

# ESTIMATION OF INFLUENCE OF VARIOUS FACTORS ON FINAL RE-ENTRY PREDICTION OF UNCONTROLLABLE SPACE OBJECTS DESCENDING FROM ORBITS

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## ABSTRACT

This paper deals with a problem of searching for ways which can increase an accuracy of the final results of re-entry prediction of space objects (SO), descending from their orbits in uncontrollable mode, and, in particular, an estimation of influence of different factors on accuracy of such predictions. On examples of the solution of the re-entry prediction problem for the 7th various space objects on the basis of processing of these SO' measurements fulfilled during 1-1.5 days before their re-entries and realization of a considerable quantity of variants of the orbit determination and re-entry prediction tasks at different initial data for these objects, including the use of different models of atmosphere, some results were obtained and certain conclusions were made, which can be used for selection of the best solutions and an estimation of a real accuracy of SO' re-entry. Five of mentioned above SOs were employed at different time as the test-objects for the IADC re-entry test campaigns and there were the officially adopted re-entry time and impact area coordinates for those objects. The reason of differences in a SO' re-entry prediction at usage of the best for today models of atmosphere GOST-2004 and NRL MSISE-2000 is detected. For the considered cases it is estimated both the value of these differences and the proximity of the relevant re-entry prediction data, obtained with using these models, to the officially adopted data on the SO' pass of an 80 km altitude atmosphere interface, confirmed by other independent sources.

## 1 INTRODUCTION

The precise prediction of the re-entry time and impact area of an uncontrollable space object descending from its orbit is a complicated scientific-and-technical problem being of theoretical interest and having the large practical importance.

This problem takes a special importance in the cases of the re-entry of so-called risk objects, that capable to create hazardous situations for ecology, economy and even for the people living at falling on the Earth. As

risk space objects we consider the space vehicles equipped with the radioactive power sources or have on board other hazardous substances, as well as large and heavy man-made space objects with the high-melting units onboard which fragments, having a large kinetic energy, can impact the Earth' surface. In these cases reaching the highest possible accuracy of a SO' re-entry prediction is actual, both from the point of view of the well-timed warning of an approach of possible hazardous situations in certain territories and from the point of view of conformation or disclaimer of the facts of such SO' fragments falling on one or another area of the Earth' surface.

However for today practically reached accuracy of an uncontrollable SO re-entry time prediction in the most cases remains enough low. The standard error in predicted re-entry time of a descending SO is estimated by the value making ~ 10-20 % from a remained ballistic lifetime. Evidently, that accuracy of a SO' re-entry parameters prediction will be increase at shortening of a remained lifetime (that is interval of a prediction). However even for a short-term forecast the achievable accuracy of the final re-entry prediction as well as the accuracy of a posteriori determination of a SO' re-entry parameters can be different and will depend on many factors such as: parameters of a final SO' orbit, a composition of measurements used at a SO' orbit determination, perturbation forces taken into account in a model of SO' motion, aerodynamic and other characteristics of a SO, an orientation of SO in space, an accuracy of the used methodic and prediction software.

One of the important factors, influencing the motion prediction parameters of a SO, is the correct taking into account of an atmospheric resistance force depending on: a) the used model of the atmosphere, b) on input in atmospheric model parameters of the solar and geomagnetic activity, c) on features and conditions of SO' flight. Whereas that at usage of different atmospheric models in a final phase of SO' flight an essentially different results in an object re-entry time prediction can be obtained, quite natural there is a problem – which model of atmosphere is preferable.

Other serious problem connected with an achievement of the highest possible accuracy of a SO' re-entry time prediction, is a selection from among available observations the most effective ones from the navigational point of view for SO' orbit determination. In the present paper certain criterions for selection of such a kind of the measurement data set are offered.

The degree of influence of the specified above factors on an accuracy of a re-entry prediction was estimated on the examples of monitoring of the uncontrollable space objects' flights, for which confirmed high accuracy of the re-entry prediction and final determination of impact areas has been reached. The results, making it possible to estimate efficiency of the offered criterions for selection of the best compositions of the measurement data for the solved problem by comparing of the SO' re-entry time, obtained with using the selected in this way measurements, with the results of exact determination of a SO' re-entry time and its impact area, that were obtained by an independent way, in particular, with usage of the US Strategic Command tools and software, are presented.

On the basis of set of measurements selected according to the indicated criterions from among those, that have

been fulfilled during last days of SO' flights, the tasks of orbit determination for each SO have been solved at using the atmospheric models GOST-2004 and NRL MSISE-2000. The differences in the re-entry time predictions for each SO at usage one and other model were established.

The reason of the indicated differences is revealed. It is caused by the fact that the atmospheric models GOST-2004 and NRL MSISE-2000 represent a different value of an atmospheric density within an altitude range 120-80 km. The degree of this difference in the represented density and its influence on the results of the SO' re-entry prediction was estimated for all considered cases.

## 2 SELECTED OBJECTS, INITIAL DATA, MODELS AND METHODS USED AT CARRYING OUT INVESTIGATIONS

For carrying out investigation on estimation of various factors' influence on the final re-entry prediction of uncontrollable space objects, descending from their orbits, 7 already re-entered SOs have been selected. Some information concerning these objects is presented in Tab. 1.

Table 1. Re-entered space objects being under post flight analysis.

No	Space object	COSPAR ID	US SSN No	Re-entry date	Country
1	SC «Coronas-F»	2001-032A	26873	06.12.2005	Russia
2	R/B «Delta-2»	2007-023B	31599	16.08.2007	USA
3	R/B «Vostok-2M»	1979-093B	11601	30.04.2010	Russia
4	SC «ROSAT»	1990-049A	20638	23.10.2011	Germany
5	SC «Phobos-Grunt»	2011-065A	37872	15.01.2012	Russia
6	R/B «Soyuz-FG»	2011-078B	38037	24.12.2011	Russia
7	SC «Sfera»	1998-067CM	38751	24.11.2012	Russia

The first five of the listed SO were chosen as test objects for the exercises in the frame of the Interagency Co-ordination Committee on space debris (IADC) test campaign. For these objects the exact data on SO' pass of an 80 km altitude atmosphere interface obtained by USSTRATCOM were officially adopted by a management of the IADC test campaigns.

The light of burning upper stage of the RB "Soyuz-FG" was observed by many Church-goers in some European cities on the Christmas eve at 17:30 CET on December, 24th, 2011.

Experimental small space craft "Sfera" was launched by the astronauts Padalka G.I. and Malchenko J.I. from the International Space Station (the 32nd expedition on the ISS) during out-of-doors space activity on August, 20th 2012 at 22:16 of Moscow time. It was supposed, that

this space craft will be used for improvement of the mathematical methods of taking into account of atmospheric resistance on the base of measurements, fulfilled by ground station means.

