RESEARCH ON LONG-TERM ORBIT PROPAGATION FOR SPACE DEBRIS IN LEO

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ABSTRACT

Space debris long-term orbit propagation is one of the main problems for the space debris environment models. The evolution of space debris in low Earth orbit (LEO) is determined by a complex interplay of different perturbations. The aim of this paper is to investigate the long-term effects of the most dominating perturbations: Earth geopotential effects, atmospheric drag, luni-solar perturbations and solar radiation pressure. The atmospheric drag is the major non-gravitational perturbation in LEO. This article provides an average method of numerical integration on one revolution for rotating and stationary atmosphere, and then one can make use of various atmospheric densities to calculate the orbital evolution. Using this method, we have analyzed the effects of rotating and stationary atmospheric drag perturbation on orbital lifetime. The results show the effects of solar activity and geomagnetic index on orbital evolution are obvious. The lifetime difference in rotating atmosphere is mostly depending on inclinations of space debris.

1 INTRODUCTION

The increasing number of space debris in the low Earth orbit (LEO) region raised a question on how to avoid the risk imposed by the objects. It is therefore necessary to build a model for describing the debris environment [1]. Space debris long-term orbit propagation is one of fundamental problems for modeling the environment [2].

The orbit of space debris in the Earth's atmosphere is also affected by the perturbation from geopotential effects, atmospheric drag, luni-solar perturbations and solar radiation pressure. The Earth gravity harmonics make the semi-major axis and eccentricity variation periodically, and the right ascension of the ascending node and argument of perigee alteration symmetrically [3]. The luni-solar gravitation is rather small in LEO and radiation pressure effects make great influence to debris which has high area-to-mass ratios [4,5].

The orbital decay of space debris in the LEO almost entirely depends upon the atmosphere. The interaction between debris and upper atmosphere is very complicated and insufficiently studied, especially the rotating effects. The intensity of solar radiation flux and the value of the geomagnetic index are all important factors for accurately estimating the perturbation of atmosphere [6]. This paper provides arithmetic for the long-term orbital propagation of space debris in LEO.

2 ORBITAL PERTURBATIONS

The components of the perturbing force per unit mass are denoted by U, N and W, where U is along the direction of the velocity vector, N is perpendicular to U and in the osculating plane of the orbit, W is normal to the osculating plane. The osculating plane is defined as the plane containing the debris's velocity vector and passing through the Earth's centre. The Gauss planetary expresses the rate of change of the orbital elements in terms of the components of the perturbing force. These equations are:

$$
\begin{aligned}
\frac{da}{dt} &= \frac{2}{n\sqrt{1-e^2}}\sqrt{1+2e\cos\theta+e^2}\,U \\
\frac{de}{dt} &= \frac{\sqrt{1-e^2}}{na\sqrt{1+2e\cos\theta+e^2}}\left[2(e+\cos\theta)U - N\sqrt{1-e^2}\sin E\right] \\
\frac{di}{dt} &= \frac{r\cos(\omega+\theta)}{na^2\sqrt{1-e^2}}W \\
\frac{d\Omega}{dt} &= \frac{r\sin(\omega+\theta)}{na^2\sqrt{1-e^2}\sin i}W \\
\frac{d\omega}{dt} &= \frac{\sqrt{1-e^2}}{nae\sqrt{1+2e\cos\theta+e^2}}\left[2U\sin\theta + N(\cos E+e)\right] - \cos i\frac{d\Omega}{dt} \\
\frac{dM}{dt} &= n - \frac{1-e^2}{nae\sqrt{1+2e\cos\theta+e^2}}\left[U\left(2\sin\theta+\frac{2e^2}{\sqrt{1-e^2}}\sin E\right) + N(\cos E - e)\right]
\end{aligned} \tag{1}
$$

Where a is semi-major axis, e is eccentricity, i is the inclination, Ω is right ascension of ascending node, ω is argument of perigee, M is mean anomaly, $n=\sqrt{\mu/a^3}$ is orbit mean motion, μ is Earth gravitational constant, θ is true anomaly, E is the eccentric anomaly, r is vector length of position.

