ANALYSIS OF OBSERVING STRATEGY FOR MONOPULSE TRACKING RADAR IN STARE MODE

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ABSTRACT

Due to high relative orbital velocities, centimeter-sized debris can seriously damage or disable an operational spacecraft. These debris are mainly detected by monopulse tracking radar in stare mode. In this paper, the orbit determination method in this situation was derived, accordingly the proper strategy of observation was analyzed, Last, the suggestion for selecting azimuth and elevation angles of radar was provided.

1 INTRODUCTION

Orbital debris is recognized as a present and growing hazard for both humans and machines in space, and the debris engineering models present a comprehensive view of the space environment to spacecraft designers and owners/operators. In order to update and improve the environmental models, measurement data of debris for verifying and validating the current models are necessary.

As we known, objects in low Earth orbit (LEO) larger than about 10-30cm in diameter are tracked by routine space surveillance system, and objects smaller than about 1mm in diameter are available from the analysis of returned surfaces from space. The objects about 1-10cm in diameter are called hazardous debris which are very dangerous to spacecrafts, they are mainly observed by high power, high sensitivity monopulse tracking radar in stare mode[1][2], such as the famous Haystack radar of NASA and the TIRA radar, and the method for observing debris in centimeter level has drawn great attention.

In this paper, aiming at observing the LEO space debris with the size of centimeter level, a proper observing strategy for monopulse tracking radar was proposed. First, the method for estimation orbital parameters of space debris based on stare mode was analyzed. Then three common observing strategies were analyzed. Last, suggestion for selection of observing parameters was proposed.

2 ORBITAL PARAMETERS ESTIMATION OF CENTIMETER LEVEL DEBRIS

The estimation of orbital parameters of space debris is first step to understand the distribution of space debris and their threat. At present, according to size of debris the orbit determination methods for space debris can be summarized as followed :

- Space debris larger than 10cm in diameter: the routine method to estimate the orbital parameters, which utilized tracking data of multiple arc section observed and obtain the orbital parameters.
- Space debris about 1-10cm: with the limitation of radar's beam width, the echo data of radar are rare and the arc of observation is short, and the specific method corresponding to this situation should be researched[3].

According to observation experiments by Haystack radar [4][5] and TIRA radar, space debris of centimeter size mainly detected by using stare mode. Different to the tracking mode, observation of stare mode only focus on the statistical characteristic of orbital parameters of space debris, such as the height, inclination, size and quantity, which does not care obtaining precise orbit parameters of each debris. With the measured distribution of space debris, the model of debris (such as ORDEM and MASTER) can be verified [6].

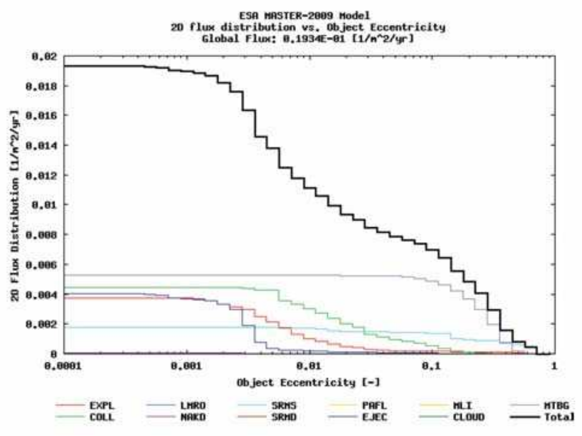

Figure1. The distribution of eccentricity of debris

At present, when the stare mode of radar is used for detect debris, two assumptions is generally adopted [3].

- The eccentricity of orbit generally assumed to be zero, according to the EAS MASTER-2009 model of debris, most of the debris are in nearly circular orbit. Figure 1 shows the

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distribution of eccentricity of space debris which proved the assumption above-mentioned.

- Debris moving through the beam center of radar. As the echo intensity of small debris are usually very weak, it's hard to precisely determine the offset between debris and center of beam, meanwhile the beam width of these radars are often narrow, for example the beam width of Haystack radar is only 0.058°. So the assumption is reasonable roughly here.

