A METHOD TO IMPROVE THE PRECISION OF PREDICTED LEO SPACE OBJECTS EPHEMERIS

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ABSTRACT

There are two major errors in predicting Space Object (SO) ephemeris (state vector): one is the error of the Initial Orbital Element (IOE) sets, the other is that of the Dynamical Model (DM). In order to get a better predicted ephemeris, a better IOE or/and DM is needed. This paper aims to seek a method of improving the precision of IOE and PM together. The basic idea is to reduce the error of semi-major axis (a) together with that of the Area-Mass ratio (A/M) by calibration. The fundamental principle is that the error of a and A/M would propagate and augmented with time symmetrical in down-track component, the necessary condition is that the precise ephemeris of the past must be obtained.

1 BACKGROUND AND MOTIVATIONS

As space objects growing rapidly, especially the accidental/natural collision between Iridium 33 and Cosmos 2251 in 2009, space objects conjunction/collision warning is widely operated presently. Due to the limitation of the precision of SO ephemeris and lack of SO's size/shape and attitude, the creditability of a predicted collision is still at a low level. In order to improve it, a database of SO's size/shape should be developed, and the precision of predicted ephemeris should be improved.

One way to improve the creditability is to increase the threshold of the acceptable warning collision possibility. If this is assumed as bigger than 10^{-3}, the combined size of the two SOs as about 5m, then the conjunction distance between the two SOs should be less than 100m. Considering the whole process period of collision avoidance (discovery/ verification/ warning/ maneuver) as one week, the practically needed precision of 7-day predicted ephemeris should be better than 100m.

This paper focuses on the improvement of the precision of predicted SO's Ephemeris. It will exam the principle of error propagation and then seek a method to reduce the errors existed in the used IOE sets and DM. Its goal is to create 7-day predicted ephemeris with precision about 100m or close to that of orbit determination, which is limited by the quality/accuracies, quantity and distributions of observations. Its motivation is to support a better operational daily SOs conjunction/collision warning service.

2 PRINCIPLE AND IDEA

2.1 Principle

Orbit prediction is based on the Initial Orbital Element (IOE) sets and the Dynamical Model (DM), thus the errors of these two factors are the major sources of the errors (the numerical procedure errors is omitted) of the predicted ephemeris (state vector). At any time t, the position errors projected:

- in Radial/Down-track/Cross-track ($\Delta U, \Delta V, \Delta W$),and

- the classical Kepler orbital element set ($\Delta a, \Delta e, \Delta i, \Delta\Omega, \Delta w, \Delta M$)

has the following relations:

$$\begin{cases} \Delta U = r\Delta a / a - a\cos f\Delta e \\ \qquad + ae\sin f\Delta M / \sqrt{1-e^2} \\ \Delta V = r\cos i\Delta\Omega + a(1+r/p)\sin f\Delta e \\ \qquad + r\Delta\omega + a^2\sqrt{1-e^2}\Delta M / r \\ \Delta W = r\sin u\Delta i - r\sin i\cos u\Delta\Omega \\ \qquad p = a(1-e^2), \quad u = w+f \end{cases} \tag{1}$$

For circular or near circular orbit, it can be simplified as the following:

$$\begin{cases} \Delta U \approx \Delta a - a\cos f\Delta e + ae\sin f\Delta M \\ \Delta V \approx a\cos i\Delta\Omega + 2a\sin f\Delta e + a\Delta\lambda \\ \Delta M \approx a(\sin u\Delta i - \sin i\cos u\Delta\Omega) \\ \qquad \lambda = \omega + M \end{cases} \tag{2}$$

In orbit prediction, the orbital element error from time t_0 to t will be propagated as:

$$\begin{cases} \Delta a_t \approx \Delta a_0 + \Delta\dot{a}dt, \Delta e_t \approx \Delta e_0 \\ \Delta i_t \approx \Delta i_0, \Delta\Omega_t \approx \Delta\Omega_0, \Delta\omega_t \approx \Delta\omega_0 \\ \Delta\lambda_t \approx \Delta\lambda_0 - 3n\left(\Delta a_0 dt + 0.5\Delta\dot{a}dt^2\right)/2a \end{cases} \tag{3}$$

