$, which shows that the position vector is observed, where $\mathbf{1}_{m\times n}$ is a full 1 matrix with m columns and n rows, and $\mathbf{0}_{m\times n}$ is a full 0 matrix with m columns and n rows.

2.2 Bayesian model of group center

In this study, we have only considered the average value of multiple debris' states to represent the group characteristics:

$$L_t = \Theta\left(X_t\right) = \frac{1}{N_{\max}} \sum_{i=1}^{N_{\max}} X_{t,i} \tag{4}$$

The state transition procedure of group center can be expressed as:

$$L_t = F_l\left(L_{t-1}\right) + Q_L = F \bullet L_{t-1} + Q_L \tag{5}$$

In addition, there are some interior connections between the group centers and the individual objects which should be explored. For example, the group center should restrain the object number from fluctuating rapidly, and keep the consistency and stability of the tracks. The group center is not only important for improving the tracking performance of individual trajectories, but also can describe the general motion tendency of the group space debris.

Furthermore, to reveal the change in group structure during the tracking procedure, let us define the existence state of multiple objects $e_t = \left[e_{t,i}, \cdots, e_{t,N_{\max}} \right]$, where $e_{t,i} \in \{0,1\}$. The variable $e_{t,i} = 1$ and $e_{t,i} = 0$ represents the existence and disappearance of i th object in X_t, respectively.

According to the observation model, we can note that the essence of group tracking is to estimate the group characteristics parameter L_t, the state existence variable e_t, and the individual motion state X_t. Bayesian model is one of the optimum filtering for estimation and tracking, which makes the maximum use of prior information based on the probability density of states. Under the Bayesian frame, the group tracking is equal to the calculated posterior density $p\left(L_t, X_t, e_t \mid Z_{1:t}\right)$ at time t based on the observations $Z_{1:t}$ from time 1 to t. By assuming a Markova state transition, the standard Bayesian filtering prediction and update steps can be given as:

$$
\begin{aligned}
&p(L_t, X_t, e_t \mid Z_{1:t}) \\
&= \frac{p\left(Z_t \mid X_t, e_t, L_t\right) p\left(L_t, X_t, e_t \mid Z_{1:t-1}\right)}{p\left(Z_t \mid Z_{1:t-1}\right)}
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
&p(L_t, X_t, e_t \mid Z_{1:t-1}) \\
&= \int \frac{p\left(L_t, X_t, e_t \mid L_{t-1}, X_{t-1}, e_{t-1}\right)}{p\left(L_{t-1}, X_{t-1}, e_{t-1} \mid Z_{1:t-1}\right)} dX_{t-1} de_{t-1} dL_{t-1}
\end{aligned} \tag{7}
$$

From Eq. 6 and Eq. 7, we can find that the calculation of state transition density $p\left(L_t, X_t, e_t \mid L_{t-1}, X_{t-1}, e_{t-1}\right)$ is the key problem for estimating the posterior density.

3 BAYESIAN SOLUTION

For solving the Bayesian tracking problem of group space debris, we first expand Eq. 7 to:

$$
\begin{aligned}
&p\left(L_t, X_t, e_t \mid L_{t-1}, X_{t-1}, e_{t-1}\right) \\
&\propto p\left(X_t, e_t \mid L_{t-1}, X_{t-1}, e_{t-1}\right) p\left(L_t \mid L_{t-1}, X_{t-1}, e_{t-1}\right) \psi\left(X_t, e_t, L_t\right)
\end{aligned} \tag{8}
$$

where $p\left(X_t, e_t \mid L_{t-1}, X_{t-1}, e_{t-1}\right)$ is the prediction model of the objects' states, $p\left(L_t \mid L_{t-1}, X_{t-1}, e_{t-1}\right)$ is the prediction model of the group center, $\psi\left(X_t, e_t, L_t\right)$ is the MRF model that describes the interaction between the group center and individuals.