With these assumptions, the orbital parameters of debris especially the height and inclination of the debris can be obtained by utilizing the ranging information. The orbit determination method is analyzed as follow.

2.1 The known condition

The altitude, latitude and longitude of the radar is H_R, ϕ and φ, and A, E, r, $\dot{r}$ is the azimuth angle, elevation angle, range value, range rate value of radar. Establish the coordinate as follow which shown is figure 2. The origin of the coordinate is O_E, which is the mass center of the Earth, XO_EY is the equatorial plane, the axis of X is in the circle of longitude of radar, and axis of Z is the direction of North Pole. D point is the position of debris, and R point is the position of radar. In this coordinate, the y coordinate of radar is zero, so the longitude of radar can be ignored.

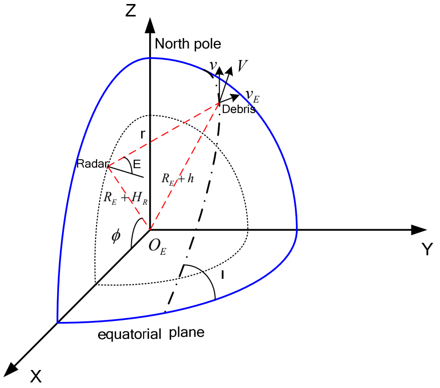


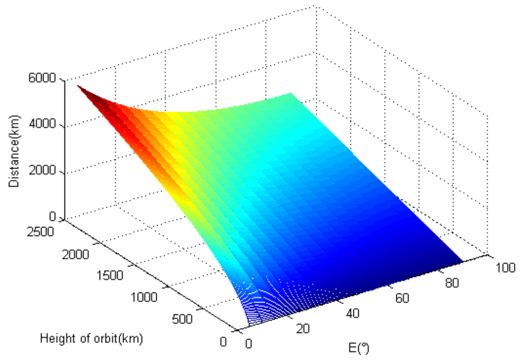
Figure 2. The coordinate for orbit determination

2.2 Obtaining the height of orbit

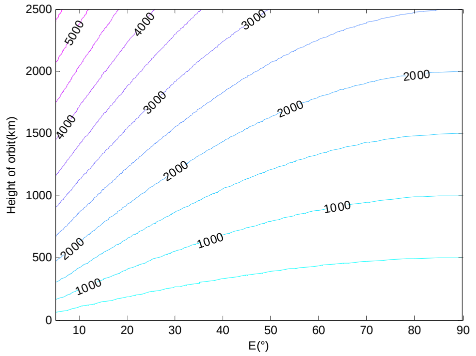
In the triangle RO_ED, based on the cosine theorem the height of orbit can be easily obtained.

$$h=\sqrt{r^2+(R_E+H_R)^2+2r(R_E+H_R)\sin(E)}-R_E \qquad (1)$$

Figure 3 shows the relationship of height, range and elevation angle. Figure 3(a) is the 3D view and figure 3(b) is contour line of figure3 (a). From these figure, we can see that if the range is fixed, the greater of the elevation angle, the greater the height of detectable debris. So if we want to detect debris of higher height, the larger elevation angle should be selected.



(a) the relationship of E, h, r



(b) the contour line of fixed range

Figure 3. The influence factors of height of orbit

2.3 Obtaining the inclination of orbit

According to the geometric relation in figure 2, the velocity of debris in inertial coordinate system $\vec{V}$ can be described as the sum of velocity $\vec{v}$ in O_E-XYZ and velocity v_E produced by Earth rotation.

$$\vec{v}=\vec{V}-\vec{v}_E \qquad (2)$$

The range rate of radar $\dot{r}$ can be represent as the projection of $\vec{v}$ in the ranging direction, so from the equation (2),

$$\begin{gathered}\vec{v}\cdot\vec{r}=\dot{r}\\ \dot{r}=\vec{r}\cdot\vec{V}-\vec{r}\cdot\vec{v}_E\end{gathered} \qquad (3)$$

$\vec{r}$ is the vector of radar R and debris D. In equation (3), there are three unknown variables: $\vec{r}$, $\vec{V}$ and $\vec{v}_E$, all of them are analyzed in order.