On the basis of available experience of activities on monitoring of various uncontrollable man-made space objects re-entries and the accumulated statistics on the results of determination of time and impact areas of descending SOs, the final flight time interval 1-1.5 days long before the SO' re-entry was considered as appropriate measured arc on which measurements the most authentic results of a SO' re-entry parameters prediction could be obtained.

As the input measured information for solving a task of SO' orbit determination (OD) there was used:

a) orbital data in the form of state vector (S/V) performed by the Russian Space Surveillance System (RSSS) on the basis of its sensors measurements,

b) orbital data in the form of TLE performed by US Space Surveillance Network (SSN) and accessible from the public sources,

c) SO' orbit parameters in TLE and S/V forms prepared by NASA and other space agencies (DLR, ESA, CNES) which were put into the IADC database REDB during the re-entry test campaign.

The orbital data performed in TLE format, were preliminary transformed in rectangular state vector of an object, referred to the standard reference frame (SRF) of J2000 epoch.

It was supposed that all using measurement data from different sources have the same precision, which was

determined by the root-mean-square errors of the object position (after transformation input data to the form of SO' state vectors in SRF):  $\sigma X = \sigma Y = \sigma Z = 1$  km.

In table 2 the sets of measurements fulfilled during 1-1.5 days before SO re-entry that were used at post flight analysis for those SO are presented. In table for each considered SO are given: duration of the measured arc (in time span and in the remaining orbits where they are spread out); the source of measurement; number and format of corresponding data. Thus, the data on the Phobos-Grunt orbit during the last day of its flight was performed by 5 sources (Russian space surveillance system - RSSS, US space surveillance network - NASA, German tracking sensors - DLR, France tracking sensors - CNES, ESA tracking sensors - ESA). The figure 0 in a column "remaining orbits" testifies that last measurement for given SO is attached to "impact" orbit.

Table 2. The orbital data used at post flight SO' re-entry analysis

No	Space object	Measured arc		Number and sources of measurement data
		Time span (UTC)	Remaining orbits before re-entry	
1	SC «Coronas-F»	05.12.2005/04:11 – 06.12.2005/17:00	24 - 0	23 (RSSS – 15 S/V, NASA – 5 TLE, ESA – 3 TLE)
2	R/B «Delta-2»	14.08.2007/01:12 – 16.08.2007/05:28	38 - 2	25 (RSSS – 12 S/V, NASA – 13 TLE)
3	R/B «Vostok-2M»	29.04.2010/02:38 – 30.04.2010/16:16	26 - 1	24 (RSSS – 11 S/V, NASA – 11 TLE, ESA – 2 TLE)
4	SC «ROSAT»	21.10.2011/07:33 – 22.10.2011/22:54	29 - 2	37 (RSSS – 9 S/V, NASA – 11 TLE, ESA – 15 TLE, CNES – 2 S/V)
5	SC «Phobos-Grunt»	14.01.2012/17:49 – 15.01.2012/17:03	17 - 0	34 (RSSS – 16 S/V, NASA – 10 TLE, ESA – 3 TLE, CNES – 1 S/V, DLR – 4 TLE)
6	R/B «Soyuz-FG»	22.12.2011/21:35 – 24.12.2011/13:41	29 - 2	17 (RSSS – 12 S/V, NASA – 5 TLE)
7	SC «Sfera»	22.11.2012/20:09 – 24.11.2012/07:13	24 - 1	21 (RSSS – 10 S/V, NASA – 11 TLE)

For the description of a SC motion in a low earth orbit the model of acting forces which took into account a gravitation field of the nonspherical Earth, an atmospheric drag and gravitational attraction of the third body (the moon and the Sun) was used. The fulfilled estimations have shown that for considered objects the influence of other natural factors, disturbing SO' motion, could be neglected.

For estimation of the influence of one or another model of a gravitational field of the Earth on the results of a navigation task solution the corresponding calculated parameters of SC' motion were compared. For this purpose the Russian model PZ-90, the American model JGM-3 and the European model GEM-T3 were chosen. The comparison of the calculated parameters of SC' motion, when different numbers of harmonics of the

selected models of geopotential were used, was done as well. The usage of different models of a gravitational field of the Earth at the final phase of a SO' flight led to the residuals (in calculated positions of a space vehicle on a prolongation interval of 1.5 day) having the values no more than 1 km (for the same number of taken into account harmonics). The difference in the calculated SO' positions on the given interval of prediction at taking into account the restricted and full number of harmonics in the used model of a geopotential could be already more considerable. In this connection at carrying out of the basic calculations, within the frame of considered investigations, the model PZ-90, in which all harmonics up to degree and order (16×16) were taken into account, was used.

The results of previous researches, in particular,

presented in [2], have allowed to conclude, that from all atmospheric models, including dynamic models GOST-2004, NRL MSISE-2000, GOST-84 (edition of 1990), Jacchia, and the Russian static model SMA-62, used for the solution of practical problems, the best ones, from the point of view of maintenance of achievement of more accurate prediction of SO' motion in low-earth-altitude orbits, are GOST-2004 and NRL MSISE-2000. Therefore, for an estimation of dependence of the time predictions when a SO pass a 80 km altitude interface and its impact the Earth on the used atmospheric model, the identical variants of the solution of a considered task were realized at use of dynamic models GOST-2004 and NRL MSISE-2000. At the same time at carrying out the estimated calculations in the indicated dynamic models of the atmosphere the actual (i.e. updated on using the real measurements) indexes of solar ( $F_{10.7}$ ) and geomagnetic ( $A_p$ ) activity, provided by both the NOAA and the Institute of a Terrestrial Magnetism, an Ionosphere and distribution of the radio waves of the Russian Academy of Sciences (IZMIRAN), were used.

At calculation of the perturbations from the Moon and the Sun the ephemerides DE403, developed by the Jet Propulsion Laboratory (JPL) were used.

A SO' motion was described by the system of the differential equations of the second degree, represented in the rectangular geocentric inertial co-ordinate system, referred to the mean equinox and equator of the standard epoch J2000. Calculation of the SO' orbital parameters for a certain epoch implemented by means of numerical integration of the indicated equations with help of high-effective method, allowing to reach any given accuracy [3]. Thereby the computation error in a problem of numerical prolongation of a SO' trajectory was negligible small.

The determination of SO' motion parameters during its flight in a near Earth orbit by means of treatment of the measurement data was fulfilled according to the methods which is described in [1, 2].

