SEMI-ANALYTICAL COMPUTATION OF PARTIAL DERIVATIVES AND TRANSITION MATRIX USING STELA SOFTWARE

Vincent Morand, Juan Carlos Dolado-Perez, Hubert Fraysse [(1)], Florent Deleflie, Jérôme Daquin [(2)], Cedric Dental [(3)]

[(1)] CNES, Toulouse Space Center, Toulouse, France, Vincent.Morand@cnes.fr
[(2)] IMCCE, Paris, France, Florent.Deleflie@imcce.fr, Université Lille 1, Lille, France
[(3)] THALES, C4I Systems Division Toulouse, France, Cedric.Dental@thalesgroup.com

ABSTRACT

The paper details the semi-analytical method implemented in the STELA software for the state transition matrix propagation. STELA (Semi-Analytical Tool for End of Life Analysis) is the CNES reference tool for long term orbit propagation.

The use of a semi-analytical method for state transition matrix propagation is particularly suited to perform orbit determination for a very large number of objects, using space debris catalogue, which will continue to grow up by the next decades. The advantages of using simplified force models in the computation of state transition matrix are presented.

1 INTRODUCTION

Space Debris mitigation has become a major concern for many space agencies around the world. Very long term orbit propagation techniques (up to one hundred years) as well as the ability to deal with increasing number of space debris are required for studies in the frame of Space Situational Awareness. Semi-analytical methods are now commonly used since they represent a fair compromise between precision, needed to get representative results, and computational efficiency.

Long term orbit propagation techniques using semi-analytical methods have been implemented in the STELA software, which is presented in the second section of this paper. It allows the propagation of a state vector in an efficient way over very long time scales. The latest STELA version propagates the orbital elements and their sensitivity (through a set of variational equations) at the same time. The transition matrix of the orbital state is computed following a semi-analytical approach that is explained in the third section. Practical examples of the benefits of semi-analytical methods for state transition matrix propagation are given in the fourth section of the paper. It is particularly suited to perform orbit determination using a space debris catalogue, with a very large number of objects, since it allows the orbit determination process to run much faster.

2 STELA SOFTWARE

2.1 General information

STELA (Semi-Analytical Tool for End of Life Analysis) is the CNES reference tool for long term orbit propagation. It has been developed to assess the compliance of disposal orbit against the French Space Operations Act, in line with IADC (Inter-Agency Space Debris Coordination Committee) recommendations, through the removal of non-operational objects from populated regions. More information on the use of STELA in the frame of the French Space Operations Act are presented in [1] and [2]

The STELA propagator is also currently used in Space Situational Awareness applications: as an early warning and crisis management tool, CNES has developed and implemented an algorithm to detect and monitor the short and middle term uncontrolled re-entry of space objects, OPERA. This algorithm takes STELA as the dynamical model, and uses it during the filtering stage of the orbit determination process as well as once the state vector has been estimated to propagate the space object up to re-entry. A practical application of OPERA is presented in the fourth section of this paper. See [3] to have more details on OPERA methodology.

STELA propagator is also used in the MEDEE software to propagate the space debris population over long time scales. MEDEE is the CNES orbital debris evolutionary model [4].

2.2 Dynamical model

The idea that underlined the STELA development was to take into account only the perturbations that have a significant effect on long term orbit evolution, with accuracy sufficient to assess the compliance with the French Space Operations Act. Therefore, three dynamical models have been established, each one adapted to one orbit type: Low Earth Orbit (LEO), Geostationary (GEO) or Geostationary Transfer Orbit (GTO). To ensure reasonable CPU integration times, the long time scale analysis is based on the numerical integration of equations of motion, where the short

Proc. '6th European Conference on Space Debris'
Darmstadt, Germany, 22–25 April 2013 (ESA SP-723, August 2013)

periodic terms have been removed by means of an analytical averaging for conservative perturbations and numerical averaging for dissipative perturbations. This allows the use of a very large integration step size, typically 24h, reducing significantly the total time of computation. As an example the typical computation time is about one minute for a 100 year propagation. The averaging approach follows methods developed in the theory of mean orbital motion ([5]) and derived for orbits with very small eccentricities, removing all divisions by the eccentricity in the mean equations of motion ([6]). The corresponding perturbation equations have been written, namely the Planetary Lagrange equations for perturbations deriving from a potential (internal gravity field, moon-sun perturbations), and the Gauss equations (for the atmospheric drag and solar radiation pressure). In both cases, the averaged forces (over the rapid variable) are inserted into the equations of motion, which are, consequently, mean equations of motion. The mean potential $\overline{U}$ (except for the J2 coefficient where a term proportional to $J2^2$ is explicitly added) is computed once for all in an analytical way, from the expression of the osculating potential U:

$$\overline{U} = \frac{1}{2\pi}\int_0^{2\pi} U dM \quad (1)$$

The mean effect of dissipative perturbations on each orbital parameter is evaluated through a numerical Simpson quadrature method considering *n* constant intervals in true or eccentric anomaly (depending on the perturbation) along one orbit.

The variational equations have been implemented from the GTO dynamical model (which is more complete and generic than LEO and GEO model, with no eccentricity limitation). Therefore, we will not present the LEO and GEO model here, see [1] for more information. The GTO mean dynamical model implemented in STELA is given in Tab. 1.

