GRAVITATION EFFECT ON A FLUX OF SPORADIC MICROMETEOROIDS IN THE VICINITY OF NEAR-EARTH ORBITS

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ABSTRACT

The new approach to gravitation effect determination in calculating the flux of sporadic micrometeoroids in the near-Earth space is proposed. The technique is based on integrating the equations of motion of sporadic micrometeoroids with allowance for bending their trajectories when particles approach the Earth. The technique and results of calculation of the gravitational focusing factor k_g for various conditions are presented.

The feature of the proposed technique for calculating coefficient k_g consists in the fact that this coefficient does not explicitly depend on the values of particles' velocity at the last point. The results of investigation of coefficient k_g have shown that for the given initial velocity of micrometeoroids the values of this coefficient depend on deflection of its trajectories from the direction to the Earth center. It is shown that for low-altitude orbits the flux density growth can reach 60 %. The distribution of probabilities of various directions of particles flying to spacecraft structural elements is found to be non-uniform.

1 INTRODUCTION

The influence of micrometeoroids on the space flight safety has drawn specialists' attention since the time of launching of first satellites. Even on the first re-entry spacecraft the plates made of various materials were installed — to estimate the degree of micrometeoroids influence. After long staying in orbit these test plates became such, as if they were ''eaten-through'' by microcraters. Investigations have shown these particles to have various sizes (from some microns up to 1000 mm) and various velocities reaching 72 km/s [1].The most informative measurements have been obtained on the American satellite LDEF (Long Duration Exposure Facility) [2,3]. It was launched into orbit in 1984and was staying in the near-Earth space (NES) for more than 5 years, after which it was returned to the Earth in1990 by the ''Shuttle'' Columbia. The uniqueness of this experiment consisted in the fact, that the satellite was stabilized, had large size (9.1_4.3 m), and its whole surface was some kind of a sensor of collisions. The surface of the LDEF satellite was carefully studied by many specialists. Some thousand craters were found, which were formed as a result of collisions with micrometeoroids and man-made space debris particles. Chemical analysis materials allowed one to separate these two types of collisions.

The American specialist B.G. Cour-Palais in 1969 has constructed the micrometeoroid flux model [4], which became a basis for further investigations and remains rather popular till now. The variety of micrometeoroid models has been developed in subsequent years [5–11].In 2009 the group of specialists prepared the survey report on comparing various models of micrometeoroids [12].Based on the results of investigations it was found that, unlike the flux of micrometeoroids, the flux of sporadic meteoroids was stationary. Besides, it was assumed that, relative to the Earth surface, the sporadic meteoroids arrive isotropically from all directions and with the same velocity. To correct for Earth's gravitational enhancement at the given distance above the Earth, the average meteoroid flux must be multiplied by the defocusing factor G_e[13].

With reference to paper [13] mentioned above, the MASTER model documentation [10] outlines the algorithm of calculating the defocusing factor which is designated now as χ (H). Here H is the altitude above the planetary surface.

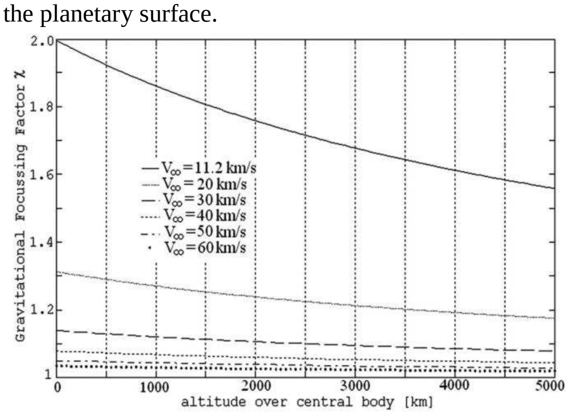

Figure 1. Gravitational focusing factor V_∞.

This algorithm is very simple:

$$\chi(H)=\left(V(H)/V_\infty\right)^2. \qquad (1)$$

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Here V_∞ is particles velocity in the deep space, $V(H)$ is particles velocity at altitude *H*.