### 3 RESULTS OF THE SOs' ORBIT DETERMINATION AND RE-ENTRY PREDICTION AT THE FINAL PHASE OF THEIR FLIGHTS

For determination of the best composition of measurements, from among available observation data fulfilled during 1-1.5 day before SO' re-entry, various samples of this information have been considered. In each variants, corresponding to a concrete sample, the problem of a SO' orbit determination, including six-measured state vector at some initial epoch  $t_0$  and ballistic coefficient of this object i.e. set  $\bar{q} = \{\bar{r}_0, \bar{V}_0, S_b\}$ , was solved. Further on the basis of

the improved orbital parameters the SO motion prediction up to the specific epoch  $T_f$ , corresponding to object' pass the altitude  $H \sim 100$  km directly prior to its atmospheric entry was implemented.

Navigation efficiency of each considered sample of the original measured information, i.e. the achievement of the highest possible accuracy of a SO' motion parameters on the base of given composition of measurements, and quality of the obtained solution was estimated by following criteria:

1. Achievement of the minimum value of a standard root mean square error  $\sigma_0$ , defined as:

$$\sigma_0 = \sqrt{\frac{\sum_k \sum_{i=1}^{M_k} P_i^k [\Psi_{i_{obs}}^k - \Psi_{i_{calc}}^k(\bar{q})]^2}{N - m}},$$

where the following denotations are used:

$\Psi_{i_{obs}}^k$  – the measured value of  $i$ -th observation of type  $k$  (i.e.  $X_i, Y_i$  or  $Z_i$ ),

$\Psi_{i_{calc}}^k(\bar{q})$  – the calculated analog of this observation value corresponding to the improved vector of the determined parameters  $\bar{q}$  having dimension  $m$ , (in our case  $\bar{q} = \{\bar{r}_0, \bar{V}_0, S_b\}$ ,  $m = 7$ ),

$P_i^k = \frac{1}{\sigma_i^k}$  – a weight of  $i$ -th observation ( $\sigma_i^k$  – a mean-square measurement error),

$M_k$  – total number of fitted measurements of  $k$ -th

type,  $N = \sum_k M_k$ .

At a good fitting of the used measurements and the adequate definition of their weights the requirement  $\sigma_0 \leq 1$  should be satisfied.

2. Minimum of a formal error of a ballistic coefficient determination -  $3\sigma S_b$ .

3. Minimum of the formal errors of a SO' position ( $3\sigma$ ) referred to the moment  $T_f$ , calculated on the basis of the covariance matrix of errors for the obtained OD solution and transferred to the orbital co-ordinate system RNB (the centre of the given system of coordinates coincides with the centre of SC' mass, the axis R is directed from the Earth' centre towards the space vehicle, the axis N is orthogonal to the R, it lies in the orbital plane and directed towards a space vehicle movement, the axis B supplements the right handed system) i.e. in form of  $3\sigma R, 3\sigma N, 3\sigma B$ .

4. Consistency of the last fulfilled measurements to other measurements from a considered composition (residuals of the observed and calculated values of the last fulfilled measurements, but not treated under least square method should be rather small).

Necessity of application of the criterion 4 may be explained by the circumstance that often enough the last measurement data corresponds to the SO' observations executed by tracking facilities in difficult conditions when SO being at a low altitude.

The solution of a task of a SO' orbit determination (OD) and re-entry prediction was carried out by usage of dynamic models of atmosphere both GOST-2004 and NRL MSISE-2000 at actual values of indexes  $F_{10.7}$  and  $A_p$ .

The results of measurements fitting and formal estimations of the improved parameters' precision for different variants of the OD task solutions on the base of measurements spread within orbital arcs of different duration - from ~6 hours up to ~1.5 day are given in tables 3-9. In addition in these tables for each variants

of OD are given: the beginning and the end of a measured interval, its duration (in hours); total number of the used sets of original orbital data; the RMS error  $\sigma_0$ ; the value of a ballistic coefficient  $S_b$  obtained in the corresponding solution with its formal error ( $3\sigma$ ); the formal errors of the SC' position ( $3\sigma$ ) referred to the moment  $T_f$  in the orbital co-ordinate system RNB. In tables' names (in brackets) officially adopted or most probable epoch of a SO' re-entry time, corresponding to other sources are presented as well.

The variants of the solutions, obtained at using atmospheric models GOST-2004 (G) and NRL MSISE-2000 (M), which have been selected as the best ones for each of considered objects according to the criteria of efficiency and qualities, described above, are marked by pink and green color respectively. At the same time it is necessary to note that the selection of the best variants of OD solutions carried out at integrated approach to applying the described above criteria.

Table 3. The main characteristics of the SC "Coronas-F" orbit determination and re-entry prediction solutions obtained on the base of various compositions of the measurement data performed during 1.5 day before re-entry (adopted re-entry time - 06.12.2005 17:34 UTC).

Var. No	Measured interval		Total number of data sets	Re-entry epoch (UTC)	RMS $\sigma_0$	Obtained value of $S_b$	Prediction errors at $T_f$		
	Beginning – end, (UTC)	Duration, hours					$3\sigma_R$ , km	$3\sigma_N$ , km	$3\sigma_B$ , km
1.	05.12.2005/04:12 – 06.12.2005/15:41	35.50	21	06.12.2005 17:07:06	4.150	0.0701±0.00048	4.870	6.282	24.568
2.	05.12.2005/19:12 – 06.12.2005/17:00	21.80	11	06.12.2005 17:34:15	0.608	0.0652±0.00018	0.797	6.198	0.875
3G.	05.12.2005/19:12 – 06.12.2005/17:08	21.93	12	06.12.2005 17:34:15	0.652	0.0653±0.00015	0.769	6.492	0.847
3M.	05.12.2005/19:12 – 06.12.2005/17:08	21.93	12	06.12.2005 17:28:31	0.736	0.0717±0.00021	0.868	7.428	0.958
4G.	06.12.2005/04:55 – 06.12.2005/17:08	12.22	8	06.12.2005 17:35:20	0.475	0.0647±0.00024	0.780	4.917	0.812
4M.	06.12.2005/04:55 – 06.12.2005/17:08	12.22	8	06.12.2005 17:27:21	0.939	0.0724±0.00057	1.550	9.783	1.604
5G.	06.12.2005/10:56 – 06.12.2005/17:08	06.20	4	06.12.2005 17:38:24	0.253	0.0634±0.00297	2.047	26.451	0.615
5M.	06.12.2005/10:56 – 06.12.2005/17:08	06.20	4	06.12.2005 17:31:40	0.235	0.0689±0.00303	1.882	24.000	0.572

Table 4. The main characteristics of the R/B "Delta-2" orbit determination and re-entry prediction solutions obtained on the base of various compositions of the measurement data performed during 1.5 day before re-entry (adopted re-entry time - 16.08.2007 09:30 UTC).

