

DETECTION OF FAINT SPACE DEBRIS ELEMENTS WITH UNKNOWN ORBITS

A.E. Kolessa

JSC "Vimpel", Moscow Institute of Physics and Technology (MIPT), kolessa.ae@mipt.ru

ABSTRACT

The extraction of faint space objects with unknown orbits from digital imagery is considered to be a problem. An image is recorded using a wide-field of view (FOV) electro-optics sensor operating in stars-tracking mode. To solve this problem an approach based on the detection of fragments of tracks with further grouping is considered. At the same time the problem of stars detection is solved. Both fragments of tracks and stars are detected within a sliding spatial window with the classification of "star / fragment of track". Signal accumulation along a hypothetical track is restricted by the spatial window. The total number of extracted track fragments to be grouped is much less than the total number of hot pixels to be associated into the tracks if the traditional approach is used, based on a low signal threshold.

1 INTRODUCTION

In this paper, the common problem (see, for example, [1], [2]) of extracting faint space objects with unknown orbits from digital imagery is analyzed. Input images are recorded using the CCD matrix of an optical sensor with a wide field of view. The sensor is operating in stars-tracking mode. The objects to be detected are moving with respect to the stars. Hence, the signal of each star is accumulated in one (or a few adjacent) pixels while the signal from each moving object is distributed along a track of unknown direction, length, position of middle point and a manner of brightness variation. The total numbers of objects are unknown.

To solve this problem a traditional approach is generally used based on the signal thresholding and the further grouping of "hot pixels" into tracks. This approach is effective for the detection of bright objects. However, the signals of faint objects can be below the threshold specified for bright objects. Reducing the threshold value can cause a considerable increase in the total number of false detections. This makes it impossible to perform data processing in real time.

A well-known alternative approach consists of implementing an appropriate parametrical model of a track which provides a direct solution to the classic problem of making a decision about the total number of space objects in an image and estimating the parameters of the track model for each object. The direct solution of

this problem for a large image (typically several tens megapixels) in real time seems feasible only with the use of a powerful supercomputer.

In this report a compromise approach is considered as the detection of short fragments of tracks with further grouping. The fragment detection algorithm is based on the direct search of local maximums of decision making statistics by sliding a window across the image. In each point of local maximum, one of the following alternatives "nothing apart background" / "star" / "fragment of track" is selected with a simultaneous estimation of the corresponding unknown parameters: mean (across current window) background value, star position and brightness, fragment direction, length, position of middle point and current brightness. The total number of detected fragments turns out to be significantly less than the total number of hot pixels detected using the traditional approach. This makes the further association of detected fragments into tracks much simpler in comparison with the classical case. To speed up the algorithm all necessary integrals are calculated in advance. This makes the algorithm capable of operating in real time on a computer with relatively modest calculation power. Dimensions of sliding window, a step of sliding and the total number of tested directions of an objects motion should be selected by the user. A similar approach for extracting static point objects from digital frames was considered in [3], [4].

It seems that particular place in integrated image processing of the method, which we propose in this paper, is the preliminary extraction of tracks of moving space objects having unknown orbits and low signal-to-noise ratio with further final full-scale processing based on signal storage along the directions of extracted tracks (similar [5]) to achieve a higher quality of track extraction and estimation.

The orbit parameters of detected objects can be estimated on the basis of tracks extracted in sequence of images by means, for example, of the algorithms considered in [6].

The algorithm is elaborated for use with the automatic optical telescopes of UN ORT (University Network of Optical Robotic Telescopes) of MIPT (Moscow Institute of Physics and Technology) and JSC "Vimpel".



Figure 1. UN ORT optical camera, 15 sm. aperture, 11°x7° FOV (Kislovodsk, Tiraspol, Blagoveschensk, Kitab)



Figure 2. UN ORT telescope, 50 sm aperture, 3.5° x 3.5° FOV (Ussuriysk)

2 MODEL OF SIGNALS

Dimension of a pixel (along each axis) is considered as scale. A hypothetical static point with position (u_0, v_0) during exposition generates in some pixel (k, l) a signal

$$S_{k,l} = A \int_{k-1}^k du \int_{l-1}^l F(u_0 - u, v_0 - v) dv, \quad (1)$$

where A - the signal "amplitude", $F(u, v)$ - normalized

Point Spread Function (PSF), $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(u, v) dudv = 1$. It

is assumed, that PSF is separable (i.e. $F(u, v) = F_u(u)F_v(v)$), on each axis, and has an effective width comparable to a pixel size. A signal generated in some pixel (k, l) by an object moving during exposition is approximated by the sum of signals from bright points with equal amplitude A distributed along the track with some period (comparable with the PSF width). A fragment of a track is characterized by the position (u, v) of its center, direction d , and amplitude A of the bright points.