2.1 Perturbations Due to atmospheric Drag

When traveling through the atmosphere the space debris experience a drag force in a direction opposite to the velocity vector. This drag force acceleration is given by the expression: [7]

$$\vec{D} = -\frac{1}{2}\frac{C_D A}{m}\rho V\vec{V} \tag{2}$$

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Where C_D is the drag coefficient, A is space debris cross-sectional area perpendicular to the direction of motion, m is the mass of space debris, ρ is the atmospheric density, $\vec{V} = \vec{v} - \vec{v}_a$ is the speed vector of space debris relative to the atmosphere, $\vec{v}$ and $\vec{v}_a$ are the speed vector of space debris and atmosphere respectively. The effect of $\vec{v}_a$ maybe potentially significant and insufficiently studied, this article will investigate the influence of stationary and rotating atmosphere conditions on the orbital lifetime.

2.1.1 Perturbation Equation for Stationary Atmosphere

For stationary atmosphere: $\vec{v}_a = 0$, $\vec{V} = \vec{v}$. Then

$$U = -\frac{1}{2}\frac{C_D A}{m}\rho v^2 = -\frac{\alpha}{2}\rho\frac{1+2e\cos\theta+e^2}{a\left(1-e^2\right)} \quad (3)$$

$$N = 0, \quad W = 0$$

The influence of stationary atmospheric drag on the semi-major axis and eccentricity can be described as follow:

$$\begin{aligned}\frac{da}{dt} &= -\frac{\alpha n a^2}{\left(1-e^2\right)^{3/2}}\left(1+2e\cos\theta+e^2\right)^{3/2}\rho \\ \frac{de}{dt} &= -\frac{\alpha n a}{\left(1-e^2\right)^{1/2}}\left(\cos\theta+e\right)\left(1+2e\cos\theta+e^2\right)^{1/2}\rho\end{aligned} \quad (4)$$

Where $\alpha = C_D \bullet A/m$, C_D is drag coefficient, A is space debris cross-sectional area perpendicular to the direction of motion, m is mass of space debris, ρ is atmospheric density.

2.1.2 Perturbation Equation for Rotating Atmosphere

For atmospheric rotating case, the situation is more complex, according to the change of satellite inclination to measure the atmosphere rotating speed, the atmosphere rotate rate not only changes with height, and there are diurnal variation and seasonal changes [8]. For simplicity this term is assumed to be the same as the earth rotation angular velocity.

$$\begin{aligned}U &= -\frac{1}{2}\beta\frac{1+2e\cos\theta+e^2}{a\left(1-e^2\right)}\rho \\ N &= 0 \\ W &= -\frac{1}{2}\chi r\cos u\sin i\left[\frac{1+2e\cos\theta+e^2}{a\left(1-e^2\right)}\right]^{1/2}\rho\end{aligned} \quad (5)$$

Where $\beta = \alpha \bullet F^2$, $\chi = \alpha n_e F$, $F = 1 - r n_e \cos i / v$, r is vector length of position, v is vector length of speed, n_e is rotational angular velocity of the earth, $u = \omega + \theta$, i is inclination.

The influence of rotating atmospheric drag on the semi-major axis and eccentricity can be described as follow:

$$\begin{aligned}\frac{da}{dt} &= -\frac{\beta n a^2}{\left(1-e^2\right)^{3/2}}\left(1+2e\cos\theta+e^2\right)^{3/2}\rho \\ \frac{de}{dt} &= -\frac{\beta n a}{\left(1-e^2\right)^{1/2}}\left(\cos\theta+e\right)\left(1+2e\cos\theta+e^2\right)^{1/2}\rho\end{aligned} \quad (6)$$

2.2 Perturbations Due to Earth Oblateness

Earth harmonics are derived from the gravity potential through the potential theory. Those harmonics are the terms of a mathematical expansion through which the deviations from a sphere can be represented. The commonly encountered harmonic is J_2, which is the largest terms of the zonal harmonics. J_2 is responsible for the secular rates of the right ascension of ascending node and the argument of perigee, these rates can be computed by the following equations [3]:

$$\frac{d\Omega}{dt} = -\frac{3}{2}\left(\frac{R_\oplus}{p}\right)^2 n J_2 \cos i \quad (7)$$

$$\frac{d\omega}{dt} = -\frac{3}{4}\left(\frac{R_\oplus}{p}\right)^2 n J_2 \left(1-5\cos^2 i\right) \quad (8)$$

Where $R_\oplus$ is Earth equatorial radius, $p = a\left(1-e^2\right)$, J_2=0.0010826.