(1) The vector $\vec{r}$

The coordinate of radar can be represented as:

$$R = \left[(R_E + H_R)\cos(\phi), 0, (R_E + H_R)\sin(\phi)\right] \tag{4}$$

And the coordinate of debris can be represent as $D = [X_D, Y_D, Z_D]$, in which

$$X_D = (R_E + H_R)\cos(\phi) - r\cos(E)\cos(A)\sin(\phi) \\ + r\sin(E)\cos(\phi)$$

$$Y_D = r\cos(E)\sin(A)$$

$$Z_D = (R_E + H_R)\sin(\phi) + r\cos(E)\cos(A)\cos(\phi) \\ + r\sin(E)\sin(\phi)$$

So the vector of R and D is

$\vec{r} = [r_X, r_Y, r_Z]$, in which

$$r_X = -r\cos(E)\cos(A)\sin(\phi) + r\sin(E)\cos(\phi)$$

$$r_Y = 0$$

$$r_Z = r\cos(E)\cos(A)\cos(\phi) + r\sin(E)\sin(\phi)$$

(2) The velocity vector $\vec{V}$ in inertial coordinate system

The plane of orbit contains the origin of the coordinate, so the plane equation can be represent as:

$$ax + by + z = 0 \tag{5}$$

Where a, b are the plane equation parameters. The inclination of orbit can be induced according to the normal vector of plane.

$$\cos I = \frac{\pm 1}{\sqrt{a^2 + b^2 + 1}} \tag{6}$$

With the known coordinated of D, from the equation 5 and 6, we can get the expression of a and b.

$$a = (-Z_D X_D \pm Y_D \sqrt{(X_D^2 + Y_D^2)\tan^2(I) - Z_D^2}) / (X_D^2 + Y_D^2)$$

$$b = (-Z_D X_D \mp Y_D \sqrt{(X_D^2 + Y_D^2)\tan^2(I) - Z_D^2}) / (X_D^2 + Y_D^2) \tag{7}$$

In the inertial coordinate system, with the law of universal gravitation, magnitude of $\vec{V}$ can be obtained:

$$\left|\vec{V}\right| = \sqrt{\frac{\mu}{(R_E + h)}} \tag{8}$$

Where u is the constant of earth gravitation, R_E is the radius of Earth. Based on the geometric relation in figure 2, the unit vector of $\vec{V}$ can be derived:

$$\hat{V} = \begin{bmatrix} (bZ_D - Y_D)\cos(I) / (R_E + h) \\ (X_D - aZ_D)\cos(I) / (R_E + h) \\ (aY_D - bX_D)\cos I / (R_E + h) \end{bmatrix} \tag{9}$$

(3) The velocity vector $\vec{v}_E$

The velocity vector $\vec{v}_E$ produced by rotation of Earth is directly related to the latitude ϕ_D of substellar point, and its magnitude is

$$\left|\vec{v}_E\right| = (R_E + h)\cos(\phi_D)\omega_E \tag{10}$$

Where ω_E is the angular velocity of earth rotation and ϕ_D is

$$\phi_D = \arcsin\left(\frac{Z_D}{(R_E + h)}\right) \tag{11}$$

And the unit vector of $\vec{v}_E$ is

$$\hat{v}_E = \left[\frac{-Y_D}{\sqrt{X_D^2 + Y_D^2}}, \frac{X_D}{\sqrt{X_D^2 + Y_D^2}}, 0\right] \tag{12}$$

After getting the expression of $\vec{r}$, $\vec{V}$ and $\vec{v}_E$, equation 3 can be derived:

$$\dot{r} = \sqrt{\mu / (R_E + h)^3}\cos(I)[(bZ_D - Y_D)\vec{r}_X + (X_D - aZ_D)\vec{r}_Y \\ + (aY_D - bX_D)\vec{r}] - (R_E + h)\cos(\phi_D)\omega_E \\ (X_D\vec{r}_Y - Y_D\vec{r}_X) / \sqrt{(X_D^2 + Y_D^2)} \tag{13}$$

3 OBSERVATION STRATEGY ANALYSIS

From the aspect of observation geometry, here we mainly concern the configuration of elevation angle and azimuth angle.