Consider a grid of nodal points (see black points in Fig.3) having a period 0.5 of pixel size, and a sliding

window $\Omega_{k,l}$ with central pixel (k, l) , composed of $N = (2n+1) \times (2n+1)$ pixels. Furthermore for the sake of simplicity the idea of the algorithm is explained for the case of 3×3 window ($n=1, N=9$). The position (u, v) of a star or the center of a track fragment, which passes through the central pixel of the window, is truncated to the position of the center of pixel (k, l) . It is assumed that the track direction d coincides with one of four reference directions (see Fig 1). Under these assumptions the signal $S_{k,l}$ in pixel (k, l) from a bright point, located at one of nodal points 1-4 (see Fig. 3), can be calculated in accordance with (1) as $S_{k,l} = A$ for nodal point 0, $S_{k,l} = A/2$ - for nodal points 1,3, и $S_{i,j} = A/4$ - for nodal point 4 (see Fig.3).

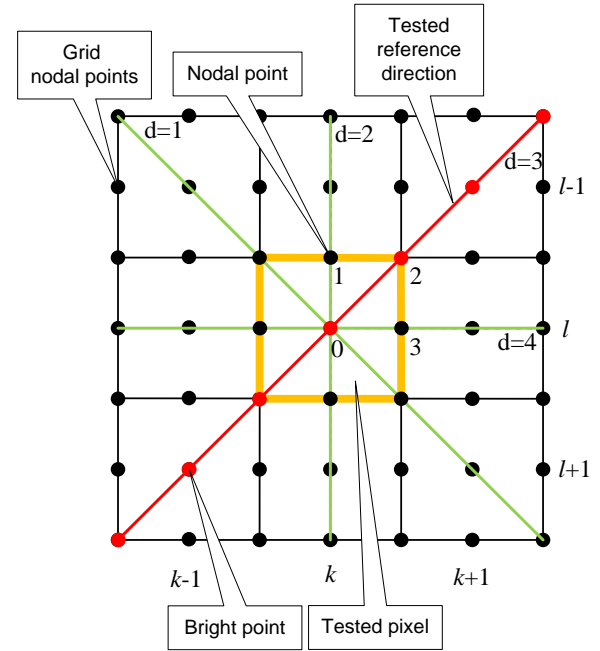


Figure 3. Sliding window 3x3.

Let B be a local (averaged across a sliding window $\Omega_{k,l}$) background of observation. If a star with signal amplitude A is located in the pixel (k, l) , then signals in pixels of $\Omega_{k,l}$ are characterized by the vector $X_1 = [B, A]$. The fragment of a track passing across the window $\Omega_{k,l}$ through the pixel (k, l) is characterized by the vector $X_2 = [B, A, D]$, where A is the amplitude of a signal of the bright points along the track, and D is the track direction. Consider also a vector $X_0 = [B]$,

characterizing a background signal in pixels of $\Omega_{k,l}$ in the absence of a star or a track.

Provided that not more than one object (star or track) can be observed in the window $\Omega_{k,l}$ the following hypotheses H_i , $i=0,1,2$ are tested on the bases of signals in a pixels $\Omega_{k,l}$:

H_1 : a star with the state vector X_1 is located in the pixel (k,l) of $\Omega_{k,l}$;

H_2 : a track fragment with the state vector X_2 passes through the central pixel (k,l) across $\Omega_{k,l}$;

H_0 : an instrumental noise and a background with the state vector $X_0=[B]$ cause the signals in pixels of $\Omega_{k,l}$.

