TRAJECTORY PREDICTIONS FOR HIGH ECCENTRICITY ORBITS OF SPACE DEBRIS OBJECTS

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ABSTRACT

Since 1957 the number of space debris has been increasing and it cause threat of collision. To calculate precisely space debris orbit we used several perturbations in our force model: geopotential, luni-solar effects, solar radiation pressure and influence of Earth's atmosphere. For satellites with altitude of perigee higher than 1000 km perturbations from the atmosphere is negligible. However for objects which reaches its lower parts is one of the most important perturbation. For the last perturbation we used NRMLMSISE-00 empirical model to calculate precise parameters for the atmosphere. For large amount of objects using numerical integration there appears to be a problem with time of calculations. For this reason, we decided to use analytical model, which is much faster and more convenient. Due to highly elliptical orbit we had to exchange the eccentricity function by the Hansen coefficients.

Key words: HEO, Highly Elliptical Orbit, Space debris, Orbit Propagation.

1. INTRODUCTION

Highly Elliptical Orbit (HEO) is an orbit with a low altitude of perigee and high altitude of apogee. The HEO orbit usually crosses the Low Earth Orbit (LEO) and reaches regions even higher than Geostationary Earth Orbit (GEO). This kind of orbit has high eccentricity which can reach $0.7 - 0.8$ value. The number of objects on HEO orbits is high and increases. HEO objects could cause hazard while in perigee due to their extremely high velocity. In such case the most threatened objects are LEO satellites. To predict and avoid a collision we have to determinate the orbit of a satellite precisely. In such case, long-term orbit propagation is useless. In our work we performed only short and medium-term predictions. For short-term propagation trajectories are calculated only for a few revolutions with high accuracy and medium-term predictions up to several weeks with less precision. To calculate precisely space debris orbits we used the force model including perturbations from: geopotential, luni-solar effects, solar radiation pressure and influence of Earth's atmosphere. For satellites with altitude of perigee higher than 1000 km perturbations from the atmosphere are negligible. However, for objects reaching very low perigee altitude atmosphere is the most important perturbation. We used NRMLMSISE-00 empirical atmospheric model in our calculations. Besides date and exact place we have to include all necessary solar activity data like local apparent solar time, 81 day average of F10.7 solar flux, daily F10.7 solar flux for previous day and daily magnetic index. One of the most influential parameter is area-to-mass ratio $\left(\frac{S}{m}\right)$. Small changes in the parameter cause significant changes in satellite orbit prediction. In this paper we have examined the influence on satellites orbits by using different value of this parameter. Also, for some objects' population, with real data from TLE Catalogue we have calculated some short and medium-term orbits using both analytical and numerical integration.

2. HEO OBJECTS IN TLE SATELLITE CATALOGUE

On 19 April 2013, there are 14941 objects in the Two/Three Line Element Set (TLE) Satellite Catalogue and 16859 objects in the Satellite Situation Raport (SSR) which are still on Earth orbit. This includes 2526 objects with $e > 0.4$. Furthermore, 653 objects have altitude of perigee higher than 1000 km and 1873 lower. Fig. 1 shows the distribution of the orbital elements for 520 HEO objects with the eccentricity $e > 0.4$, perigee altitude $q > 800$ km and apogee altitude $Q < 52000$ km. Semi-major axis definitely concentrates on value around 26500 km. The majority of objects have the eccentricity around 0.55 and 0.72. The number of objects have the inclination from $0°$ to $30°$, but most of them concentrate around the critical inclination.