Var. No	Measured interval		Total number of data sets	Re-entry epoch (UTC)	RMS $\sigma_0$	Obtained value of $S_b$	Prediction errors at $T_f$		
	Begin – end, (UTC)	Duration, hours					$3\sigma_R$ , km	$3\sigma_N$ , km	$3\sigma_B$ , km
1G.	14.08.2007/19:58 – 16.08.2007/05:28	33.50	15	16.08.2007 09:29:19	2.927	0.0729±0.00042	1.845	18.798	2.860
1M.	14.08.2007/19:58 – 16.08.2007/05:28	33.50	15	16.08.2007 10:03:45	5.013	0.0717±0.00069	3.201	32.610	4.905
2G.	15.08.2007/05:50 – 16.08.2007/05:28	23.50	10	16.08.2007 09:14:58	1.122	0.0715±0.00030	0.932	9.987	1.365
2M.	15.08.2007/05:50 – 16.08.2007/05:28	23.50	10	16.08.2007 09:41:32	1.577	0.0703±0.00042	1.315	14.289	1.917

3G.	15.08.2007/10:15 – 16.08.2007/05:28	17.10	7	16.08.2007 09:30:07	0.998	0.0684±0.00045	1.038	13.155	1.656
3M.	15.08.2007/10:15 – 16.08.2007/05:28	17.10	7	16.08.2007 09:46:07	1.139	0.0680±0.00048	1.191	15.129	1.890
4G.	15.08.2007/18:06 – 16.08.2007/05:28	11.50	6	16.08.2007 09:29:02	1.027	0.0684±0.00132	1.170	22.767	1.750
4M.	15.08.2007/18:06 – 16.08.2007/05:28	11.50	6	16.08.2007 09:32:33	1.028	0.0699±0.00132	1.176	23.007	1.754
5G.	15.08.2007/23:27 – 16.08.2007/05:28	06.00	3	16.08.2007 09:39:27	0.555	0.0651±0.00258	1.613	20.172	1.275
5M.	15.08.2007/23:27 – 16.08.2007/05:28	06.00	3	16.08.2007 09:40:56	0.615	0.0669±0.00291	1.803	22.677	1.415

Table 5. The main characteristics of the R/B "Vostok-2M" orbit determination and re-entry prediction solutions obtained on the base of various compositions of the measurement data performed during 1.5 day before re-entry (adopted re-entry time - 30.04.2010 16:54 UTC).

Var. No	Measured interval		Total number of data sets	Re-entry epoch (UTC)	RMS $\sigma_0$	Obtained value of $S_b$	Prediction errors at $T_f$		
	Begin – end, (UTC)	Duration, hours					$3\sigma_R$ , km	$3\sigma_N$ , km	$3\sigma_B$ , km
1.	29.04.2010/02:38 – 30.04.2010/16:16	36.65	24	30.04.2010 16:47:48	1.081	0.0622±0.00009	0.406	4.965	0.995
2G.	29.04.2010/16:44 – 30.04.2010/16:16	23.53	17	30.04.2010 16:48:13	1.055	0.0621±0.00018	0.495	5.949	1.077
2M.	29.04.2010/16:44 – 30.04.2010/16:16	23.53	17	30.04.2010 16:39:02	0.871	0.0679±0.00015	0.408	4.980	0.889
3G.	30.04.2010/00:37 – 30.04.2010/16:16	15.65	12	30.04.2010 16:50:17	0.689	0.0615±0.00021	0.447	5.187	0.926
3M.	29.04.2010/00:37 – 30.04.2010/16:16	15.65	12	30.04.2010 16:39:54	0.836	0.0676±0.00030	0.546	6.357	1.127
4G.	30.04.2010/04:27 – 30.04.2010/16:16	11.80	8	30.04.2010 16:51:04	0.653	0.0612±0.00030	0.540	4.917	1.014
4M.	30.04.2010/04:27 – 30.04.2010/16:16	11.80	8	30.04.2010 16:40:52	0.898	0.0672±0.00048	0.745	6.804	1.394
5G.	30.04.2010/08:51 – 30.04.2010/16:16	06.50	5	30.04.2010 16:52:28	0.424	0.0604±0.00036	0.994	3.423	1.021
5M.	30.04.2010/08:51 – 30.04.2010/16:16	06.50	5	30.04.2010 16:42:53	0.831	0.0662±0.00078	1.952	6.708	2.001

Table 6. The main characteristics of the SC "ROSAT" orbit determination and re-entry prediction solutions obtained on the base of various compositions of the measurement data performed during 1.5 day before re-entry (adopted re-entry time - 23.10.2011 01:57 UTC).

Var. No	Measured interval		Total number of data sets	Re-entry epoch (UTC)	RMS $\sigma_0$	Obtained value of $S_b$	Prediction errors at $T_f$		
	Begin – end, (UTC)	Duration, hours					$3\sigma_R$ , km	$3\sigma_N$ , km	$3\sigma_B$ , km
1G.	21.10.2011/10:32 – 22.10.2011/22:55	36.00	29	23.10.2011 02:05:25	3.627	0.0481±0.00024	1.313	15.606	2.625
2G.	22.10.2011/19:08 – 22.10.2011/18:33	23.58	15	23.10.2011 01:42:18	1.564	0.0478±0.00042	1.282	23.550	1.556
2M.	22.10.2011/19:08 – 22.10.2011/18:33	23.58	15	23.10.2011 02:03:22	1.455	0.0461±0.00039	1.192	22.005	1.457
3G.	22.10.2011/20:35 – 22.10.2011/20:09	23.43	15	23.10.2011 01:57:40	1.057	0.0468±0.00027	0.867	15.171	1.064
3M.	22.10.2011/20:35 – 22.10.2011/20:09	23.43	15	23.10.2011 02:15:35	1.080	0.0456±0.00027	0.888	15.603	1.091
4G.	22.10.2011/04:13 – 22.10.2011/20:00	15.93	14	23.10.2011 01:57:50	0.915	0.0468±0.00036	0.782	11.232	1.014
4M.	22.10.2011/04:13 – 22.10.2011/20:00	15.93	14	23.10.2011 02:08:00	0.883	0.0460±0.00036	0.757	10.917	0.982

5G.	22.10.2011/09:59 – 22.10.2011/22:55	12.00	10	23.10.2011 02:07:43	0.889	0.0460±0.00042	0.740	8.901	1.246
5M.	22.10.2011/09:59 – 22.10.2011/22:55	12.00	10	23.10.2011 02:03:01	0.962	0.0462±0.00048	0.804	9.741	1.349
6G.	22.10.2011/16:08 – 22.10.2011/22:55	06.78	4	23.10.2011 02:01:54	0.501	0.0469±0.00114	0.886	17.778	1.768
6M.	22.10.2011/16:08 – 22.10.2011/22:55	06.78	4	23.10.2011 01:53:06	0.432	0.0477±0.00099	1.523	15.390	1.525

Table 7. The main characteristics of the SC "Phobos-Grunt" orbit determination and re-entry prediction solutions obtained on the base of various compositions of the measurement data performed during 1 day before re-entry (adopted re-entry time - 15.01.2012 17:53 UTC).