Figure 1 depicts the curves for χ as a function of altitude and varying particles velocity V_∞, having values in the range of 11.2 to 60 km/s. Assuming the encounter with particles to occur at the lower boundary of the velocity spectrum (11.2 km/s), the focusing factor may be increased up to a factor of two.

The technique of defocusing factor calculation given above is characterized by the fact that it does not account for a diversity of trajectories of particles approaching the Earth. This circumstance is characterized by the data of figure 3. Intuitively, it is clear that the effect of defocusing factor on particles approaching the spacecraft orbit of radius R at points *a* and *b* is different. In the first case the orbit is not bent.

2 TECHNIQUE

One of basic characteristics of considered sporadic micrometeoroids is the density (cross sectional area flux) at the far (infinitely long) distance from the Earth. According to the materials of report [4], for particles larger than 0.01 cm in size this estimate equals $Q_0 \approx 10^{-7}$ 1/m^{2}sec. This estimate has been updated subsequently. The specific value of Q_0 has no principal significance for our analysis. Of importance is the assumption, that at the long distance from the Earth all directions of approaching of sporadic meteoroids of some particular size are equally probable and are characterized by the constant value of the cross sectional area flux Q_0. The statistic distribution of the values of velocity $V_\infty$is assumed to be known.

Consider the Near-Earth Space (NES) region close to the Earth (for example, at distances up to 200000 km). In this region the main disturbing factor is the gravitational attraction of the Earth. We shall construct the arbitrarily directed axis *x* with origin at the Earth center and the arbitrary axis *y* perpendicular to *x*. Among the majority of particles one can always find such ones, whose trajectory lies in the plane (*x0y*).

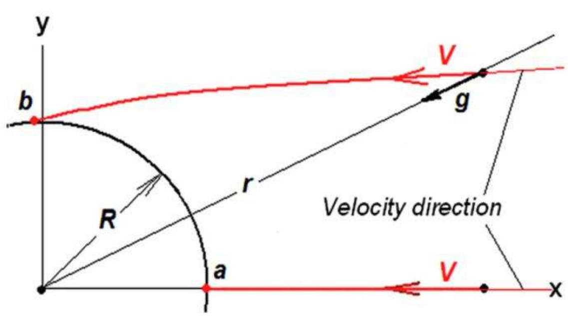

Figure 2. Various particles' approaching trajectories.

We assume that at the initial time instant the micrometeoroid is located at the point with coordinates $(x_0, y_0, z_0 = 0)$ and that projections of its velocity on the axes of the selected coordinate system are: $V_{x0} = -V_0$, $V_{y0} = 0$, $V_{z0} = 0$. The gravitational acceleration vector $\boldsymbol{g}$ is directed to the Earth center (Fig.2). Its value is equal to

$$g = \frac{\mu_E}{r^2}, \qquad r^2 = x^2 + y^2 \tag{2}$$

Differential equations of particles motion are as follows:

$$\begin{aligned} \frac{dx}{dt} &= V_x ; \\ \frac{dy}{dt} &= V_y ; \\ \frac{dV_x}{dt} &= -g\frac{x}{r} ; \\ \frac{dV_y}{dt} &= -g\frac{y}{r} ; \end{aligned} \tag{3}$$

It is obvious from these equations that, in the general case, the gravitation has double effect on the motion of particles: both on the value of their velocity (*V*), and on the form of trajectory (in the general case it is bent while approaching to the Earth center). An exception is the specific case, when $y_0 = 0$. Then $V_{y0} = 0$, and the particle trajectory does not deflect from axis *x*, i.e. it represents a direct line.

We accept that the geocentric distance r_∞ corresponds to particle position in the "deep" space. The sum of kinetic and potential energy of a particle at this point is equal to

$$E_\infty = \frac{1}{2} m V_\infty^2 - m\frac{\mu_E}{r_\infty} \tag{4}$$

At the point with the geocentric distance *r* the total energy of a particle is

$$E_r = \frac{1}{2} m V_r^2 - m\frac{\mu_E}{r} \tag{5}$$

From the energy conservation one can easily determine the velocity of a particle at its approaching to the Earth at the distance *r*:

$$V_r = \sqrt{V_\infty^2 + 2\mu_E\left(\frac{1}{r} - \frac{1}{r_\infty}\right)} \tag{6}$$

For $r_\infty = \infty$ and $r = R_E + H$ this velocity coincides with $V(H)$ velocity from formula (1).