The following model of random signals $Y_{k+k',l+l'}$, $k',l'=-1,0,+1$ is accepted in pixels of the window $\Omega_{k,l}$:

$$Y_{k+k',l+l'} = S_{k+k',l+l'}(X_i, I) + \varepsilon_{k+k',l+l'}, \quad (2)$$

where I is the random number of the hypothesis H_i , taking *a priori* the values $i=0,1,2$ respectively with probabilities P_0, P_1, P_2 , where $(P_0 + P_1 + P_2 = 1)$, $S_{k+k',l+l'}(X_i, i)$ - is the signal in pixel $(k+k', l+l')$ for the hypothesis H_i , described by the vector of parameters X_i , $\varepsilon_{k+k',l+l'}$ - is the noise of the CCD camera with zero mean and known (estimated empirically) local variance σ^2 . Noise signals in different pixels are assumed to be uncorrelated. The value $S_{k,l}/\sigma$ is taken as the signal-to-noise ratio for the pixel (k,l) . The experimental distribution (see Figure 4) of the instrumental noise of the CCD camera, which was used for the algorithm testing, justifies the choice of Gaussian model of probability distribution of $\varepsilon_{k,l}$.

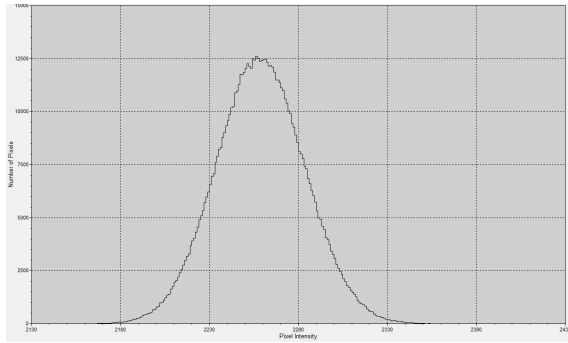


Figure 4. Experimental probability distribution of instrumental noise value for ProLine PL 11002 (Sensor KAI-11002).

The factors $\Phi_{k',l'}$ and $\Psi_{k',l'}(d)$, $k',l'=-1,0,+1$, $d=1,\dots,4$ are assigned in accordance with Table 1. Under these assumptions the model of a signal $Y_{k+k',l+l'}$ in pixel $(k+k', l+l')$, $(k',l'=-1,0,+1)$ can be written as

$$Y_{k+k',l+l'} = \begin{cases} B + \varepsilon_{k+k',l+l'} & \text{if } i=0 \\ A\Phi_{k',l'} + B + \varepsilon_{k+k',l+l'} & \text{if } i=1 \\ A\Psi_{k',l'}(D) + B + \varepsilon_{k+k',l+l'} & \text{if } i=2 \end{cases} \quad (3)$$

Table 1.

k'	-1	0	+1	l'
$\Phi_{k',l'}$	0	0	0	-1
	0	1	0	0
	0	0	0	+1
$\Psi_{k',l'}(1)$	1.5	0.25	0	-1
	0.25	1.5	0.25	0
	0	0.25	1.5	+1
$\Psi_{k',l'}(2)$	0	2	0	-1
	0	2	0	0
	0	2	0	+1
$\Psi_{k',l'}(3)$	0	0.25	1.5	-1
	0.25	1.5	0.25	0
	1.5	0.25	0	+1
$\Psi_{k',l'}(4)$	0	0	0	-1
	2	2	2	0
	0	0	0	+1

3 DECISION MAKING STATISTICS

Consider the problem of making a decision j about the unknown number I of the hypothesis H_i ($I=0,1,2$) on the basis of a realization $y = \|y_{k+k',l+l'}\|$ of the random signals $Y = \|Y_{k+k',l+l'}\|$, $k',l'=-1,0,+1$ in pixels of the sliding window $\Omega_{k,l}$ with a further calculation of estimate \hat{x}_j corresponding to state vector X_j .

For the synthesis of the algorithm for solving this problem, the Bayesian approach is used, according to which the optimal decision is made on the basis of minimization on ν and $\hat{x}_\nu \in \omega_\nu$ of a posteriori risk [7]

$$R_a(y, \hat{x}_\nu, \nu) = \sum_{i=0}^2 P_i \int g(\nu, \hat{x}_\nu | i, x_i) p_{Y, X_i | I} (y, x_i | i) dx_i, \quad (4)$$

where ω_ν - is a domain of a priori permissible values of \hat{x}_ν , $p_{Y, X_i | I} (y, x_i | i)$ - is the joint probability density of Y and X_i under condition $I = i$,