Var. No	Measured interval		Total number of data sets	Re-entry epoch (UTC)	RMS $\sigma_0$	Obtained value of $S_b$	Prediction errors at $T_f$		
	Begin – end, (UTC)	Duration, hours					3 $\sigma_R$ , km	3 $\sigma_N$ , km	3 $\sigma_B$ , km
1G.	2012.01.14/17:49 – 2012.01.15/17:03	23.23	34	15.01.2012 18:00:00	0.759	0.0136±0.00003	0.314	4.137	0.561
1M	2012.01.14/17:49 – 2012.01.15/17:03	23.23	34	15.01.2012 17:39:17	0.740	0.0161±0.00003	0.313	4.041	0.547
2.	2012.01.14/17:49 – 2012.01.15/12:34	18.73	25	15.01.2012 18:09:06	0.566	0.0134±0.00005	0.263	5.109	0.447
3.	2012.01.14/23:10 – 2012.01.15/17:03	17.88	24	15.01.2012 17:58:33	0.561	0.0137±0.00003	0.324	3.396	0.471
4.	2012.01.14/17:49 – 2012.01.15/06:27	12.62	19	15.01.2012 18:14:46	0.449	0.0133±0.00011	0.228	6.852	0.423
5G.	2012.01.15/04:59 – 2012.01.15/17:03	12.07	17	15.01.2012 17:57:04	0.470	0.0138±0.00004	0.447	3.492	0.492
5M	2012.01.15/04:59 – 2012.01.15/17:03	12.07	17	15.01.2012 17:40:17	0.668	0.0159±0.00007	0.634	4.926	0.697
6.	2012.01.15/02:05 – 2012.01.15/13:59	11.90	12	15.01.2012 18:01:33	0.574	0.0136±0.00012	0.552	7.893	0.570
7.	2012.01.14/17:49 – 2012.01.15/02:05	8.25	15	15.01.2012 17:58:02	0.427	0.0135±0.00026	0.260	10.818	0.522
8G.	2012.01.15/07:54 - 2012.01.15/13:59	6.08	7	15.01.2012 17:55:47	0.605	0.0138±0.00042	0.978	10.941	0.897
8M	2012.01.15/07:54 - 2012.01.15/13:59	6.08	7	15.01.2012 17:31:29	0.613	0.0165±0.00051	0.993	11.040	0.906
9.	2012.01.15/02:05 - 2012.01.15/07:54	5.83	7	15.01.2012 18:35:48	0.553	0.0130±0.00063	0.659	29.226	0.633
10G.	2012.01.15/12:31 - 2012.01.15/17:03	4.30	11	15.01.2012 17:55:15	0.310	0.0141±0.00012	0.474	2.676	0.630
10M.	2012.01.15/12:31 - 2012.01.15/17:03	4.30	11	15.01.2012 17:44:12	0.364	0.0158±0.00016	0.555	3.081	0.740

Table 8. The main characteristics of the R/B "Soyuz-FG" orbit determination and re-entry prediction solutions obtained on the base of various compositions of the measurements performed during 1.5 day before re-entry (more probable re-entry time - 24.12.2011 16:30 UTC).

Var. No	Measured interval		Total number of data sets	Re-entry epoch (UTC)	RMS $\sigma_0$	Obtained value of $S_b$	Prediction errors at $T_f$		
	Begin – end, (UTC)	Duration, hours					3 $\sigma_R$ , km	3 $\sigma_N$ , km	3 $\sigma_B$ , km
1.	22.12.2011/21:35 – 24.12.2011/13:41	40.10	17	24.12.2011 16:12:33	1.911	0.0802±0.00021	1.643	15.279	1.953
2G.	23.12.2011/12:43 – 24.12.2011/13:41	24.97	10	24.12.2011 16:22:16	1.167	0.0774±0.00036	1.357	12.999	1.684
2M.	23.12.2011/12:43 – 24.12.2011/13:41	24.97	10	24.12.2011 16:34:15	1.261	0.0836±0.00045	1.464	14.127	1.811
3G.	23.12.2011/17:15 – 24.12.2011/13:41	19.43	7	24.12.2011 16:24:19	1.352	0.0770±0.00075	1.801	22.149	2.163
3M.	23.12.2011/17:15 – 24.12.2011/13:41	19.43	7	24.12.2011 16:30:19	1.397	0.0844±0.00087	1.858	23.109	2.228

4G.	23.12.2011/20:23 – 24.12.2011/13:41	17.30	5	24.12.2011 16:25:43	1.591	0.0768±0.00114	2.737	30.270	2.930
4M.	23.12.2011/20:23 – 24.12.2011/13:41	17.30	5	24.12.2011 16:29:18	1.684	0.0847±0.00132	2.987	32.370	3.099

Table 9. The main characteristics of the SC "Sfera" orbit determination and re-entry prediction solutions obtained on the base of various compositions of the measurement data performed during 1.5 day before re-entry (our own estimation of re-entry time - 24.11.2012 08:40 UTC).