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Where:

$dt = t - t_0$, $n = 2\pi / P$, and

P is the orbital Period

Then, we get:

$$\begin{cases} \Delta U_t \approx \Delta U_0 + \Delta \dot{a} dt + e \sin f \Delta V_t \\ \Delta V_t \approx \Delta V_0 - \dfrac{3n}{2}\left(\Delta a_0 dt + \dfrac{1}{2} \Delta \dot{a} dt^2 \right) \\ \Delta W_t \approx \Delta W_0 \end{cases} \qquad (4)$$

which is the principle of errors propagation in orbit prediction.

2.2 Idea

Equation (4) shows that only the down-track error may change quickly along with time, and Δa_0 and $\Delta \dot{a}$:

- are the major errors which cause the augmentation of position error in down-track component, and

- are propagated in the same way with time both in forward and backward direction

Due to the sign and value of ΔV_0, Δa_0 and $\Delta \dot{a}$, we can get 6 kind of curves of ΔV_t. All of them can meet the limitation of observations used in determine them (see Fig.1), but there may be significant difference among predicted ephemeris. A further test should be made to improve the above parameters to ensure a better prediction.

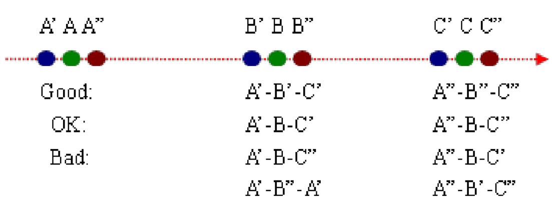

Fig 1 O-C of orbit determination

Based on above characteristics and the fact that the precise ephemeris of the past (backward direction) can be obtained (in the Space Surveillance Center), by comparing the difference of down-track component between the determined ephemeris (here as benchmark) and the backward predicted state vectors, a calibration of the IOE sets can be carried and the errors of a and $\dot{a}$ existed in them may be reduced.

In order to reduce the effect caused by periodical items and random sample, we will (see Fig 2):

- compare the down-track difference of K circles (K Period) separated by N circles

- select about points well- proportioned over each circle (about 1 points/minute)

to create an average value as the result (but how to select t, N, K need to be studied). When we get a better IOE sets, a better predicted ephemeris can be expected.

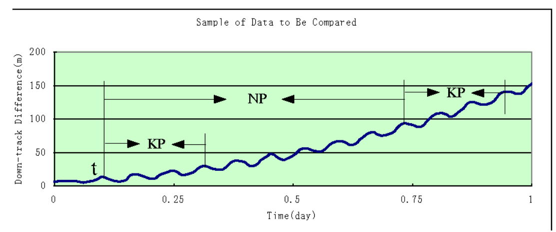

Fig 2 Sample of the Data to be Compared

Notice that:

- in order to get a better predicted ephemeris, in prediction, the same DM and configuration parameters used in determining the IOE sets should be used

- Δa_0 and $\Delta \dot{a}$ reflects the errors of IOE and DM or parameter such as Square-to-Mass ratio (S/M). It is difficult to separate them, partially because DM is used in determining the IOE sets

3 METHOD AND STEPS

3.1 Difference of In-track error

According to function (4), the down-track error between time t and t+NP is:

$$\Delta V(t + NP) - \Delta V(t) =$$

$$-3\pi N \left(\Delta a_0 + \Delta \dot{a}(t - t_0) + \frac{\Delta \dot{a}}{2} NP \right)$$

Let's compare the time interval $t_1 \sim t_1 + KP$ and $t_2 \sim t_2 + KP$ ($t_2 = t_1 + NP$) as follows, we get:

$$\sum_{t_j = t_1}^{t_j = t_1 + KP} \left[\Delta V(t_j + NP) - \Delta V(t_j) \right] =$$

$$-3\pi k N \left\{ \Delta a_0 + \Delta \dot{a}(t_1 - t_0) + \frac{\Delta \dot{a}}{2}(N + K) P \right\} \qquad (5)$$

Where:

k is the number of points (in order to illuminate the periodical items, k =100K is suggested, which means divided each circle into 100 sections. For LEO, about 1 point per minute).