3.1 Dynamic prediction model of individuals

The current multiple objects' states can be derived from the prediction of previous individuals' states. Here, we do not take into account the previous group center's influence on individuals' states. Thus, the prediction model of multiple objects' states can be written as:

$$
\begin{aligned}
&p\left(X_t, e_t \mid L_{t-1}, X_{t-1}, e_{t-1}\right) = \\
&p\left(X_t, e_t \mid X_{t-1}, e_{t-1}\right) = p\left(X_t \mid X_{t-1}, e_t, e_{t-1}\right) p\left(e_t \mid e_{t-1}\right)
\end{aligned} \tag{9}
$$

where $p\left(e_t \mid e_{t-1}\right)$ represents the prediction of the existence state, which can also be regarded as a transition of simplified group structure, and $p\left(X_t \mid X_{t-1}, e_t, e_{t-1}\right)$ is the state prediction of individuals' motion based on the existence state.

3.1.1 Transition model of objects' existence states

In this formulation, X_t is regarded as a fixed dimensional quantity with $N_{\max}$ elements [12], each of which is active or inactive according to its existence variable $e_{t,i}$. This is a reasonable framework given that practical systems have computational and storage limitations. Thus, the existence state transition model is:

$$
p\left(e_t \mid e_{t-1}\right) = \prod_{i=1}^{N_{\max}} p\left(e_{t,i} \mid e_{t-1,i}\right) \tag{10}
$$

where an empirical value is set for the existence or disappearance probability of each object. The concrete parameters' value should be selected according to the practical background. However, from our experience, we found that the tracking result is not sensitive to the parameters' value.

3.1.2 Transition model of objects' motion states

The transition models of objects' motion state are not the same with respect to the different combinations of e_t and e_{t-1}. Let Υ_b denote the set of objects with $e_{t,i} = 1$ and $e_{t-1,i} = 0$, which represent the object's birth. In the same way, let Υ_d denote the set of objects with $e_{t,i} = 0$, which represents the object's disappearance, and Υ_u denote the set of objects with $e_{t,i} = 1$ and $e_{t-1,i} = 1$, which represents that the objects need to be updated. Furthermore, N_{Υ_b}, N_{Υ_d}, and N_{Υ_u} are the corresponding object number in the set, respectively.

$$
\begin{aligned}
&p\left(X_t \mid X_{t-1}, e_t, e_{t-1}\right) = \\
&p_u\left(X_{t,\Upsilon_u} \mid X_{t-1,\Upsilon_u}\right) \prod_{i \in \Upsilon_b} p_b\left(X_{t,i} \mid X_{t-1,\Upsilon_u}\right) \prod_{i \in \Upsilon_d} p_d\left(X_{t,i}\right)
\end{aligned} \tag{11}
$$

The details of the calculation of Eq. 11 can be found in [12].

3.2 Prediction model of group center

When the existence state e_{t-1} and the motion state X_{t-1} of the group objects are obtained, the state of group center can be predicted. The previous group center L_{t-1} is used to describe the group characteristics and constrain the individuals at the previous time. This information is involved in multiple objects states X_{t-1} through inference and interaction. Here, we directly predict the current group center according to the previous individuals' states, ignoring the influence of L_{t-1}. Thus, we can get $p\left(L_t \mid L_{t-1}, X_{t-1}, e_{t-1}\right) \approx p\left(L_t \mid X_{t-1}, e_{t-1}\right)$.

The group center has a more reliable motion tendency and higher detection probability than the individuals. The prediction of group center should maintain the consistency of orbit. At time $t-1$, the probability of group existence state $e_{t-1}=h$ is $q\left(e_{t-1}=h\right)$. The different existence states of objects will lead to different group structures. Thus, a plethora of group centers will exist. To prevent divergence of the tracking result, the group structure whose probability is smaller than a threshold P_G is removed. P_G can be chosen to have a small value between 0 and 0.5; e.g. in this study, its value is 0.3:

$$q'\left(e_{t-1}=h\right)=\begin{cases}0 & \text{if } q\left(e_{t-1}=h\right) \leq P_G \\ q\left(e_{t-1}=h\right) & \text{otherwise}\end{cases} \tag{12}$$

Subsequently, the probability of group structure is normalized as follows:

$$\hat{q}\left(e_{t-1}=h\right)=\frac{q'\left(e_{t-1}=h\right)}{\sum_{h=1}^{N_{\gamma}\left(e_{t-1}\right)} q'\left(e_{t-1}=h\right)} \tag{13}$$