3 ARITHMETIC FOR PERTURBATION EQUATIONS

The effects of upper atmosphere drag are very complicated. The change in the semi-major axis and eccentricity is obtaining by integrating over a complete revolution. During one revolution, the influences of atmosphere drag become:

$$\begin{aligned}D_a &= \frac{1}{T}\int_0^T\left(\frac{da}{dt}\right)dt = \frac{1}{T}\int_0^{2\pi}\left(\frac{da}{d\theta}\frac{d\theta}{dt}\right)d\theta \\ D_e &= \frac{1}{T}\int_0^T\left(\frac{de}{dt}\right)dt = \frac{1}{T}\int_0^{2\pi}\left(\frac{de}{d\theta}\frac{d\theta}{dt}\right)d\theta\end{aligned} \quad (9)$$

Where T is the orbital period, $d\theta/dt = \left(1+e\cos\theta\right)^2\sqrt{\mu/p^3}$, $p = a\left(1-e^2\right)$.

Thus the changes of semi-major axis and eccentricity are as follows:

$$\begin{aligned}a &= a_0 + D_a \bullet \Delta t \\ e &= e_0 + D_e \bullet \Delta t\end{aligned} \quad (10)$$

Where a_0 and e_0 are initial semi-major axis and eccentricity. During one period moving, assumed the semi-major axis and eccentricity are constant, so D_a and D_e are the definite integration of true anomaly θ.

4 UPPER ATMOSPHERE DENSITY

An accurate prediction of the lifetime of space debris under the influence of drag requires a good density model of the upper atmosphere. Through long term research, various density models have been developed with varying degrees of complexity and fidelity. These models commonly take into account the diurnal variation, the 27-day fluctuation (sun's rotation period), the annual variation, and 11-year solar cycle.

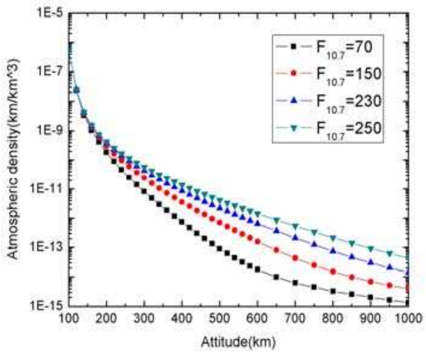


Figure.1 Average atmospheric density of MET model for different $F_{10.7}$ (A_p=4)

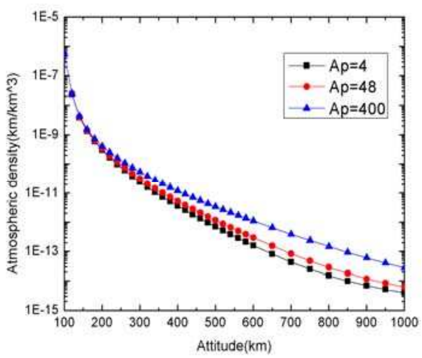
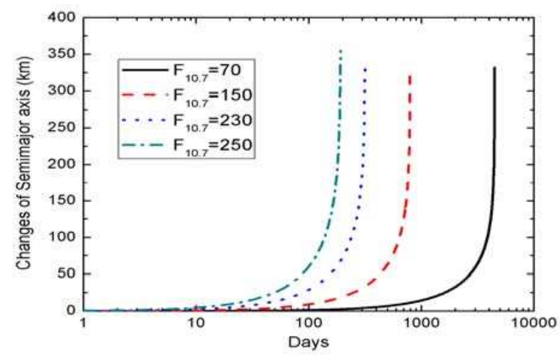


Figure.2 The average atmospheric density of MET model for different A_p ($F_{10.7}$=150)