Consider now the gravitation effect on characteristics of the planar flux of particles, namely, at small initial deflection in the coordinate *z*. The flux represents the number of particles intersecting some plate of area S per unit of time. If the normal to the plate is parallel to the particle velocity, then the flux is equal to the product of the cross sectional area flux Q by the area S:

$$P = QS \tag{7}$$

Fig. 4 presents two trajectories of particles, for which the initial value of coordinate y differ in small quantity Δy_0. At this initial point the value of a flux through the plate of area $S_0 = \Delta y_0 \cdot \Delta z$ is equal to

$$P_0 = Q_0 \Delta y_0 \Delta z \tag{8}$$

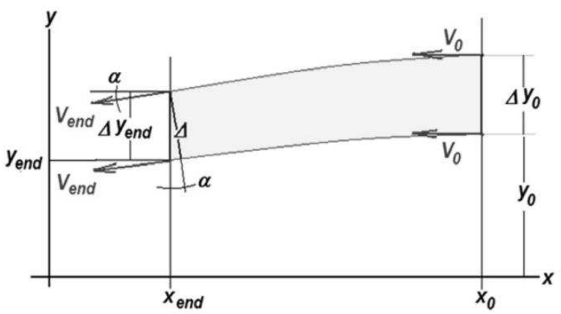

Figure 3. Planar flux of particles in the interval between two trajectories. The blue color indicates the region, where the particles move with initial values of coordinate y in the range of $(y_0, y_0 + \Delta y_0)$.

Here Q_0 is the value of cross sectional area flux of particles at the great distance from the Earth.

When approaching the Earth, the trajectory characteristics at points with coordinates x_{end} change: the velocity increases, the velocity vector deflects from the direction of axis x by some angle (α), the values of coordinate y decrease. We denote the distance between trajectories at the last point by Δy_{end}. The distance between trajectories in the direction perpendicular to the velocity vector is equal to

$$\Delta = \Delta y_{end} \cos\alpha \tag{9}$$

The planar flux value in the interval between two trajectories at the last point is equal to

$$P_{end} = Q_{end} \Delta y_{end} \cos\alpha \Delta z \tag{10}$$

For determining the cross sectional area flux Q_{end} at the last point the condition is used, that in the interval between two considered trajectories on the time interval of particles' motion from point x_0 to point x_{end}, the particles do not disappear and do not arise. *Therefore, the flux values at the initial and last points are equal:*

$$P_{end} = P_0 \tag{11}$$

It follows from the condition (13) that the value of the planar flux density at the last point is equal to

$$Q_{end} = Q_0 \frac{dy_0}{dy_{end} cos\alpha} = Q_0 * k_{g1} \tag{12}$$

Here k_{g1} is the first gravitational focusing factor, which is calculated by formula

$$k_{g1} = \frac{dy_0}{dy_{end} cos\alpha} \tag{13}$$

This coefficient indicates how the density of a planar flux of particles changes on the interval of their motion from point x_0 to point x_{end}. An important feature of the gravitational focusing factor (13) consists in the fact that it does not depend on the particle velocity, but depends on the bending of particle trajectories only. This circumstance distinguishes the considered technique from the traditional approach (formula (1)).