Let Υ_e be the set of objects with $e_{t-1,i}=1$, and N_{Υ_e} be the corresponding object number. The group center is set as the average value of the objects' states $\bar{X}_{t-1}=\frac{1}{N_{\Upsilon_e}} \sum_{i \in \Upsilon_e} X_{t-1,i}$. If the group has no member, there is $\bar{X}_{t-1}=NaN$. The trajectory of group center can be predicted according to the mean value $\bar{X}_{t-1}$ at time $t-1$ as:

$$p\left(L_t \mid \bar{X}_{t-1}\right)=\begin{cases}NaN & \text{if } \bar{X}_{t-1}=NaN \\ N\left(F \bullet \bar{X}_{t-1}, Q_L\right) & \text{otherwise}\end{cases} \tag{14}$$

where Q_L is the orbital prediction covariance of the group center. The probability distribution of the predicted group center can be rewritten as:

$$p\left(L_t \mid X_{t-1}, e_{t-1}, L_{t-1}\right)=\hat{q}\left(e_{t-1}=h\right) p\left(L_t \mid \bar{X}_{t-1}\right) \tag{15}$$

3.3 Association probability between group center and individuals

Both the group center and the individuals' states reflect the kinetic characteristics of the group space debris. Certainly, there are some connection and interaction between them. Here, the association probability $\psi\left(X_t, e_t, L_t\right)$ is used to describe the relationship in the form of MRF. When the objects' motion state X_t, the existence state e_t, and the group center L_t have been inferred at time t, the matching degree between the predicted group center and the average value of the predicted individuals' states can be used to evaluate the association probability.

Let $\tilde{\Upsilon}_e$ be the set of objects that have $e_{t,i}=1$, and $N_{\tilde{\Upsilon}_e}$ be the corresponding object number. The mean value of the multiple objects' states is $\tilde{L}_t=\frac{1}{N_{\tilde{\Upsilon}_e}} \sum_{i \in \tilde{\Upsilon}_e} X_{t,i}$. If no object is contained, the value is $\tilde{L}_t=NaN$, and the corresponding association probability is also set as NaN. Otherwise, the association probability between the group center and the individuals can be calculated with JPDA filtering [14] as follows:

$$\begin{aligned}&\psi\left(X_t, e_t, L_t\right)= \\ &\sum_{\forall \Omega} p_{JPDA}\left(\tilde{L}_t \mid L_t, \Omega, Q_L, p_{fa}, p_d, \lambda\right) p_{JPDA}(\Omega)\end{aligned} \tag{16}$$

In Eq. 16, Ω denotes the validation matrix of joint association event, Q_L is the observation covariance, p_{fa} indicates the probability of false alarm, p_d signifies the detection probability, and λ is the expected number of false alarm in the observation area.

According to the MRF model $\psi\left(X_t, e_t, L_t\right)$, which achieves a prior information fusion between the group and individuals, the overall tendency of the group and individuals' trajectories can be maintained with consistency and robustness even in the case of low detection probability and highly dense clusters.

3.4 The likelihood probability of observation

The probability density of observation $p\left(Z_t \mid X_t, e_t, L_t\right)$ was calculated using the JPDA algorithm. As the group center L_t cannot be directly observed, only the position information was assumed to be consisted in the observation Z_t. Let us denote the position vector in X_t as $\boldsymbol{r}_t$. The observation probability density can be obtained from Eq. 17:

$$p\left(Z_t \mid X_t, e_t, L_t, G_t\right) = p\left(Z_t \mid \mathbf{r}_t, e_t\right) = \sum_{\forall \Omega'} p_{JPDA}\left(Z \mid \mathbf{r}_t, e_t, \Omega', Q_\omega, p'_{fa}, p'_d, \lambda'\right) p_{JPDA}(\Omega') \tag{17}$$

where the calculation procedure and parameters setting in Eq. 17 is similar to Eq. 16.

4 CALCULATION OF BAYESIAN TRACKING BASED ON MCMC-PARTICLE ALGORITHM

MCMC algorithm [15] is an efficient tool for the inference and integral of probability distribution during a nonlinear process. However, it usually has a serious computationally burden. For releasing the computation resources, we have used the improved MCMC-Particle algorithm [12] to solve the Bayesian tracking problem presented in Section 3.