3. FORCE MODEL FOR HEO ORBITS

Space debris objects moving in the HEO region are strongly perturbed by geopotential and atmospheric drag

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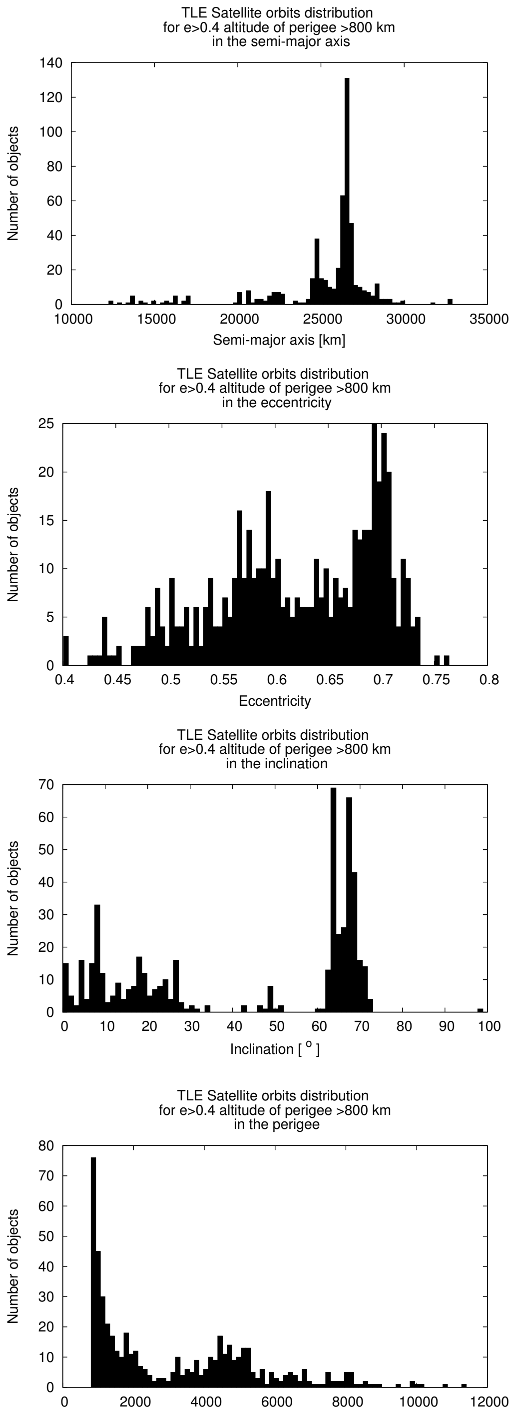

Figure 1. Distribution of the semi-major axis, inclination, eccentricity and altitude of perigee for 520 HEO objects with the eccentricity $e > 0.4$ and with the perigee altitude $q > 800$ km and apogee altitude $Q < 52000$ km.

when they are close to their perigee, and by luni-solar effects and solar radiation pressure in apogee. Precise modeling of the HEO orbits is therefore very complicated task both in the case of analytical and numerical prediction method applied. To keep an appropriate accuracy level of trajectory prediction, all perturbing forces have to be included and modeled with very high precision. Prediction algorithms for HEO have to be used in a specific way. In the case of analytical methods, one of the most difficult cases is the calculation of the eccentricity function $G_{lpq}(e)$ values, for $e = 0.7 \sim 0.8$ and for high indices.

4. THE APPLIED ORBITAL MODEL

Predicted HEO orbits were calculated with the use of orbital software tool developed in the Astronomical Observatory of the Adam Mickiewicz University in Poznan, Poland [3]. The tool works both in analytical and numerical modes and includes influence of the following perturbing forces:

$$\ddot{\vec{r}} = (\vec{F}_0 + \vec{F}_G + \vec{F}_M + \vec{F}_S + \vec{F}_{SR} + \vec{F}_A)/m, \qquad (1)$$

where :

$\vec{F}_0$	is the keplerian term,
$\vec{F}_G$	is the perturbation force due to the geopotential,
$\vec{F}_M$	is the perturbation force due to the Moon gravity,
$\vec{F}_S$	is the perturbation force due to the Sun gravity,
$\vec{F}_{SR}$	is the perturbation force due to the solar radiation pressure,
$\vec{F}_A$	is the perturbation force due to the atmospheric drag.

To get high precision of calculations we used the NRLMSISE-00 atmosphere model. Unfortunately in solar radiation pressure we didn't use the shadow function.