$$x_0 = [b], \quad x_1 = [b, a], \quad x_2 = [b, a, d], \quad (5)$$

$$x_0 \in \omega_0 \Leftrightarrow (b \geq 0),$$

$$x_1 \in \omega_1 \Leftrightarrow (b \geq 0, a \geq 0),$$

$$x_2 \in \omega_2 \Leftrightarrow (b \geq 0, a \geq 0, d = 1, \dots, 4).$$

The choice of loss function of the form

$$g(\nu, \hat{x}_\nu | i, x_i) = \begin{cases} g_{\nu, \nu} [1 - \delta(x_\nu - \hat{x}_\nu)] & npu \quad \nu = i \\ g_{\nu, i} & npu \quad \nu \neq i \end{cases} \quad (6)$$

with $g_{0,i} = g_{1,i} = g_{2,i} = g_i$ for $i = 0, 1, 2$, makes equivalent minimization on \hat{x}_ν and ν of a posteriori risk (4) and maximization on \hat{x}_ν and ν of the value

$$r_a(y, \hat{x}_\nu, \nu) = g_\nu P_\nu p_{Y | I, X_i} (y | \nu, \hat{x}_\nu) p_{X_i | I} (\hat{x}_\nu | \nu), \quad (7)$$

where $p_{Y | I, X_i} (y | \nu, x_\nu)$ - is the conditional probability density (likelihood function) of Y provided fixed values $I = \nu$, $X_\nu = x_\nu$, and $p_{X_i | I} (x_\nu | \nu) = p_{X_\nu} (x_\nu)$ - a priori probability density of X_ν .

At uniform on a certain region Ω_ν a priori probability density

$$p_{X | I} (x_\nu | \nu) = \frac{I_{\Omega_\nu} (x_\nu)}{V_\nu},$$

$$V_\nu = \int_{\Omega_\nu} dx, \quad I_{\Omega} (x) = \begin{cases} 1 & npu \quad x \in \Omega \\ 0 & npu \quad x \notin \Omega \end{cases},$$

Eq. (7) takes the form

$$r_a(y, \hat{x}_\nu, \nu) = c_\nu I_{\Omega_\nu} (\hat{x}_\nu) p_{Y | I, X_i} (y | \nu, \hat{x}_\nu), \quad (8)$$

where $c_\nu = g_\nu P_\nu / V_\nu$. The domain Ω_ν can always be chosen sufficiently large to satisfy the condition

$$\begin{aligned} \hat{x}_\nu^* &= \arg \max_{\hat{x}_\nu \in \omega_\nu} I_{\Omega_\nu} (\hat{x}_\nu) p_{Y | I, X} (y | \nu, \hat{x}_\nu) = \\ &= \arg \max_{\hat{x}_\nu \in \omega_\nu} \ln p_{Y | I, X} (y | \nu, \hat{x}_\nu) \end{aligned} \quad (9)$$

In this case, the decision rule takes the following form. The values \hat{x}_ν^* , $\nu = 0, 1, 2$ are calculated due to Eq. (9). Then, the decision is made about the order number of observable hypothesis in accordance with the following formula

$$j = \arg \max_{\nu=0,1,2} c_\nu p_{Y | I, X_i} (y | \nu, \hat{x}_\nu^*), \quad (10)$$

and \hat{x}_j^* is taken as the estimate of the corresponding state vector.

According to Eq. (3)

$$\ln p_{Y | I, X_\nu} (y | \nu, x_\nu) = const - \frac{1}{2\sigma^2} \rho_\nu (x_\nu), \quad (11)$$

where function $\rho_\nu (x_\nu)$, taking into account Eq.(5), is described as

$$\rho_0(b) = (y - b\bar{1})^T (y - b\bar{1}), \quad (12)$$

$$\rho_1(b, a) = (y - b\bar{1} - a\Phi)^T (y - b\bar{1} - a\Phi), \quad (13)$$

$$\rho_2(b, a, d) = (y - b\bar{1} - a\Psi(d))^T (y - b\bar{1} - a\Psi(d)), \quad (14)$$

$\bar{1}$ - is a vector of order N with unit components, Φ and $\Psi(d)$ - is a column of vectors of coefficients $\Phi_{k', l'}$ and $\Psi(d)$, $k', l' = -1, 0, +1$ respectively, defined in Table 1.