Var. No	Measured interval		Total number of data sets	Re-entry epoch (UTC)	RMS $\sigma_0$	Obtained value of $S_b$	Prediction errors at $T_f$		
	Begin – end, (UTC)	Duration, hours					$3\sigma_R$ , km	$3\sigma_N$ , km	$3\sigma_B$ , km
1G.	23.11.2012/04:42 – 24.11.2012/04:47	24.08	9	24.11.2012 08:35:30	1.1	0.2576±0.0009	0.8202	11.52	1.9989
2G.	23.11.2012/07:40 – 24.11.2012/07:13	23.55	7	24.11.2012 08:24:47	2.19	0.2622±0.0015	1.6527	18.615	4.668
3G.	23.11.2012/07:40 – 24.11.2012/05:46	22.10	7	24.11.2012 08:37:20	1.28	0.2573±0.0009	1.3101	12.576	2.5467
3M.	23.11.2012/07:40 – 24.11.2012/05:46	22.10	7	24.11.2012 08:39:00	1.60	0.2620±0.0012	1.8231	15.645	2.9082
4G.	23.11.2012/19:29 – 24.11.2012/07:13	11.73	7	24.11.2012 08:19:25	2.04	0.2659±0.0033	1.9134	17.955	3.978
5G.	23.11.2012/19:29 – 24.11.2012/05:46	10.28	7	24.11.2012 08:41:19	1.16	0.2543±0.0030	1.4106	14.295	2.0469
5M.	23.11.2012/19:29 – 24.11.2012/05:46	10.28	7	24.11.2012 08:46:03	1.07	0.2561±0.0027	1.3989	13.203	1.5498
6G.	23.11.2012/19:29 – 24.11.2012/04:47	9.30	6	24.11.2012 08:37:10	0.89	0.2566±0.0027	0.9858	11.868	1.7985
6M.	23.11.2012/19:29 – 24.11.2012/04:47	9.30	6	24.11.2012 08:42:55	0.83	0.2582±0.0024	1.0974	11.046	1.3518
7G.	24.11.2012/00:24 – 24.11.2012/07:13	6.82	6	24.11.2012 08:10:19	0.54	0.2787±0.0018	0.7332	5.556	0.8397
7M.	24.11.2012/00:24 – 24.11.2012/07:13	6.82	6	24.11.2012 08:11:08	0.54	0.2842±0.0018	0.7125	5.601	0.8628
8G.	24.11.2012/00:24 – 24.11.2012/05:46	5.37	6	24.11.2012 08:45:05	1.19	0.2502±0.0093	1.7103	23.226	1.8126
8M.	24.11.2012/00:24 – 24.11.2012/05:46	5.37	6	24.11.2012 08:48:28	1.12	0.2529±0.0090	1.6554	22.155	1.5948

As follows from the tables 3-9, in the most cases the best solutions in sense of the highest accuracy of a SO' motion prediction can be obtained on the base of measurements distributed in trajectory arcs from ~0.5 till ~1 day before its re-entry. Good solutions can be obtained on shorter tracking arcs as well, for example on arc ~6 hours. However a selection of such solutions as the final re-entry prediction result may be interfered with an uncertainty of their reliability, especially if a total number of the measurements, distributed on short arcs is a small quantity and there are inconsistency of residuals of the last fulfilled measurements and residuals of all other data being treated by least square method.

In table 10 the information on main orbital parameters, obtained in the best OD solutions of the considered SOs, referred to the initial epoch  $t_0$  and duration of a prolongation interval (in orbits) up to these objects re-entry are presented.

Table 10. The orbital parameters of considered SOs at the initial epoch  $t_0$ , obtained in the best their OD solutions

№	Space object	e	$H_{max}$ km	$H_{min}$ km	$S_b$	Duration of prediction in orbits
1.	SC «Coronas-F»	0.00117	188	161	0.065	8
2.	R/B «Delta-2»	0.00603	228	150	0.068	15
3.	R/B «Vostok-2M»	0.00032	181	159	0.061	8
4.	SC «ROSAT»	0.00065	187	170	0.047	15
5.	SC «Phobos-Grunt»	0.00174	154	135	0.014	8
6.	R/B «Soyuz-FG»	0.00394	224	176	0.077	19
7.	SC «Sfera»	0.00191	215	192	0.254	9

As one can see from the table 10, duration of the final prediction interval corresponding to the best solution for considered objects, made up 8-19 orbits, in dependence on object orbit, accuracy of the measurements, flight conditions etc.



Comparing the results of the best solutions obtained at usage of different atmospheric models: GOST-2004 and NRL MSISE-2000, it is possible to establish certain differences both in fitting of the same compositions of the measurement data and in a SO' re-entry time prediction.

In table 11 for the best solution for each considered objects the following data are presented: the root mean square error  $\sigma_0$  of measurements fitting for variants of

using models GOST-2004 and model NRL MSISE-2000; the differences in the SO' re-entry time for these variants of OD solutions  $\Delta t = T_{re}^G - T_{re}^M$  ( $T_{re}^G$  - re-entry time at using model GOST-2004 (G),  $T_{re}^M$  - re-entry time at using model NRL MSISE-2000 (M)); the deviation of  $T_{re}^G$  and  $T_{re}^M$  from the officially adopted (or the most probable) re-entry time  $T_{re}^{Ad}$  for each SO. The times  $T_{re}^{Ad}$  are presented in last column of the table 11.

Table 11. Differences in the re-entry times of the considered SOs, predicted at using the dynamic atmospheric models GOST-2004 and NRL MSISE-2000.

No	Space object	RMS $\sigma_0$		$\Delta t =$	$\Delta t =$	$\Delta t =$	$T_{re}^{Ad}$ (UTC)
		GOST	MSISE	$T_{re}^G - T_{re}^M$	$T_{re}^G - T_{re}^{Ad}$	$T_{re}^M - T_{re}^{Ad}$	
1.	SC «Coronas-F»	0.48	0.94	8 min.	1 min.	-7 min.	06.12.2005 17:34
2.	R/B «Delta-2»	1.00	1.14	-16 min.	0 min.	16 min.	16.08.2007 09:30
3.	R/B «Vostok-2M»	0.65	0.90	10 min.	-3 min.	-13 min.	30.04.2010 16:54
4.	SC «ROSAT»	0.92	0.88	-10 min.	1 min.	11 min.	23.10.2011 01:57
5.	SC «Phobos-Grunt»	0.47	0.67	17 min.	4 min.	-13 min.	15.01.2012 17:53
6.	R/B «Soyuz-FG»	1.17	1.26	-12 min.	-8 min.	4 min.	24.12.2011 16:30*
7.	SC «Sfera»	1.16	1.07	-5 min.	-	-	

As follows from the table, the difference in a SO' re-entry time predictions –  $\Delta t = T_{re}^G - T_{re}^M$  may achieve a significant values at using the different atmospheric models. As one can see in the most cases the results of the re-entry predictions obtained with the use of model GOST-2004 were closer to the officially adopted ones for relevant SO.

On an example of the SC "Phobos-Grunt" it is demonstrated how some differences in the re-entry time prediction at using different atmospheric models (namely GOST-2004 and NRL MSISE-00) can be transformed in the displacement of the SO' impact area on the Earth surface. In this case the difference in the re-entry time prediction about 17 minutes generated large enough difference in a predicted impact window as it is shown in a figure 1

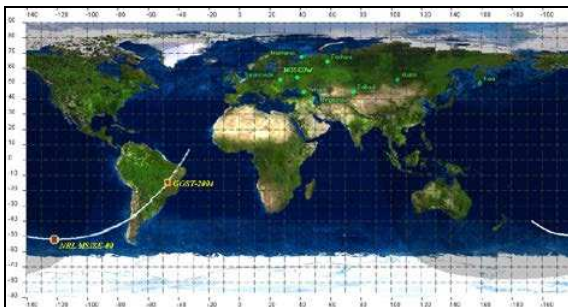


Figure 1. Differences in the SC "Phobos-Grunt" re-entry prediction, calculated for the best solutions at using atmospheric models GOST-2004 and NRL MSISE-00.