At the m th MCMC-Particle iteration, the procedure is absolutely same as the details in [12] except that the group center proposal should be done firstly in this paper. Here, only the group center proposal is introduced. Firstly, all the particles $\left\{X_{t-1,p}, e_{t-1,p}\right\}$ at time $t-1$ should be clustered. Commonly, the particles will converge around the real value after carrying out MCMC iteration for certain number of times. The classic K-means algorithm [16] can easily accomplish the clustering. After deleting the clustering sets of particles with minor proportion (e.g. 5%), the estimated states are labelled. The set of particles belonging to the n th, $n = 1, 2, \cdots, N_{\max}$ object is denoted as $\Upsilon_{e,n}$. The average value of the particles' states $X_{t-1,n} = \bar{X}_{t-1,\Upsilon_{e,n}}$ is taken as the state estimation of the corresponding object, while $e_{t-1,n} = 1$. If there is no particle that belong to the n th object, we have $e_{t-1,n} = 0$. Subsequently, we propose an existence state variable $e_{t-1} = h$ with a probability $q'\left(e_{t-1} = h\right)$ at time $t-1$. Let $\tilde{\Upsilon}_e$ represent the set of objects with $e_{t-1,n} = 1$ based on the existence state $e_{t-1} = h$. The group center is proposed from the Gauss distribution $L^*_{t-1} \sim N\left(\bar{X}_{t-1,\tilde{\Upsilon}_e}, Q_L\right)$. If the value of all the existence states is zero, then we set $L^*_{t-1} = NaN$.

We executed MCMC-Particle iteration for a certain number of times and burnt-in partial old particles. The distribution of converging particles can be approximately represented as the posterior density of the group space debris' states.

5 SIMULATIONS AND PERFORMANCE ANALYSIS

Let us consider the scenario of four closed-space debris combined to a group. The maximum object number is set as $N_{\max} = 4$. The orbital parameters of the four space debris at initial time are listed in Tab. 1.

Table 1. Orbital parameters of space debris in the scenario

	Semi-major Axis（km）	Eccentricity	Inclination(°)	Right ascension of ascending node(°)	Argument of perigee(°)	Eccentric Anomaly(°)
Object 1	7500	0.01	1	0	0	0.4
Object 2	7500	0.01	1	0.4	0	0
Object 3	7501	0.01	1	0.4	0	0
Object 4	7501	0.01	1	0	0	0.4

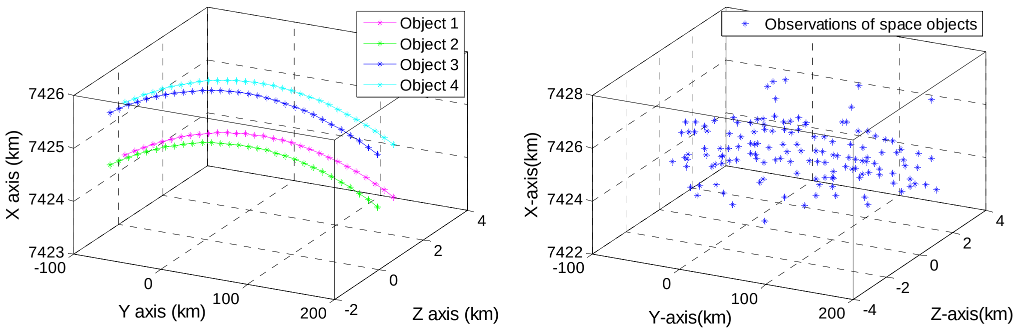
The tracking parameters for the simulation scenario are presented in Tab. 2.