Denoting

$$\hat{x}_0^* = [\hat{b}_0], \quad \hat{x}_1^* = [\hat{b}_1, \hat{a}_1], \quad \hat{x}_2^* = [\hat{b}_2, \hat{a}_2, \hat{d}], \quad (15)$$

and taking into account (10), it is clear, that finding \hat{x}_ν^* from (9) is equivalent to calculation $\hat{b}_0, \hat{a}_1, \hat{b}_1, \hat{a}_2, \hat{b}_2, \hat{d}$ from

$$\hat{\rho}_0 = \rho_0(\hat{b}_0) = \min_{b \geq 0} \rho_0(b), \quad (16)$$

$$\hat{\rho}_1 = \rho_1(\hat{b}_1, \hat{a}_1) = \min_{b \geq 0, a \geq 0} \rho_1(b, a), \quad (17)$$

$$\hat{\rho}_2 = \rho_2(\hat{b}_2, \hat{a}_2, \hat{d}) = \min_d \min_{b \geq 0, a \geq 0} \rho_2(b, a, d). \quad (18)$$

Minimization in Eqs. (16)-(17) on $b \geq 0, a \geq 0$ gives:

$$\hat{b}_0 = s, \quad (19)$$

$$\hat{a}_1 = \begin{cases} \frac{y_{k,l} - s}{1 - \frac{1}{N-1}} & \text{if } y_{k,l} \geq s \\ 0 & \text{else} \end{cases}, \quad (20)$$

$$\hat{b}_1 = \begin{cases} \frac{s - \frac{y_{k,l}}{N}}{1 - \frac{1}{N-1}} & \text{if } y_{k,l} \geq s \\ s & \text{else} \end{cases}. \quad (21)$$

$$\hat{\rho}_0 = e - Ns^2, \quad (22)$$

$$\hat{\rho}_1 = e - Ns^2 - \hat{a}_1(y_{k,l} - s), \quad (23)$$

where the coefficient $s = \frac{1}{N} \bar{1}^T y$ is calculated as

$$s = \frac{1}{N} \sum_{k',l'} y_{k+k',l+l'}, \quad (24)$$

The values $\tilde{\rho}_2$, $\tilde{b}_2 = \tilde{b}_2(d)$, $\tilde{a}_2 = \tilde{a}_2(d)$ defined by the relationship

$$\tilde{\rho}_2 = \rho_2(\tilde{b}_2, \tilde{a}_2, d) = \min_{b \geq 0, a \geq 0} \rho_2(b, a, d) \quad (25)$$

can be found as

$$\tilde{a}_2 = \begin{cases} \frac{\mu - \alpha s}{w - \frac{\alpha^2}{N}} & \text{if } \alpha s \leq \mu \\ 0 & \text{else} \end{cases} \quad (26)$$

$$\tilde{b}_2 = s - \frac{1}{N} \alpha \tilde{a}_2, \quad (27)$$

$$\tilde{\rho}_2 = e - Ns^2 - \tilde{a}_2(\mu - \alpha s), \quad (28)$$

where

$$e = \sum_{k',l'} (y_{k+k',l+l'})^2, \quad (29)$$

and the coefficients $\mu = \mu(d) = \Psi^T y$, $\alpha = \alpha(d) = \bar{1}^T \Psi$, $w = w(d) = \Psi^T \Psi$ are calculated as

$$\alpha = \begin{cases} 5.5 & \text{for } d = 1, 3 \\ 6.0 & \text{for } d = 2, 4 \end{cases}, \quad (30)$$

$$w = \begin{cases} 9.25 & \text{for } d = 1, 3 \\ 12 & \text{for } d = 2, 4 \end{cases}, \quad (31)$$

$$\mu = \begin{cases} 1.5(y_{k-1,l-1} + y_{k,l} + y_{k+1,l+1}) + S_0 & \text{if } d = 1 \\ 2(y_{k,l-1} + y_{k,l} + y_{k,l+1}) & \text{if } d = 2 \\ 1.5(y_{k+1,l-1} + y_{k,l} + y_{k-1,l+1}) + S_0 & \text{if } d = 3 \\ 2(y_{k-1,l} + y_{k,l} + y_{k+1,l}) & \text{if } d = 4 \end{cases}, \quad (32)$$

$$S_0 = 0.25(y_{k,l-1} + y_{k-1,l} + y_{k+1,l} + y_{k,l+1}), \quad (33)$$

In Eqs. (24), (29) the sum is over $k', l' = -1, 0, 1$.