- The azimuth angle of radar in stare mode is generally selected two values: 180°and 90°, which correspond to south-stare mode observation and east-stare mode.
- The elevation angle of radar is between 0° and 90°.

When the elevation angle is 90°, which means zenith observation, it can detect highest height of orbit, but the range rate is identically equals to 0. Only the routine orbit determination method can be used in this situation, and as the available data are rare, the precision of orbit determination is low. In this paper, we only discuss the elevation angle under 90°.

3.1 South-staring mode (A=180°)

When the azimuth angle is 180°, the equation 3 can be simplified as follow:

$$\dot{r} = \mp\sqrt{\frac{\mu}{(R_E+h)^2}}\cdot\cos(I)\cdot(R_E+H_R)\cdot\cos(E)\cdot\sqrt{X_D^2\tan^2(I)-Z_D^2}\,/\,X_D \quad (14)$$

Obviously, there are two inclination angles satisfying the equation, and these two angles are supplementary angles.

Figure 4 shows the relationship between range rate and height of orbit with different inclination angles. From this figure, we can see that the curve of inclination of 20° and 160° are the same. So additional judgment conditions such as the change rate of angle measurement should be used to distinguish the difference. Here H_R=0, ϕ =45°, E=15°.

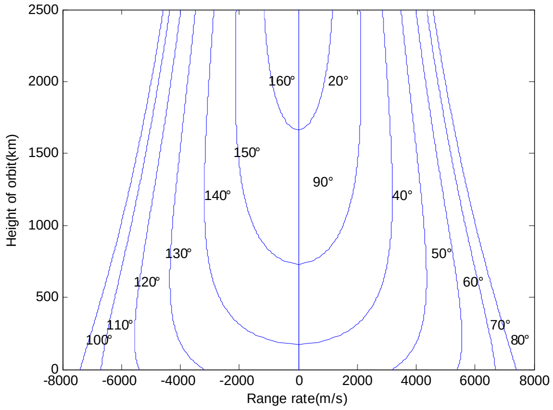


Figure 4. The relationship between range rate and height of orbit with different inclination in south stare mode

For the south stare mode, it can detect debris with low inclination angle, but the range is increased, especially for the low elevation.

When the south-staring observation is adopted, we prefer to select low elevation angle. 15° is often selected as the proper value of elevation angle, this elevation angle can bring large range of detectable inclination, for example when H_R=0, ϕ =45° and the height of orbit is 1500km, the range of inclination is about 20° to 160°, but the range value dramatically increased with lower elevation angles.

Figure 5 shows the relationship between range rate and inclination with different elevation angles. The height of orbit is fixed as1500km. We can find that the low elevation angle can obtain a wide range of inclination. Meanwhile, with the same range rate, the low elevation angle can obtain two supplementary angles with large difference.

When selecting the elevation angle, two thing should be considered, first one is the fuzzy of inclination, and second one is the inclination range of debris which can be detected. Elevation should be selected according to the actual observation conditions and requirement.