Presented results have been obtained for the orbit of object No. 23840 - ATLAS 2A CENTAUR R/B. The epoch of initial orbital elements was 2013/4/8.4214020700. We used exact values of orbital elements from TLE Satellite Catalogue, which are shown in Tab. 1. As we can see the eccentricity of this object is really high, therefore, we have applied the value of index $q = 50$.

Table 1. Set of initial input orbital elements.

a (km)	24656.4396648861
e	0.7008709
i (deg)	21.0075
ω (deg)	89.2578
Ω (deg)	175.2247
M (deg)	5.5452
n (rev/day)	2.24209852

4.1. The eccentricity function

To calculate precisely orbital elements of the HEO objects we have to use the Hansen coefficients instead of Kaula's eccentricity function $G_{lpq}(e)$ which is related to the Hansen function by the following relation:

$$G_{lpq}(e) = X_{l-2p+q}^{-(l-1),(l-2p)}(e) \tag{2}$$

We calculated the Hansen function using the following formula [1] [2]:

$$X_{n,m}^{(k)} = \frac{1}{(1+\beta^2)^{n+1}} \sum_{l=-\infty}^{\infty} E_{k-l}^{n,m} J_l(ke), \tag{3}$$

where:

$$\beta = \frac{e}{1+\sqrt{1-e^2}}, \tag{4}$$

and

$$E_{k-l}^{n,m} = (-\beta)^{k-l-m} \sum_{s=0}^{\infty} C_{n-m+1}^{k-l-m+s} C_{n+m+1}^{s} \beta^{2s}, \tag{5}$$

and $J_l(ke)$ are the Bessel functions of the first kind. The Bessel function $J_0(ke)$ is calculated directly from the definition for small values of ke ($ke < 20$), and from Jacobi asymptotic formula for the higher values. All other Bessel functions $J_l(ke), l > 0$ are calculated by successive multiplication of $J_0(ke)$ by p_s factors, that are determined from the following recursive relation:

$$p_s = \frac{1}{2s/ke - p_{s+1}}. \tag{6}$$

Then,

$$J_{s+1}(ke) = p_s J_s(ke). \tag{7}$$

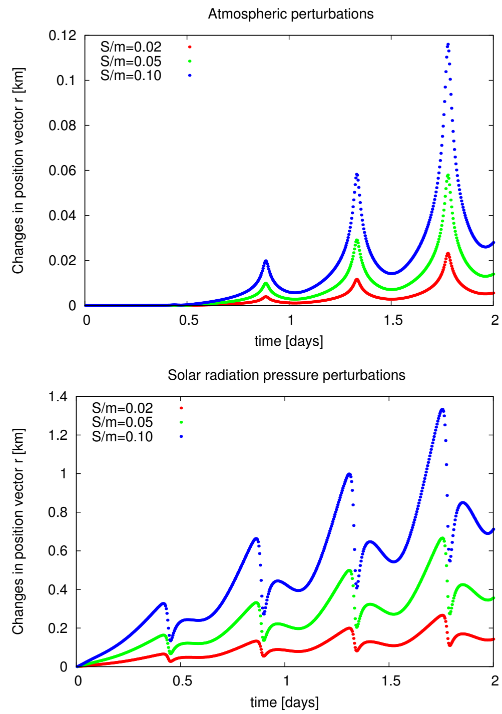

Figure 2. Changes in position vector due to the atmospheric drag and solar radiation pressure perturbations which are dependent on S/m parameter. Each color presents different S/m parameter value.

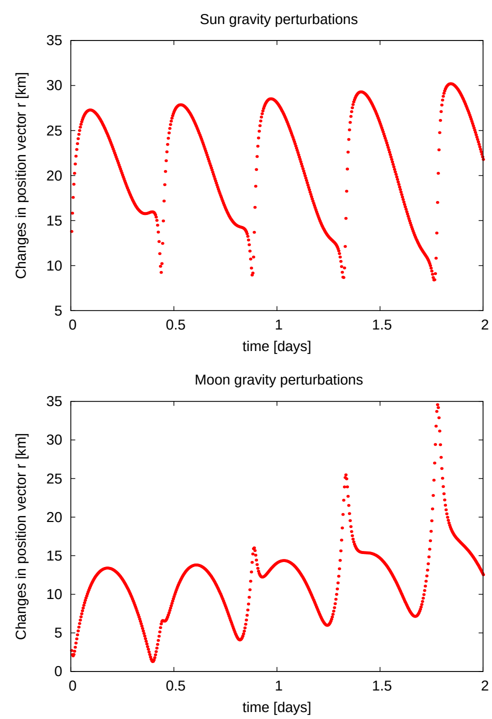

Figure 3. Changes in position vector due to the Sun and Moon gravity perturbations.