At the same time, as have shown calculations, differences in a SO' parameters of motion prediction, obtained on the base of the same measurement sets, but at using the different models of atmosphere, for the orbital part of flight are not so significant.

In table 12 the differences in the predicted parameters of the considered SOs' motion, corresponding to the epoch of the ascending nodes of the last full closed orbits (the next ones will be "impact" orbits), for variants of the best solutions, obtained at using the atmospheric models GOST-2004 and NRL MSISE-2000 are presented. Differences are referred to the orbital co-ordinate system, i.e. to a radial direction –  $\Delta R$ , to a direction along SO' track –  $\Delta N$ , and to a direction across track –  $\Delta B$ . Here for each case the altitude above the Earth' equator at passing the ascending node for the comparison orbit is presented as well.

Data resulted in the Tab. 12 testify, that at long enough SO' orbital motion prediction (which duration according to Tab. 10 could achieve 19 orbits) the differences in the predicted parameters of a SO' motion at the beginning of final closed orbit of flight for variants in which models GOST-2004 and NRL MSISE-2000 were used, in the most cases did not exceed units of kilometers.

Table 12. Differences in the orbital co-ordinates referred to an ascending node of last closed orbit at usage atmospheric model GOST-2004 and MSISE-00.

No	Space object	Altitude $H_{eq}$ for last closed orbit, km	$\Delta R$ km	$\Delta N$ km	$\Delta B$ km
1.	SC «Coronas-F»	124.5	-0.4	-0.3	1.1
2.	R/B «Delta-2»	122.0	3.2	8.4	-0.5
3.	R/B «Vostok-2M»	135.0	-0.4	0.7	-0.1
4.	SC «ROSAT»	128.7	3.1	29.5	-1.7
5.	SC «Phobos-Grunt»	119.9	-0.3	1.1	-0.1
6.	R/B «Soyuz-FG»	134.9	1.7	12.4	-0.7
7.	SC «Sfera»	146.5	0.7	4.7	-0.1

The obtained results concerning the significant differences in the predictions of the SO' re-entry parameters at using different atmospheric models when the predictions of the motion parameters within an orbital phase of SO' flight for both cases of the same using models are enough close made us search the cause of this problem.

As a result of such research it has been established that the models GOST-2004 and NRL MSISE-2000 produce the atmospheric density in different ways for different altitude level within a so-called "impact" (or unclosed) orbit when SO concludes its flight.

In Fig. 2-4 the dynamics of a density ratio  $\rho_{MSISE} / \rho_{GOST}$  calculated on the base of models NRL MSISE-2000 and GOST-2004 accordingly in space points through which SC "ROSAT", RB "Vostok-2M" and SC "Phobos-Grunt" passed during a final phase of their flights are shown. In these figures the curves representing altitudes of the indicated points of space over the Earth' surface are shown as well. On an abscissas' axis of the represented graphics in these figures the remaining lifetime till the moment when the descending object has reached the certain altitude is put aside. For the ROSAT and the Vostok-2M it was the moment  $t^0$ , corresponding to the achievement by object of zero altitude over the Earth' surface, and for the Phobos-Grunt – the moment  $t^{80}$ , in which descending object has reached altitude  $H=80$  km.

As one can see from the figures, in all cases a densities ratio  $\rho_{MSISE} / \rho_{GOST}$  for altitudes of flight over 120 km has complicated enough oscillating character, correlated with an altitude changing in corresponding points of space. Thus the amplitude of oscillation of a density ratio  $\rho_{MSISE} / \rho_{GOST}$  on this phase of flight for all objects was insignificant, and the total change of this ratio remains in limits: 0.85-1.1 - for the SC "ROSAT", 0.8-1.07 - for the RB "Vostok-2M" and 0.75-1.07 - for the SC "Phobos-Grunt". However at descending of the SO, since an altitude band of 120-110 km, the character of the ratio  $\rho_{MSISE} / \rho_{GOST}$  became sharply changed. The value of this ratio fast increased during a process of the altitude decreasing, having reached the maximum value

of the ratio  $\rho_{MSISE} / \rho_{GOST}$  at  $H \sim 100$  km. After that the densities ratio also promptly began to decrease, having reduced to the value  $\sim 1.1$  at the altitude  $H \sim 80-60$  km.

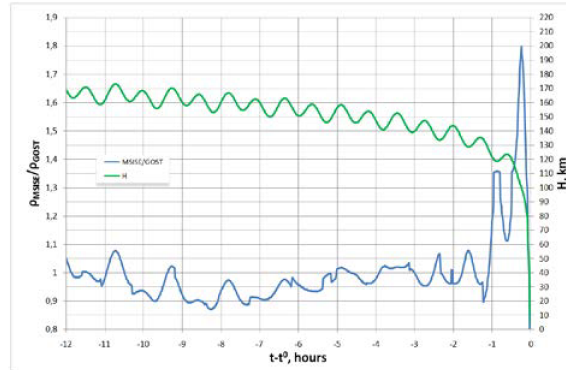


Figure 2. Dynamics of a density ratio  $\rho_{MSISE} / \rho_{GOST}$  and an altitude changing during the final phase of the SC "ROSAT" flight.

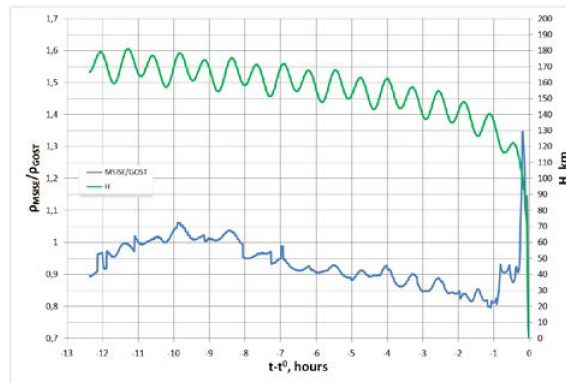


Figure 3. Dynamics of a density ratio  $\rho_{MSISE} / \rho_{GOST}$  and an altitude changing during the final phase of the RB "Vostok-2M" flight.

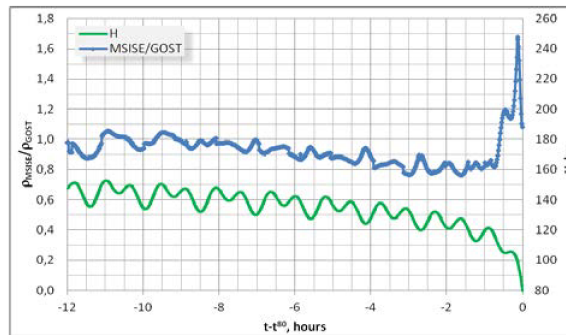


Figure 4. Dynamics of a density ratio  $\rho_{MSISE} / \rho_{GOST}$  and an altitude changing during the final phase of the SC "Phobos-Grunt" flight.