Finally,

$$\hat{\rho}_2 = \rho_2(\hat{b}_2, \hat{a}_2, \hat{d}) = \min_d \rho_2(\tilde{b}_2(d), \tilde{a}_2(d), d). \quad (34)$$

By Eq. (11) the inequality

$$c_v p_{Y|I, X_v}(y|v, \hat{x}_v^*) \geq c_i p_{Y|I, X_i}(y|i, \hat{x}_i^*)$$

is equivalent to

$$l_{v,i} \geq 2\sigma^2 (\ln c_i - \ln c_v) = h_{v,i}. \quad (35)$$

where

$$\hat{\rho}_i = \rho_i(\hat{x}_i^*), \quad (36)$$

$$l_{v,i} = l_{v,i}(x_v^*, x_i^*) = \hat{\rho}_i - \hat{\rho}_v. \quad (37)$$

Taking into account Eqs. (22)-(28), the decision making statistics (37) can be written as

$$l_{1,0} = \hat{a}_1(y_{k,l} - s), \quad (38)$$

$$l_{2,0} = \hat{a}_2(\mu - \alpha s), \quad (39)$$

$$l_{2,1} = l_{2,0} - l_{1,0}. \quad (40)$$

Thus, the following decision making procedure can be formulated:

- The inequalities $l_{1,0} \geq h_{1,0}$ and $l_{2,0} \geq h_{2,0}$ are verified;
- If both of them aren't valid, the decision $I = j = 0$ (there is no any object within the window) is made;
- If the first of them is valid and the second isn't, the decision $I = j = 1$ (there is a star within the window) is made;
- If the first of them isn't valid and the second is, the decision $I = j = 2$ (there is a track within the window) is made;
- If both inequalities are valid, the next inequality $l_{2,1} \geq h_{2,1}$ is verified; and if it is valid, the decision $I = j = 2$ is made, otherwise $I = j = 1$.

The thresholds $h_{1,0}$, $h_{2,0}$ are chosen on the basis of an acceptable level of false detection of a star or a track

fragment, the threshold $h_{2,1}$ - on the basis of an acceptable level of probability of false detection of a track fragment instead a star. This method allows for the selection of thresholds without assigning definite values to loss coefficients, prior probabilities P_i of hypotheses and the volumes V_i of the domains Ω_v , $i = 0, 1, 2$.

4 IMAGE PROCESSING

To detect all objects of different types within an image the relief of optimal value for decision making statistics $l_{v,i}$ (with corresponding v, i selected for each window position (k, l) , $k = 1, \dots, n_x - 2$; $l = 1, \dots, n_y - 2$) is used. Each local maximum of the relief relates to an object of corresponding type. Verification of the local maximum condition avoids the duplication of "phantoms" in adjacent points. Figure 5 shows the simulated image, which contains 100 stars with signal-to-noise ratio 50σ , four tracks with 200 pixels in length and a constant signal-to-noise ratio assuming values $1.5\sigma, 2.0\sigma, 2.5\sigma, 3.0\sigma$ for the tracks in top-down order. Instrumental noise $\sigma = 50$. Background value $B = 2000$.

Figure 6 shows detected track fragments (green squares) and stars (red squares).

Figure 7 shows track fragments associated into tracks (yellow rectangles). A consideration of associating algorithm is beyond the scope of this report.

It is seen from Figures 6 and 7 that the algorithm is capable of extracting tracks with signal-to-noise ratio near 1.5.