When analyzing the particle flux in the 3-dimensional space it's necessary to take into account that, with the fixed direction of axis x, axes y are arbitrary. So, the region of space for the given values of y_0 and Δy_0 has the form of a circle (figure 4) of area $S_0 = 2\pi \cdot y_0 \cdot \Delta y_0$. When approaching the Earth the area of the normal cross section of a flux decreases due to its approaching to axis x and decreasing of "thickness" of the layer Δy. At the last point the cross section of the flux of "thickness" Δ is equal to

$$S_{end} = 2\pi\, y_{end} \Delta \tag{14}$$

The application of estimates of the cross sectional area S_0 and S_{end}, as well as of the condition (11) results in the following estimate of flux density at the last point:

$$Q_{end} = Q_0 \frac{\Delta y_0}{\Delta} \frac{y_0}{y_{end}} = Q_0\, kg_1\, kg_2 = Q_0\, kg_{sum} \tag{15}$$

Here

$$kg_2 = \frac{y_0}{y_{end}} \tag{16}$$

$$kg_{sum} = kg_1\, kg_2 \tag{17}$$

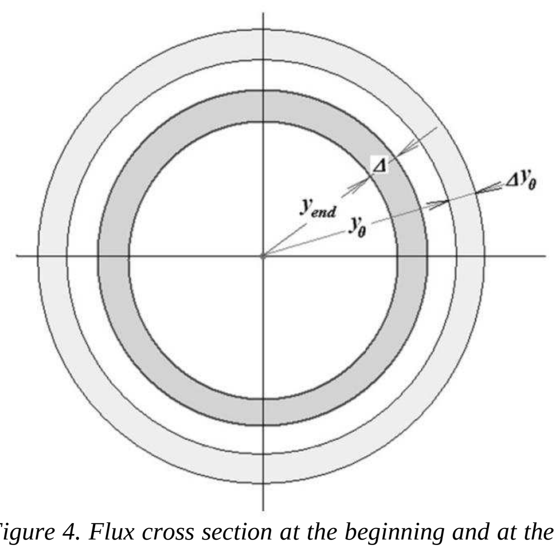

Figure 4. Flux cross section at the beginning and at the end. The light color indicates the initial region of particles position, and the dark color — the region of their position when approaching the Earth.

An important feature of the presented technique for calculating the coefficient k_{gsum} is the fact, that this coefficient does not depend in the explicit form on the values of particles velocity at the last point, but depends on the trajectory disposal from the direction to the Earth center (coordinate y_0).

Eq. (3) can easily be integrated numerically. The integration termination conditions are:

$$\sqrt{x^2 + y^2} < R \text{ or } x < x_{end} \tag{18}$$

Here R and x_{end} are specified values. If for x_{end} = 0 the first of conditions (20) it is not fulfilled, the particle flies by the sphere of radius R.

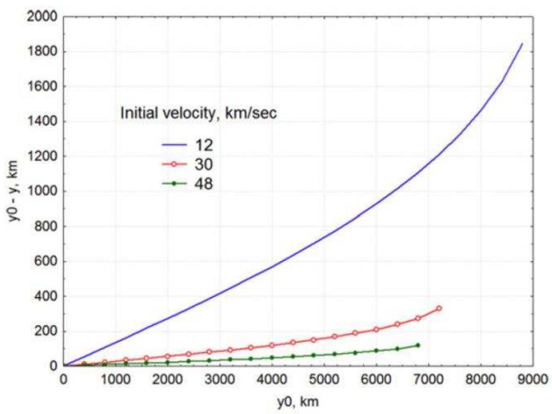

Figure 5. Change of coordinate y with respect to its initial value y_0.

To determine the form of function $y = F(y_0, V_0)$ the system of Eq. (3) was integrated with various initial conditions (y_0 and V_0) and specified R and x_{end}. The data of calculations of function $y = F(y_0, V_0)$ are given in Fig. 5. It is seen from these data that in all cases the values of coordinate y decrease in comparison with the initial value of y_0 and the greater, the lower the initial velocity V_0.

Presented results illustrate the nature of focusing factor origination. All the matter lies in the fact that, as particles approach the Earth, the values of coordinate y decrease, and, besides, the nonlinearity of function $y = F(y_0)$ has an effect. By this reason at the last point the area of the normal cross section of a flux decreases, and the flux density, accordingly, grows.

Fig. 6 presents the estimates of gravitational focusing factor k_{gsum} for various orbits and various values of initial velocity V_0. The form of this figure is the same as that for figure 2. It is obvious from the comparison of the data of these figures that our estimates of the gravitational focusing factor are much lower, than corresponding estimates calculated by the generally accepted technique [13] developed 60 years ago.