Table 2.The tracking parameters for the simulation results

	symbol	value		symbol	value
Observation time interval	τ	1 s	Motion model error of objects	Q_u	$\begin{bmatrix} 50\boldsymbol{I}_{3\times3} & \boldsymbol{0} \\ \boldsymbol{0} & 10\boldsymbol{I}_{3\times3} \end{bmatrix}$
Maximum number of objects	$N_{\max}$	4	Prediction covariance of object's state	Q_p	$\begin{bmatrix} 300\boldsymbol{I}_{3\times3} & \boldsymbol{0} \\ \boldsymbol{0} & 100\boldsymbol{I}_{3\times3} \end{bmatrix}$
Observation covariance	Q_ω	$100\boldsymbol{I}_{3\times3}$	Prediction covariance of group center	Q_L	$\begin{bmatrix} 150\boldsymbol{I}_{3\times3} & \boldsymbol{0} \\ \boldsymbol{0} & 50\boldsymbol{I}_{3\times3} \end{bmatrix}$
Probability of false alarm of object	p'_{fa}	2e-005	Probability of false alarm of group center	p_{fa}	0.01
Probability of detection for one object	p'_d	0.65	Probability of detection for group center	p_d	0.98
Expected observed number of false alarm objects at each time step (Poisson distribution)	λ'	2	Expected number of false alarm of group centers at each time step (Poisson distribution)	λ	1

The four orbits propagate after 30 seconds from the initial time in our scenario. They form a steady group in the tracking area.

The four orbits propagate after 30 seconds from the initial time in our scenario. They form a steady group in the tracking area. We obtained the observation at each one second. The real trajectories of space debris and the observations are shown in Fig. 5.



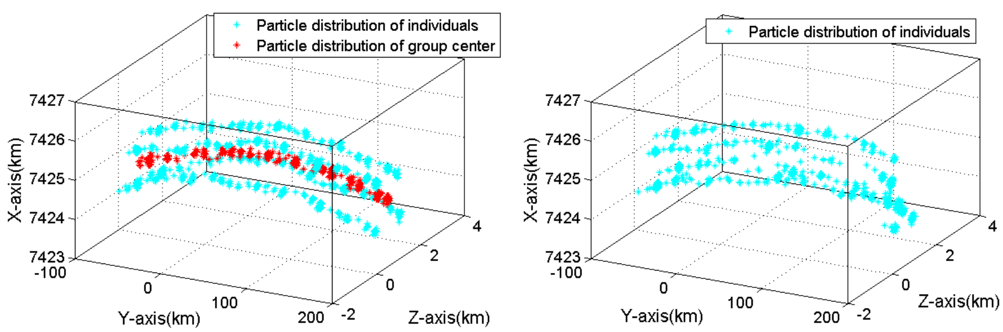
(a) (b)

Fig. 5. (a) Real trajectories of space debris; (b) The observations

We assumed that the real value exists around the observations in the form of normal distribution. Thus, the track is initiated by proposing the particles from the observations. At every time step, 10000 MCMC iterations of both the joint and individual proposals are performed. The initial 1000 iterations are used for burn-in, and 2000 MCMC outputs were kept as particle approximation to the posterior probability distribution.

The performance superiority of Bayesian tracking has been verified in [11][12] based on the MCMC algorithm. In the present study, we primarily analyzed the group center's impact on the performance of group tracking.

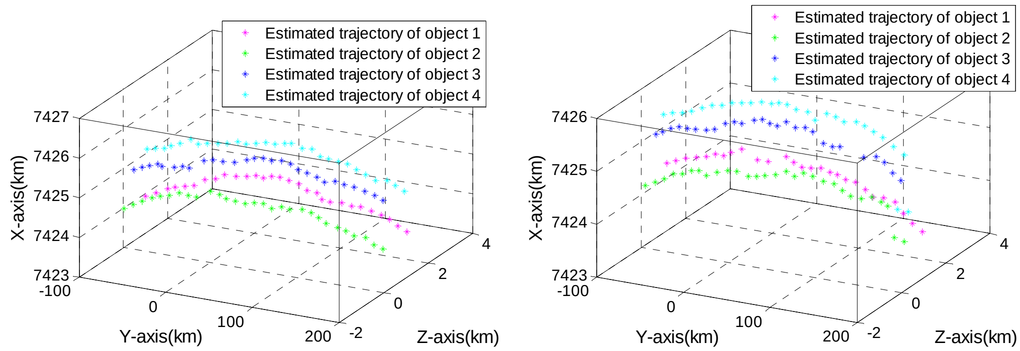
We considered the simulations according to the above-mentioned tracking parameters. The original state particles were distributed as shown in Fig. 6.