3.2 Relations between Δa_0 and $\Delta\dot{a}$:

Assuming the IOE sets is determined by the observations from $t_{obs-first}$ to $t_{obs-last}$. In order to keep the accuracy of orbit determination, the following restrictive condition is used:

$$\Delta M_{obs} = -\frac{3n}{2a}\left(\Delta a_0 dt_{obs} + \frac{1}{2}\Delta\dot{a} dt_{obs}{}^2\right) = 0$$

Where:

$$dt_{obs} = t_{obs-last} - t_{obs-first}$$

Then, we get:

$$\Delta\dot{a} = -\frac{2\Delta a_0}{dt_{obs}} \qquad (6)$$

3.3 Solution of Δa_0 and $\Delta\dot{a}$:

Substitute Eq(6) into Eq(5), we get:

$$\sum_{t_j=t_1}^{t_j=t_1+KP}\left[\Delta V(t_j+NP) - \Delta V(t_j)\right] = -3\pi kN\Delta a_0 \times \frac{dt_{obs} - 2(t_1 - t_0) - (N+K)P}{dt_{obs}}$$

Then:

$$\Delta a_0 = -\frac{\sum_{t_j=t_1}^{t_j=t_1+KP}\left[\Delta V(t_j+NP) - \Delta V(t_j)\right]}{3\pi kN} \times \frac{dt_{obs}}{dt_{obs} - 2(t_1 - t_0) - (N+K)P} \qquad (7)$$

And $\Delta\dot{a}$ can be derived from Eq(6).

Mostly, Area-to-Mass ratio (A/M) is used to instead of $\Delta\dot{a}$. In this case, we need to transform $\Delta\dot{a}$ into A/M. For LEO, $\Delta\dot{a}$ is mainly caused by the atmospheric drag. According to:

$$\dot{a} = -2C_d \frac{A}{M}\frac{a^2}{P}\rho_p\sqrt{\frac{\pi H}{2ae}} \qquad (8)$$

We get:

$$\Delta\left(\frac{A}{M}\right) = \frac{\Delta\dot{a}}{\dot{a}}\left(\frac{A}{M}\right) \qquad (9)$$

3.4 The improved orbital element sets

Define:

$$\begin{aligned}\Delta\vec{r} &= \vec{r}_{ephemeris} - \vec{r}_{prediction} \\ &= \left((\Delta\vec{r})_U, (\Delta\vec{r})_V, (\Delta\vec{r})_W\right)\end{aligned}$$

$$(t, a, e, i, \Omega, w, M)_{obs-first} = f(t, \vec{r}, \dot{\vec{r}})_{obs-first}$$

$$(t, a, e, i, \Omega, w, M)_{obs-last} = f(t, \vec{r}, \dot{\vec{r}})_{obs-last}$$

Δa_0 and $\Delta\dot{a}$ are what calculated by Eq(7), Eq(6) or Eq(9). Note $\dot{a}_{obs-first} = \dot{a}_{obs-last}$, we get:

$$\begin{cases} a^{new}_{obs-first} = a_{obs-first} + \Delta a_0 \\ a^{new}_{obs-last} = a_{obs-last} - \Delta a_0 \end{cases} \qquad (10)$$

And:

$$\dot{a}^{new}_{obs-first} = \dot{a}^{new}_{obs-last} = \dot{a}_{obs-first} + \Delta\dot{a}$$

$$\begin{cases} \left(\frac{A}{M}\right)^{new}_{obs-first} = (1 + \frac{\Delta\dot{a}}{\dot{a}})\left(\frac{A}{M}\right)_{obs-first} \\ \left(\frac{A}{M}\right)^{new}_{obs-last} = \left(\frac{A}{M}\right)^{new}_{obs-first} \end{cases} \qquad (11)$$