The detailed analysis of calculation of the gravitation effect coefficient k_{gsum} is presented in [14].

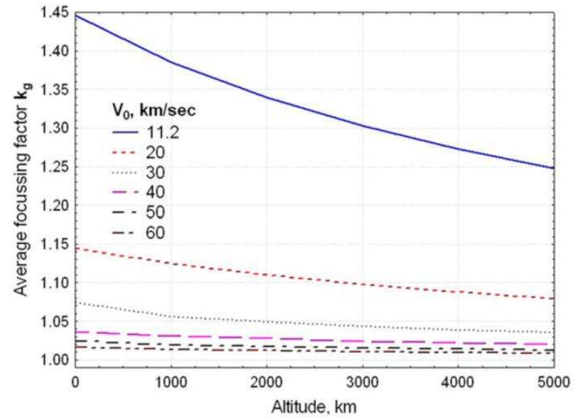

Figure 6. Average gravitational focusing factor values (k_{gsum}) under various conditions.

3 SDPA SPORADIC MICROMETEOROIDS MODEL ANALYSIS

The technique described above was applied in the Russian space debris model SDPA. The tests of the SDPA micrometeoroids model were carried out in two stages. At the first stage of tests the basic characteristics of a micrometeoroid flux in the near-Earth space at altitudes up to 36000 km were determined. Namely, the estimates of a micrometeoroid flux at various altitudes were calculated, and statistical distributions of their velocity were found.

At the second stage of tests the characteristics of a micrometeoroid flux with respect to cube faces were determined for three types of orbits: LEO (ISS), MEO (GPS, GLONASS) and GEO.

In all cases the results were compared with the corresponding data calculated by other models.

3.1. Basic characteristics of a micrometeorite flux of the SDPA model

Table 1 presents the estimates of a micrometeoroid flux in the deep space related to particles of various sizes, obtained with using the MASTER model [10].

d, cm	>0.012	>0.025	>0.05	>0.10	>0.25
Q, 1/m^{2}year	12.0	0.92	0.0727	0.0058	0.0002

Table 1. Density of a flux of particles of various sizes

It is seen from this estimate that, as the particle size increases, the micrometeoroid flux density sharply decreases. For particles of about 0.1 cm in size it becomes close to the density of an artificial space debris flux. With further decreasing of size the man-made particles become a basic source of near-Earth space contamination.

The data on micrometeoroid velocity values are presented in table 2. As particles approach the Earth, their velocity grows. The corresponding calculation results are presented in table 2.

H [km]	V_0=12 [km/sec]	V_0=36 [km/sec]	V_0=72 [km/sec]
36000	12.64	36.22	72.11
20000	13.06	36.37	72.18
1000	15.46	37.18	72.58
450	15.65	37.23	72.60

Table 2. Micrometeoroid velocity values

Table 3 presents the basic characteristics of a flux of micrometeoroids of mass greater than 10^{-6}g (of size larger than 0.0124 cm) for four types of orbits with altitudes of 450 km, 1000 km, 20000 km (MEO) and 36000 km (GEO). Namely, the table presents:

- The averaged-per-revolution flux density Q (1/m^2 year);
- The average value of relative velocity *Vrel* (km/s).

The flux density estimates given in table 3 take into account the effect of three factors:

1. Particle flux shadowing by the Earth;
2. Particles velocity increase under the Earth gravitation effect;
3. The influence of gravitation effect (coefficient k_g) as a result of trajectory bending.

The first of mentioned factors results in decreasing flux density. This is clearly seen from the data of the second line of table 3. Two other factors have opposite effect – they increase the flux density.

This is clearly seen also from table's data. The highest influence of a gravitation effect takes place for low-orbit satellites.

H [km]	450	1000	20000	36000
Q(D>0.0124 cm) (1/m^2 year);	15.17	14.52	13.10	12.74
Vrel (km/sec)	22.5	23.3	21.1	20.1

Table 3. Flux density estimates for LEO, MEO, GEO orbits

Figures 7 and 8 present statistical distributions of directions of velocity at which micrometeoroides approach the spacecraft in various orbits.