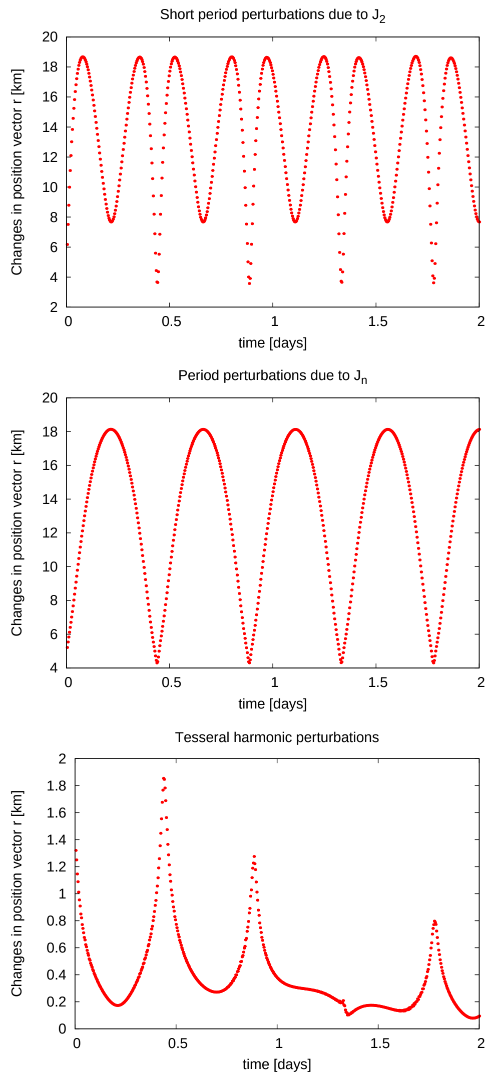

Figure 4. Changes in position vector due to the zonal and tesseral harmonics perturbations.

5. RESULTS

We have calculated the changes in position vector due to perturbing forces for two days time span. The changes in the position vector are between non-perturbed and perturbed orbit. As we can see either for object with altitude of perigee higher than 1000 km there are some slight changes in the vector due to the Earth's atmosphere if the area to mass ratio (S/m) is different. Much more significant changes are observed on this altitude due to solar radiation pressure. In only two days, changes are reaching up to 1.4 km for higher value of S/m parameter.

In case of changing orbit orientation the influence of the Sun's and Moon's gravity is also changing. The changes in position vector due to the Sun's gravity reaches from 7 to 30 km. The Influence of the Moon's gravity is different. While Sun's position is nearly constant and the distance is high, the Moon is constantly moving on its orbit and perturbing satellites orbits in a different way. It is well presented in the Fig. 3. In less than 5 revolutions changes in position vector has changed from 15 to over 35 km.

Much higher changes are observed in short period perturbation due to J_2 harmonic and long period perturbations due to J_n which reaches up to 19 km. The influence of tesseral harmonics is smaller, up to 2 km, but is still much higher than influence of the Earth's atmosphere. Characteristic periodical repetitions are directly related to the mean motion and perigee and apogee.

6. CONCLUSIONS

As we can see the HEO satellites are very specific type of objects. They are crossing all types of orbit causing the threat of collision and also causing propagation problems. In perigee, which is very low, the atmosphere and gravity field model are the most important to take into account. While in apogee, the solar radiation pressure and lunar and solar gravity dominates. To provide more accuracy calculation the analytical methods must be taken into account.

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