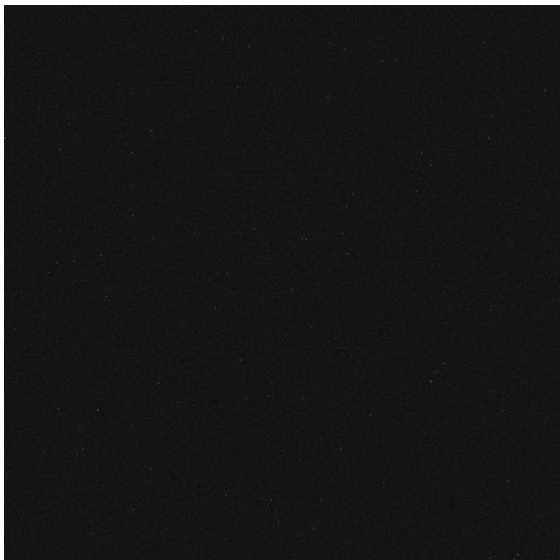


Figure 5. Input image

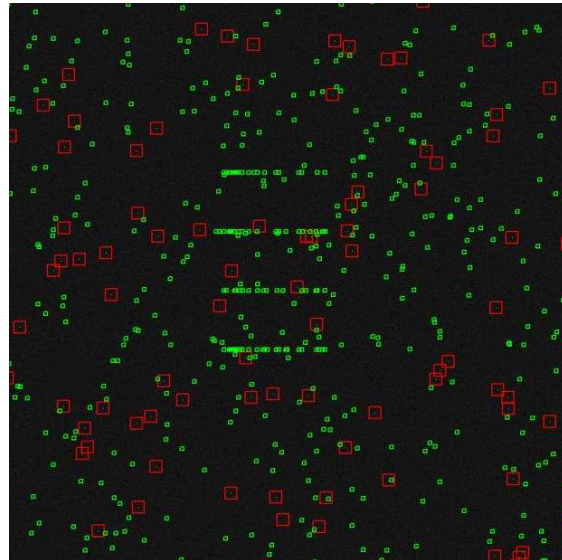


Figure 6. Image after processing: detected track fragments (green squares) and stars (red squares).

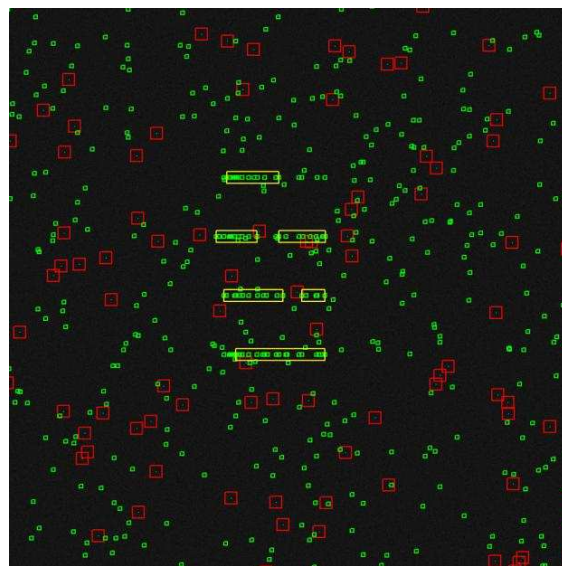


Figure 7. Image after processing: track fragments associated into the tracks (yellow rectangles).

The time taken to process the image composed of approximately 10^6 pixels on an IBM T60 notebook was about 5 seconds.

5 CONCLUSIONS

The considered algorithm can be used for the preliminary extraction of tracks of moving space objects with unknown orbits and low signal-to-noise ratio.

To achieve a higher quality of track extraction and estimation, the algorithm allows the extracted tracks to be subjected to final full-scale processing based on signal storage along the directions of extracted tracks (similar [5]).

The algorithm can be easily modified for a window with $n > 1$, a greater number of reference track directions, a higher resolution (for example 0.5 of pixel size) of position determination. This will improve the quality of the algorithm, but will cause corresponding growth of its computational complexity.

6 REFERENCES

1. Yanagisawa T., et al. (2005). Automatic Detection Algorithm for Small Moving Objects. *Publ. Astron. Soc. Japan*, 57. P.399-408.
2. Gural P.S., Larsen J.A., Gleason A.E. (2005). Matched Filter Processing for Asteroid Detection. *The Astronomical Journal*, 130: 1951-1960.
3. Kolessa A.E., Repin V.G. (2009). Robust adaptive algorithm of extraction of blips of objects in digital image. *Space Based Informational and Control Systems*. 3.
4. Kolessa A.E., Maltcev A.V., Repin V.G. (2009). Extraction and classification of unresolved grouped objects in digital optical image. *International Conference "Astronomy and World Heritage: Across Time and Continents", August 19-26, Kazan, Russia, Astrokazan-2009, Reports*.
5. Kolessa A.E. (2009). Correction of Point Target Track Parameters by means of Joint Processing of Sequential Images. 4'2009
6. Kolessa A.E., Pruglo A.V., Ravdin S.S. (2005). Estimation of Orbits Based on Angles Measurements Only. *Radio Engineering*, 10.
7. Tartakovsky G.P. (2005). *Informational Systems Theory*. Moscow, FM.