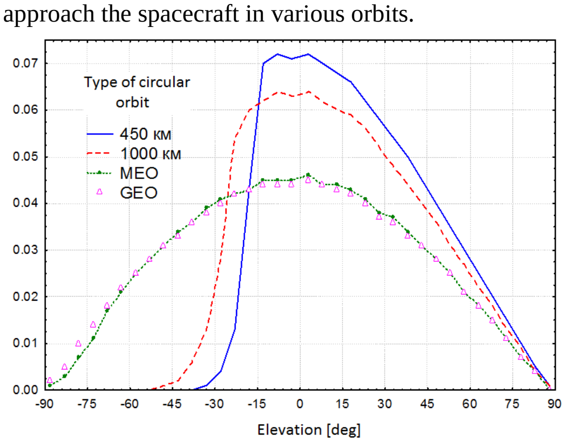

Figure 7. Distribution of possible values of micrometeoroids' approaching place angle

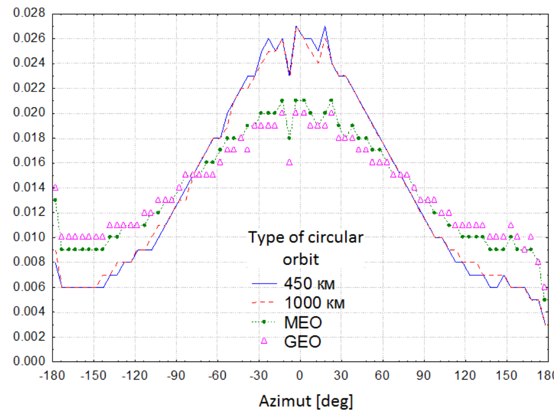

Figure 8. Distribution of possible values of micrometeoroids' approaching azimuth

The data of figure 7 clearly indicate the shadowing effect influence, which grows with decreasing orbital altitude. As a result, for the ISS (International Space Station) orbit (of altitude 450 km) particles' approaching directions are absent in the vicinity of direction to the Earth. The influence of this effect on the particle flux relative to the spacecraft at high-altitude orbits (MEO and GEO) is virtually absent.

The azimuthal dependences, presented in figure 8, are "similar" for all considered types of orbits. For lower orbits the distribution maximum is more strongly pronounced.

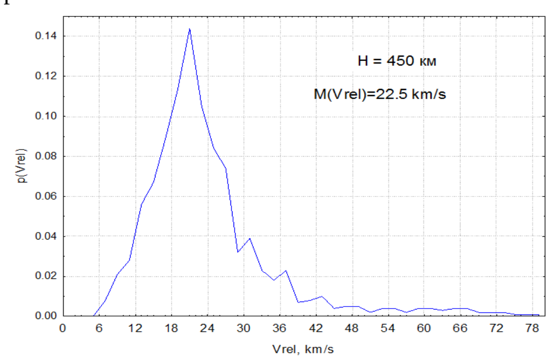

Figure 9. Distribution of p(Vrel)

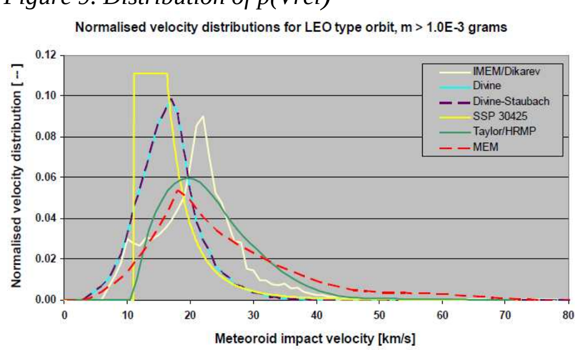

Figure 10. p(Vrel) according to the IADC data

Figure 9 presents the statistical distribution of possible values of a relative particles' velocity (P (Vrel)) for spacecraft at the altitude of 450 km, and figure 10 – similar distributions according to the IADC report data [12].