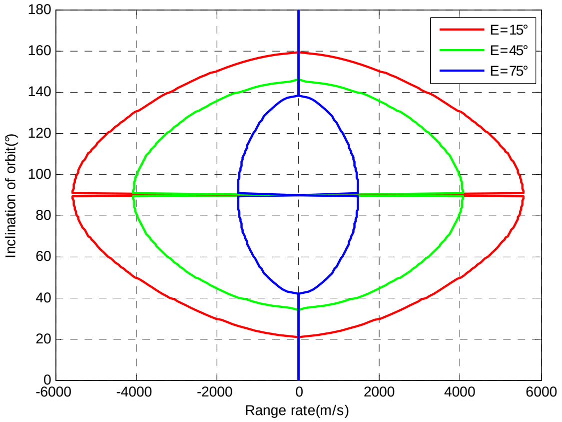


Figure 5. The relationship between range rate and inclination with different elevation angles in south stare mode

3.2 East-staring mode (A=90°)

When the azimuth angle is 90°, the equation (3) can be simplified as follow:

$$\dot{r} = \sqrt{\frac{\mu}{(R_D+h)^3}}\cdot\cos(I)\cdot(R_E+H_R)\cdot\cos(E)\cdot[\cos(\phi)-a\sin(\phi)]-\omega_E\cdot(R_E+H_R)\cdot\cos(E)\cdot\cos(\phi) \quad (15)$$

Similar with figure 4, figure 6 shows the same relationship in east-staring observation. Here E=75°.

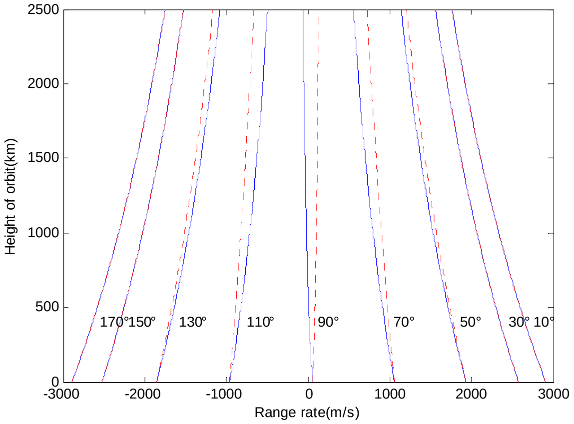


Figure 6. The relationship between range rate and height of orbit with different inclination in east stare mode

From this figure, we can find that the inclination angles are still fuzzy, high elevation angles get small fuzzy of inclination. But with the elevation increased, the fuzzy of inclination is small, which can be seen in Figure 7.

For the east stare mode, It can obtain a precise inclination of orbit in the condition of high elevation. When this mode is adopted, we prefer to select high elevation angle. And 75° is the typical value of elevation angle, because if the elevation is larger, the range rate is decreased dramatically and the arc section observed is short, which take disadvantage for orbit determination.

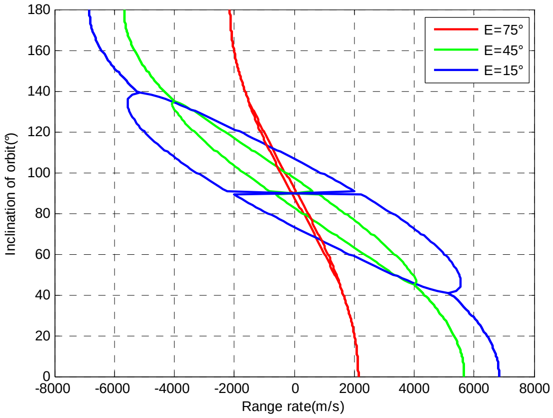


Figure 7. The relationship between range rate and inclination with different elevation angles in east stare mode

When selecting the elevation angles, two things should be considered, first one is the maximal height of debris which can be detected, and the second one is the range of range rate.

4 CONCLUSION

According to orbit determination method for observing debris in stare mode, the strategy of observation especially the selection of azimuth and elevation angles of radar is analyzed. There are two main ways to detect debris in stare mode, one is south-staring observation and another is east-staring observation. When selecting south-staring observation low elevation should be adopted, and when selecting east-staring observation high elevation should be adopted.

In the actual situation for making observation strategy, the factor of range distance, fuzzy of inclination, range rate should be comprehensively considered. Next, a cost function including these factors may be established to evaluate the observation strategy in a quantitative way.

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