(a) (b)

Fig. 6. Distribution of original estimated state particles: (a) with the group center; (b) without the group center

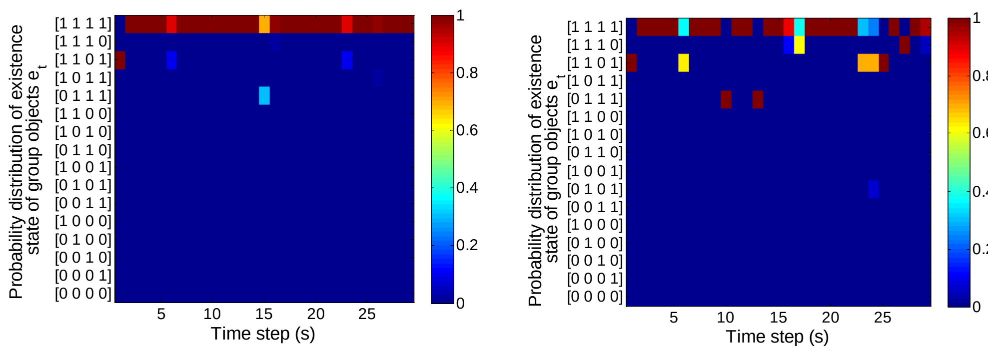
Fig. 6(a) and 6(b) shows the group tracking result with the group center and the multi-objects tracking result without the group center, respectively. According to the group center tracking, not only the group motion tendency can be obtained, but also the trajectories can be reconstructed more explicitly using the prior information of the group. Subsequently, the state particles were clustered using the K-means algorithm [16], and the outlier clusters were deleted. According to the labels of particles, the average values of the particles' states were taken as the state estimation of the corresponding objects. The estimation results are shown in Fig. 7.



(a) (b)

Fig. 7. The tracking results of group space debris: (a) with the group center; (b) without the group center

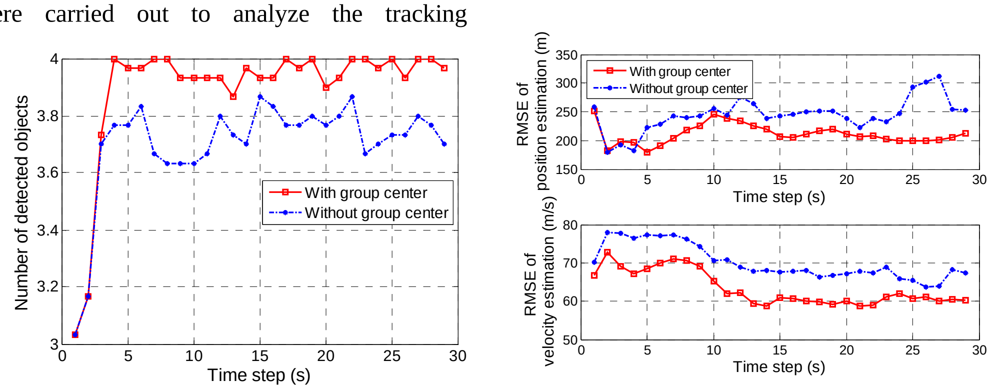
The comparisons for the distribution of the estimated existence state at each time step are shown in Fig. 8.



(a) (b)

Fig. 8. Distribution of estimated existence state for the group objects: (a) with the group center; (b) without the group center

From the simulation results, we can see that the group tracking with group center can improve the stability of individuals' existence states, when compared with that without the group center. In this study, 30 Mont Carlo runs were carried out to analyze the tracking performance. The average number of detected objects and the RMSE of the estimated objects' states at each time step were statistically calculated, as shown in Fig. 9.



(a) (b)

Fig. 9: Tracking performance of group space debris based on 30 Mont Carlo runs: (a) Average number of detected objects; (b) RMSE of the estimated objects' states

Based on the Mont Carlo runs, it is suggested that the estimated number of objects using our presented group tracking method is closer to the actual number than that obtained using the traditional multiple objects tracking method. The group tracking method can efficiently depress the miss alarm and reduce the estimation error of the objects' states. In addition, by tracking the group center, the group evolution can be exhibited clearly. In fact, in the case of low detection probability, the group tracking method can greatly improve the accuracy of the detected number and states of the objects mainly because the group center has a more consistent state and higher detection probability than the individuals. According to the mutual constraint between the group center and individuals, the group information improves the accuracy of the individuals' trajectories, while the individuals' tendencies also assist in the inference and adjustment of the group's evolution. In this study, the detection probability for group center was set as 0.98, while the individual's detection probability was only 0.65. Thus, by incorporating the group center, the tracking performance could be certainly improved due to the application of prior information of the group.