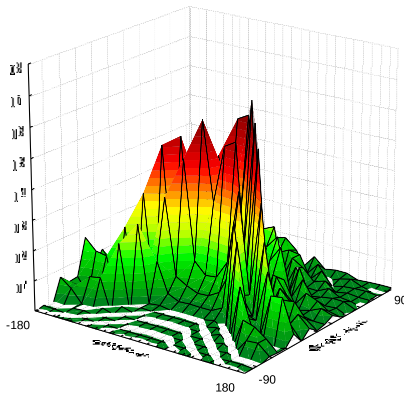

Figure 11. Distribution of p (Az, Elev)

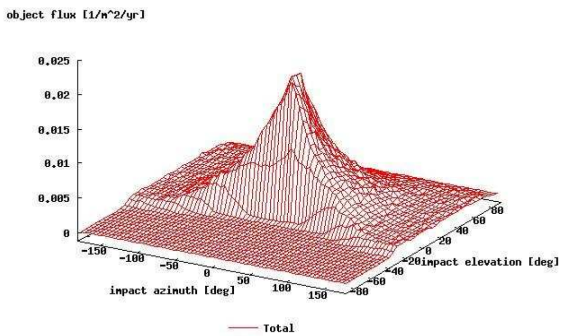

Figure 12. p (Az, Elev) according to the MASTER data

Figure 11 presents the two-dimensional distribution of particles' approaching directions (*p(Az,Elev)*) relative to the spacecraft at the altitude of 450 km, and figure 12 – the similar (but not normalized) distribution according to the MASTER model data. The distributions rather well agree in shape. The basic distinction consists in the fact, that our distribution is less smooth. This is explained by the fact that the SDPA model uses the discrete distribution of micrometeoroides velocity with a rather large step (6 km/s) and without interpolation. Taking into account high distinctions of velocity estimates in various models, the given disadvantage is not significant.

The results of calculations by the SDPA model, stated above, rather well agree with the known data. This is expressed in the fact that:

- flux density estimates (table 1) agree with IADC report's materials [12];
- the shadowing was taken into account according to the standard technique;
- estimates of particles velocity at approaching the Earth were calculated by the well-known formula (6).

Existing rather small distinctions are caused by application of an advanced technique for gravitation effect estimation.

3.2. Micrometeoroids' flux relative to structural elements for satellites in LEO, MEO, GEO orbits.

In estimating the consequences of spacecraft' structural elements collisions with space debris particles of significance are the values of the angle of incidence *(γ)* and relative velocity *(Vrel).* To take these values into account correctly, it is necessary, in the general case, to use the two-dimensional distribution *p(γ,Vrel)*. Calculations have shown that the relative velocity distributions only slightly depend on the angle of incidence. By this reason the aforementioned two-dimensional distribution can be calculated by the simplified formula $p(\gamma,Vrel)= p(\gamma)\cdot p(Vrel)$.

In determining the micrometeoroid flux relative to spacecraft, situated on particular orbits, the characteristics of a flux relative to cube faces were determined (figure 13). For a circular orbit axis R is directed along the radius vector, axis T – along the velocity vector, and axis W – along the binormal. Table 4 presents the results of calculations of a flux for particles of mass greater than 10^{-6} g relative to the ISS orbit and their comparison with IADC report's materials [12]. It is also useful to compare the distribution of directions of particles' approaching to various faces, calculated by SDPA models.

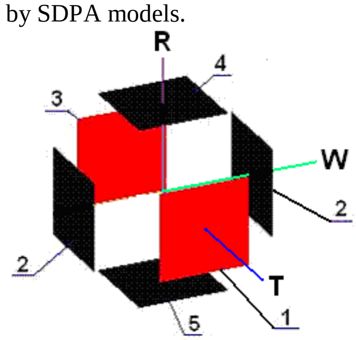

Figure 13. Cube faces orientation relative to the RTW axes

	Faces					Sum
	1	2	3	4	5	
Q(1/m^{2}year), SDPA	7.27	3.81	1.49	5.47	0.55	22.40
Same, Divine	6.3	3.3	0.71	4.3	-	17.91
Vrel (км/с), SDPA	24.9	21.7	17.0	21.7	21.4	-
Same, Divine	20.1	17.5	13.2	17.3	-	-
Same, MEM	23.1	23.2	23.6	23.1	-	-