The revealed regular appropriateness in character of densities ratio  $\rho_{MSISE}/\rho_{GOST}$  along flight trajectory of a SO during its descending from an orbit was confirmed in other cases. In Fig. 5 the curves of densities ratio  $\rho_{MSISE}/\rho_{GOST}$  in dependence on an altitude for all investigated objects are presented.

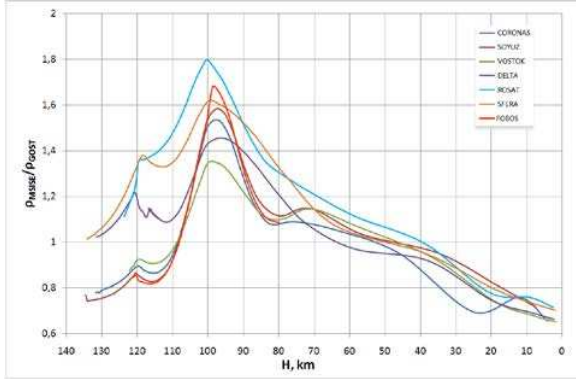


Figure 5. Dependence of a density ratio  $\rho_{MSISE}/\rho_{GOST}$  on an altitude along flight trajectories of the investigated SOs.

The revealed differences in the values of atmospheric densities represented by dynamic models GOST-2004 and NRL MSISE-2000 in the areas of low altitudes of SO' flight, with sharp let in a character of behavior of the ratio  $\rho_{MSISE}/\rho_{GOST}$  on a descending phase in an altitudes range 110-80 km is, most likely, the reason of a significant difference in a SO' re-entry time predicted at use of one and other model, even when both these models provided close enough results in the description of SO' motion on an orbital phase at identical initial data.

Analyzing Fig. 5 it is easy to establish, that the maximum values of atmospheric densities ratio –  $\max(\rho_{MSISE}/\rho_{GOST})$  have different values for the different considered cases, corresponding a different SO. The maximum value  $\max(\rho_{MSISE}/\rho_{GOST})$  corresponds to a case of the SC "ROSAT" re-entering, here  $\max(\rho_{MSISE}/\rho_{GOST}) \approx 1.8$ , and minimum value - to a case of the Vostok-2M re-entering, when  $\max(\rho_{MSISE}/\rho_{GOST}) \approx 1.35$ .

The reason of such differences in values  $\max(\rho_{MSISE}/\rho_{GOST})$  in dependence on SO' characteristics and conditions of its flight demands a special investigation. Very likely that first of all it should be established a correlation between values  $\max(\rho_{MSISE}/\rho_{GOST})$  and level of solar and geomagnetic activity during the periods of a SO descends from its orbit. In Tab. 13 data about values  $\max(\rho_{MSISE}/\rho_{GOST})$ , revealed at considered in the present paper SO re-entries, and values of solar ( $F_{10.7}$ ) and geomagnetic ( $A_p$ ) activity indexes during the end of life of these SOs.

Table 13. The maximum values of densities ratio  $\rho_{MSISE}/\rho_{GOST}$  and values of solar and geomagnetic activity indexes used in dynamic models of atmosphere, at re-entering of the considered SO.

No	Space object	max $\rho_{MSISE}/\rho_{GOST}$	Current $F_{10.7}$	Mean $F_{10.7}$		$A_p$
				GOST	MSISE	
1.	SC «Coronas-F»	1.53	92	83	86	3
2.	R/B «Delta-2»	1.45	68	71	69	7
3.	R/B «Vostok-2M»	1.35	77	80	75	5
4.	SC «ROSAT»	1.80	164	126	145	4
5.	SC «Phobos-Grunt»	1.70	130	142	127	5
6.	R/B «Soyuz-FG»	1.59	142	144	137	3
7.	SC «Sfera»	1.63	126	122	118	7

The data, presented in the Tab. 13, indicate on a certain relation between maximum value of densities ratio  $\rho_{MSISE}/\rho_{GOST}$  and level of solar and geomagnetic activity indexes. However most likely it takes place influences of maximum value of densities ratio by other factors. It can be, for example, flight conditions of a SO on a final phase: flight over land or over ocean, was SO on light or in shadow, a season when SO finished its flight, etc.

#### 4 CONCLUSION

As a result of the carried out investigations on influence of various factors on the final re-entry prediction of uncontrollable space objects descending from orbits on examples of the SOs for which officially adopted high accuracy of re-entry time and impact coordinates predictions has been achieved, following outputs are obtained.

1. It is established that the accuracy of the final results of re-entry prediction depends on compositions of the measurement set used for SO' OD task that are fulfilled within 1-1.5 day before a SO' re-entry.
2. Some criterions for selection of the best solution of OD for SO' re-entry prediction from the possible variants are offered.
3. The cause of differences taking place in the SO' re-entry time prediction at using the best atmospheric models GOST-2004 and NRL MSISE-00 is revealed. For the 7 considered have been re-entered objects such a kind differences made 8-17 minutes. Only part of indicated differences depends on values of solar and geomagnetic activity indexes.
4. The main reason of the revealed differences in the re-entry time prediction is in fact that the atmospheric models GOST-2004 and NRL MSISE-00 produce different density when SO moves in altitudes range 120 - 80 km.
5. On the base of the fulfilled investigations it is possible to make a conclusion that in the most considered cases the best results were obtained by using model GOST-2004.

## 5 REFERENCES

1. Ivanov, N.M., Kolyuka, Yu.F., Afanasieva, T.I. & Gridchina, T.A. (2007). Coronas-F Orbit Monitoring and Re-Entry Prediction. In *Proc. 20<sup>th</sup> ISSFD 'International Symposium on Space Flight Dynamics'*, GSFC, NASA/CP-2007-214158, Annapolis, MD, USA.
2. Kolyuka, Yu.F., Afanasieva, T.I., Gridchina, T.A. & Oleinikov I.I. (2012). Arrangement and results of the Phobos-Grunt emergency flight monitoring and its re-entry impact window estimation in Russian Mission Control Center. In *Proc. 23<sup>th</sup> ISSFD 'International Symposium on Space Flight Dynamics'*, JPL, Pasadena, California, USA.
3. Kolyuka Yu. F., Margorin O.K. (1995). The new high-effective method for numerical integration of space dynamics differential equation. In *Proc. 'Spaceflight Dynamics conference'*, CNES, Cepadues-Editions, Toulouse, France.