6 CONCLUSIONS

Group tracking of space debris is a potential and foremost requirement in the domain of space surveillance. It has an important significance in the surveillance of space debris clouds.

In this study, we first proposed a concept of group tracking. Subsequently, the kinetic model of group space debris was introduced. According to the motion characteristics of space debris, the Bayesian group tracking procedure with the group center was presented. The MCMC-Particle algorithm was employed to solve the intractable Bayesian integral. Based on these procedures, we carried out the simulation of tracking multiple closely spaced orbital objects. The results verified the effectiveness of our presentation in the case of low detection probability and high dense clutter. When compared with the traditional multi-objects tracking method without group center, our proposed method exhibited improved consistency and accuracy of individuals' trajectories, as well as derivation of the overall group's evolution.

7 REFERENCE

[1] Huang, J., Hu, W. D., Xin, Q., et al.. (2012). A novel data association scheme for LEO space debris surveillance based on a double fence radar system. *Advances in Space Research*, 50, 1451–1461.

[2] Johnson, N. L. (2004). The world state of orbital debris measurements and modelling. *Acta Astronautica*, 54, 267–272.

[3] Matney, M. J., Anz-Meador, P., Foster, J. L. (2004). Covariance correlations in collision avoidance probability calculations. *Advances in Space Research*, 34, 1109–1114.

[4] Reid, D. B. (1979). An algorithm for tracking multiple targets. *IEEE Trans. Autom. Control*, AC-24(6), 84-90.

[5] Fortmann, T. E., Bar-Shalom, Y., Scheffe, M. (1983). Sonar tracking of multiple targets using joint probabilistic data association, *IEEE J. Ocean. Eng.,* OE-8, 173-184.

[6] Mahler, Ronald P. S. (2003). Multitarget bayes filtering via first-order multi-target moments. *IEEE Trans. Aerosp. Electron. Syst.,* 39, 1152-1178.

[7] Vo, B. & Ma, W. (2006). The Gaussian mixture probability hypothesis density filter. *IEEE Trans. Signal Process*, 54, 4091-4104.

[8] Mahler, Ronald P. S. (2007). PHD filters of higher order in target number, *IEEE Trans. Aerosp. Electron. Syst.*, 43, 1523-1543.

[9] Vo, B., Vo, B., Cantoni, A. (2007). Analytic implementations of the cardinalized probability hypothesis density filter. *IEEE Trans. Signal Process*, 55, 3553-3567.

[10] Ng, W., Li, J., Godsill, S. J. et al. (2007). Multitarget initiation, tracking and termination using Bayesian Monte Carlo methods. *The Computer Journal,* 50, 674-693.

[11] Khan, Z., Balch, T., Dellaert. F. (2005). MCMC-Based Particle Filtering for tracking a variable number of interacting targets. *IEEE Trans. Pattern Anal. Mach. Intell*, 27, 1805-1819.

[12] Pang, S. K., Li, J., Godsill, S. J. (2011). Detection and tracking of coordinated groups. *IEEE Trans. Aerosp. Electron. Syst*., 47, 472-502.

[13] Der, G. J. (1997). An elegant state transition matrix. *Journal of the Astronautical Sciences*, 45, 371-390.

[14] Bar-Shalom, Y., Daum, F., Huang, J. (2009). The probabilistic data association filter: estimation in the presence of measurement origin un-certainty. *IEEE Control Syst. Mag.,* 29, 82-100.

[15] Gilks, W. R., Richardson, S., Spiegelhalter, D. J. (1996). *Markov Chain Monte Carlo in practice*. Lynnfield, MA: Chapman and Hall/CRC.

[16] MacQueen, J. (1967). Some methods for classification and analysis of multivariate observations. *In Proc. of the 5th Symp. on Mathematical Statistics and Probability*, Berkeley, University of California Press, 281-297.