Table 4. Flux density estimates for ISS cube faces

Figure 14 presents distributions of angles of particles' incidence to each of faces *(γ)*, which well agree with the data of other models. These distributions are characterized by the fact that they are asymmetrical for the frontal and upper faces. For the frontal face the most significant is the fraction of particles with angles of incidence of 20º - 40º, and for the upper face – those with the angle of incidence of 40º - 60º. The lower face is characterized by the absence of particles' approaching directions with angles of incidence smaller than 55º. This is a consequence of their shadowing by the Earth.

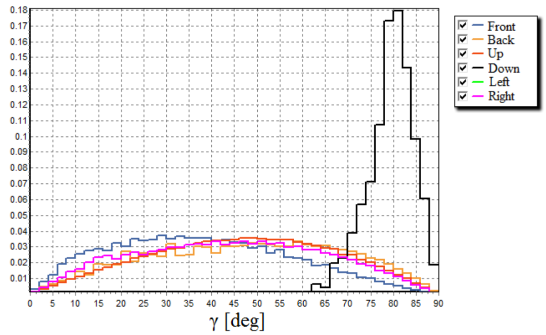

Figure 14. Distribution of angles of incidence for cube faces on the ISS orbit.

Table 5 below gives the flux density estimates for particles of mass larger than 10^{-6} g for cube faces for the orbits from the GEO region.

Characteristic	Faces					Sum
	1	2	3	4	5	
Q(1/m^{2}yr) SDPA	4.35	3.18	2.19	3.14	3.07	19.11
Same, Divine	6.3	3.3	0.71	4.3	-	17.91
Same, MASTER	1.36	1.09	0.68	0.89	1.03	6.14
Vrel (km/s), SDPA	21.7	20.6	19.4	20.7	20.7	-
Same, Divine	20.1	17.5	13.2	17.3	-	-
Same, MEM	23.1	23.2	23.6	23.1	-	-

Table 5. Flux density estimates for GEO cube faces

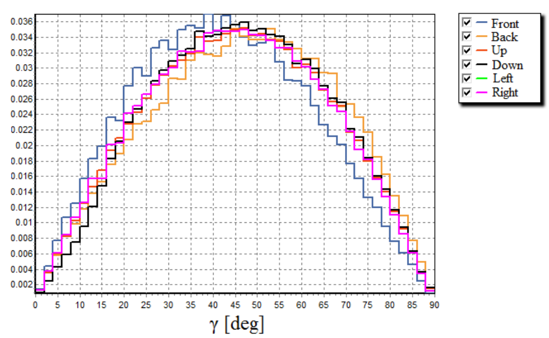

Figure 15. Distribution of angles of incidence for cube faces on the GEO orbit

Distributions of directions of particles' approaching to various faces, calculated by the SDPA model for spacecraft in the GEO region, are presented in figure 15. The common feature of these distributions is the fact that they only slightly differ for various faces. This is their principal distinction from corresponding distributions for spacecraft with altitude of 450 km presented in figure 14. The distributions differ especially greatly for the lower face. Various particles' approaching directions are possible for a geostationary satellite. The reason of this distinction is slight influence of shadowing effect on the geostationary satellite orbit.

4 CONCLUSIONS

The new approach to determination of a gravitation effect for the near-Earth space is proposed, which is based on integration of the equations of motion of sporadic micrometeoroids with allowance for bending their trajectories when particles are approaching the Earth. The technique and results of calculation of the gravitational focusing factor k_g for various conditions are presented. It is shown that for low-altitude orbits the flux density growth can reach 60 %. The results of investigation of coefficient k_g have shown that the values of this coefficient depend on the initial velocity of micrometeoroids and on the deflection of its direction from the direction to the Earth center. It is found that the distribution of probabilities of various directions of particles' approaching spacecraft's structural elements is non-uniform.

The analysis and comparison of the sporadic micrometeoroids SDPA model is performed, which uses the new technique of determining coefficient k_g that characterizes the gravitational effect.

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