Then transform orbital elements into state vector:

$$(t, \vec{r}, \dot{\vec{r}})^{new}_{obs-last} = g\left(a^{new}, e, i, \Omega, w, M\right)_{obs-last},$$

and use the following improved parameters to predict:

$$(t, \vec{r}, \dot{\vec{r}})^{new}_{obs-last}, (S/M)^{new}_{obs-last}$$

3.5 Steps

- While doing the improvement orbit determination, export the final ephemeris ($t, \vec{r}, \dot{\vec{r}}$) and A/M from $t_{obs-first} - 7days$ ~ $t_{obs-last} + 7days$ at a sampling rate of 1 point/minute
- Get the ephemeris (from Database) during time $t_{obs-first} - 7days$ ~ $t_{obs-first}$ at the same sampling rate
- Calculate Δa_0, $\Delta\dot{a}$ or $\Delta(A/M)$ and get new

orbital element sets at $t_{obs-last}$

- Predict new ephemeris from $t_{obs-last}$ □ $t_{obs-last}+7days$ 1 points/minute
- Compare them to find the difference and analyze it when necessary

4 DISCUSSION AND EXPLAINATION

4.1 Discussion

Why can we expect a better result from this method? Because:

- more information (the precise ephemeris of the past) of longer time-span is used

- the optimization (Least Square) criterion of fitting the observations is broken, but

- the Mean Anomaly at time $t_{obs-first}$ and $t_{obs-last}$ .are restricted to keep it acceptable

In fact, in order to make the predicted ephemeris more precise, the residual of the Improved is bigger than that of the Initial. Let's look into it as follows.

As for the new orbital element sets ($t_{obs-first}, a^{new}_{obs-first}, \left(e,i,\Omega,w,M\right)_{obs-first}, \left(A/M\right)^{new}_{obs-first}$), due to the change of $a^{new}_{obs-first}$ and $\left(A/M\right)^{new}_{obs-first}$, during $t_{obs-first} \sim t_{obs-last}$, ΔM (the Mean Anomaly) changes as following:

$$\Delta M_t = -\frac{3n}{2a}\left(\Delta a_0 dt - \frac{\Delta a_0}{dt_{obs}} dt^2\right)$$

At $dt = dt_{obs}/2$, it reaches its maximum:

$$\left(\Delta M_t\right)_{\max} = \Delta M_{dt_{obs}/2} = -\frac{3n}{2a}\times\frac{dt_{obs}}{4}\Delta a_0$$

Define:

$N_{obs} = \dfrac{dt_{obs}}{P}$ (Number of circles between $t_{obs-first} \sim t_{obs-last}$)

We get:

$$a\left(\Delta M_t\right)_{\max} = a\Delta M_{dt_{obs}/2} = -\frac{3\pi N_{obs}\Delta a_0}{4}$$

It shows that in the time span of the observations, the position error get bigger (maximum in the middle). And this sacrifice will benefit the long time prediction to meet our goal. Fig 3 is the theoretic down-track error propagation of the Initial vs. the Improved.

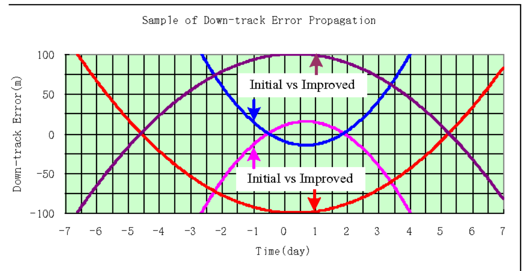

Fig 3 Theoretic down-track error propagation of the Initial vs. the Improved

4.2 Explanation

- This is just an idea which has not been testified. Lots of follow-on study (such as how to determine t, N, K and how the Solar-flux and Geomagnetic-index affect) should be carried out using operational data
- Instead of comparing ΔV, maybe we can compare Δu as following (the rest are the same) to get a similar results:

$$\Delta u = \Delta(w+f) = -\frac{3n}{2a}\int \Delta a dt = -\frac{3n}{2a}\left(\Delta a_0 dt + \frac{1}{2}\Delta\dot{a}dt^2\right)$$

- If more precise benchmark ephemeris obtained, the rest orbital element parameters can be calibrated too.