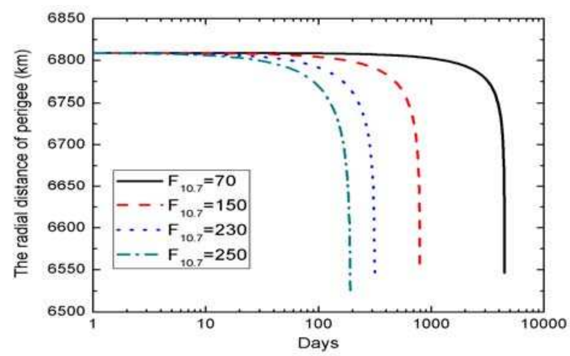
The NASA Marshall engineering thermosphere (MET) model which has been used to describe the propertied of the neutral atmosphere is based on Jacchia's empirical models [9]. The intensity of solar radiation flux $F_{10.7}$ and the value of the geomagnetic index A_p are all important factors for accurately estimating the perturbation effects of atmosphere drag. Geomagnetic storms driven by solar eruptions are known to have significant effects on the total density of the upper atmosphere in the altitude range 250~1000km. Fig. 1 and 2 depict the average atmospheric density of MET model between 100 and 1000km for different $F_{10.7}$ and A_p.

5 LIFETIME OF SPACE DEBRIS

Initial orbital elements of space debris: semi-major axis a = 6878km, eccentricity e = 0.01, inclination i = 45°, the ratio of area to mass A/m = 0.01 (m^2/kg). Four solar radiation flux $F_{10.7}$ = 70, 150, 230, 250 with an average geomagnetic index A_p = 4 and three geomagnetic index A_p = 4, 48, 400 with an average solar radiation flux $F_{10.7}$ = 150 are used for analysis and comparison. Figs. 3 and 4 illustrate how the solar radiation and geomagnetic index influence the lifetime of space debris.



(a)

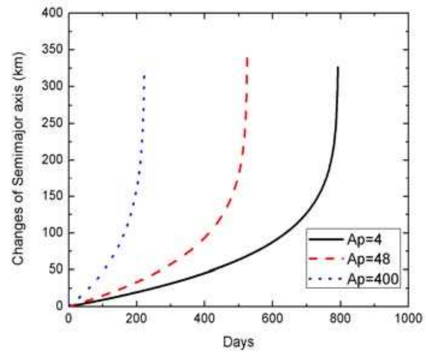


(b)

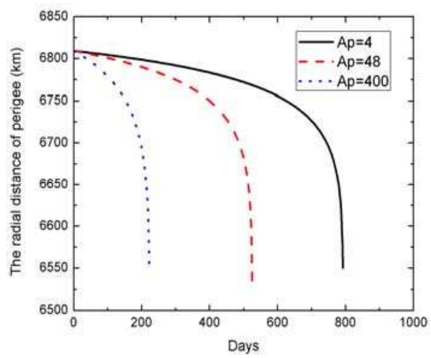
Figure.3 The changes of Δa and r_p for different $F_{10.7}$ (A_p=4, rotating atmospheric)

Compared perturbation equations for stationary and rotating atmosphere, showed by Eqs. (4) and (6), the inclination is an important factor effecting the orbital attenuation. Fig.5. shows how the inclinations influence the lifetime of space debris with solar radiation flux $F_{10.7}$= 150 and geomagnetic index A_p = 4. When the initial orbital elements are certain , the lifetime is invariable for stationary atmosphere. For rotating atmosphere, when $0° \le i < 90°$ which means space debris is in anterograde orbit, the lifetime is longer than stationary atmosphere situation, when $90° < i \le 180°$

which indicates space debris is in retrograde orbit, the lifetime is shorter than stationary atmosphere case. When $i = 90°$, the result is the same as stationary atmosphere.



(a)



(b)

Figure.4 The changes of Δa and rp for different Ap (F10.7=150, rotating atmospheric)

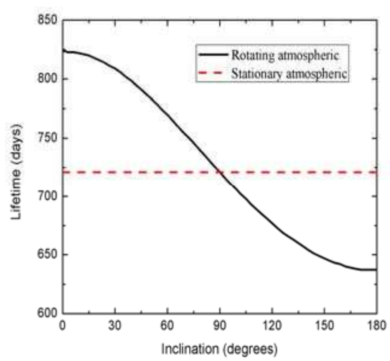


Figure.5 The changes of lifetime for different inclinations

6 SUMMARY

Atmosphere drag is an important factor which effects the lifetime of space debris in LEO. Using perturbation equations for stationary and rotating atmosphere, we have analysed the lifetime effecting by solar radiation flux and geomagnetic index based on MET model. For atmosphere is rotating, the inclination is an important element which influences the orbital decay, the results show that the lifetime is longer than stationary atmosphere situation when space debris is in anterograde orbits, and shorter when space debris is in retrograde orbits.

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