Table 1: Mean dynamical model

Perturbation	GTO orbit type
Earth's gravity field	Complete 7x7 model (Including $J2^2$. Tesseral terms taken into account only when tesseral resonances are detected)
Solar and Lunar gravity	Yes (up to degree 5)
Atmospheric drag	Yes (MSIS00 atmospheric model, rotating atmosphere)
Solar radiation pressure (SRP)	Yes (including cylindrical Earth shadow)

Note that the effect on orbit eccentricity vector of additional zonal terms of the geopotential has to be considered when the inclination is close to the critical inclination (63.4 deg for prograde orbits).

The equations have been written in a generic way that allows low and high eccentricity values and any inclination except 180 deg. The set or orbital elements is:

$$E = \begin{pmatrix} a \\ \Omega + \omega + M \\ e\cos(\Omega + \omega) \\ e\sin(\Omega + \omega) \\ \sin \frac{i}{2} \cos \Omega \\ \sin \frac{i}{2} \sin \Omega \end{pmatrix} \quad (2)$$

With a standing for the semi-major-axis, e the eccentricity, i the inclination, Ω the Right Ascension of Ascending Node, ω the Argument of Perigee and M the mean anomaly.

Osculating elements are needed for comparison with results coming from numerical integration as well as in the drag force perturbation computation or to retrieve the "true" spacecraft position. Therefore, an explicit analytical transformation from mean to osculating elements and conversely has been developed through the set of orbital elements E. The GTO model of short periodic terms contains the perturbations as described in Tab. 2.

Table 2: Short period model

	GTO orbit type
Short periodic terms	J2, Solar and Lunar gravity (degree 2)

More information on STELA and its validation are given in [1].

3 VARIATIONAL EQUATIONS

The latest STELA version (v2.4.2) propagates the orbital elements and their sensitivity at the same time, through a set of variational equations that are detailed in this section.

We can write the following ordinary differential equation (ODE)

$$\frac{dE}{dt} = f(E,t) \quad (3)$$

With E being the set of orbital mean elements and f is the mean elements rate taking into account the dynamical model described in Tab. 1.

The variational equations give the influence of the initial conditions $E(t_0)$ and force model parameters k_d and k_p on the current state E. k_d and k_p are multiplying factors for drag force and solar radiation pressure respectively, commonly used in an orbit determination process as correction factors for drag and solar radiation pressure coefficients.

The variational equations are:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial E(t_0)}\right) = \frac{\partial f}{\partial E}\frac{\partial E}{\partial E(t_0)}$$
$$\frac{d}{dt}\left(\frac{\partial E}{\partial k_{p,d}}\right) = \frac{\partial f}{\partial E}\frac{\partial E}{\partial k_{d,p}} + \frac{\partial f}{\partial k_{d,p}} \quad (4)$$

Note that second set of equations is not needed if we are interested only in the influence of initial conditions E_0. The variational equations are solved throughout the orbit propagation to preserve consistency. The dimension of the state vector is then 54 (6 orbital elements plus 48 partial derivatives). We can arrange the propagated partial derivatives $\frac{\partial E}{\partial E(t_0)}$ and $\frac{\partial f}{\partial k_{p,d}}$ to form a matrix Φ referred to as the state transition matrix

$$\Phi = \begin{bmatrix} \frac{\partial E_1}{\partial E_1(t_0)} & \cdots & \frac{\partial E_1}{\partial E_6(t_0)} & \frac{\partial E_1}{\partial k_d} & \frac{\partial E_1}{\partial k_p} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{\partial E_6}{\partial E_1(t_0)} & \cdots & \frac{\partial E_6}{\partial E_6(t_0)} & \frac{\partial E_6}{\partial k_d} & \frac{\partial E_6}{\partial k_p} \end{bmatrix} \quad (5)$$

The solve-for vectors have the following initial conditions:

$$\frac{\partial E}{\partial E(t_0)}(t_0) = I_6$$
$$\frac{\partial E}{\partial k_{p,d}}(t_0) = \{0\} \quad (6)$$

With I_6 identity matrix and {0} null matrix (explained considering that a change in the force model parameters does not affect the initial value of mean orbital elements).

3.1 Force model parameters

The f derivatives with respect to a force model parameter are straight forward (assuming a non-zero value):

$$\frac{\partial f}{\partial k_{d,p}} = \frac{1}{k_{d,p}} f_{d,p} \quad (7)$$

3.2 Dissipative perturbations

For dissipative perturbations the Gauss equations G are used and the mean elements rate is evaluated through a numerical quadrature method along the orbit (using either v, u or M, respectively true, eccentric and mean anomaly). Calling F the dissipative acceleration we have:

$$f = \frac{dE}{dt} = \frac{1}{2\pi}\int_0^{2\pi} G.F\, d \begin{vmatrix} v \\ u \\ M \end{vmatrix} \quad (8)$$

We can permute the partial derivative sign and integration sign since the quadrature is done on a finite number of intervals, we obtain:

$$\frac{\partial f}{\partial E} = \frac{1}{2\pi}\int_0^{2\pi} \frac{\partial G}{\partial E} F + G\frac{\partial F}{\partial E} d \begin{vmatrix} v \\ u \\ M \end{vmatrix} \quad (9)$$

The Gauss equations G have been compute analytically, so we can easily compute their first derivatives as well. The dissipative acceleration partial derivatives may be more tedious to compute analytically. Let us consider the drag acceleration:

$$\vec{F}_{drag} = -\frac{1}{2}\rho\frac{S}{m}CdVr\overrightarrow{Vr} \quad (10)$$

With:
ρ: Atmospheric density
S: Cross sectional area
m: Spacecraft mass
Cd: Drag coefficient
Vr: Spacecraft velocity relative to the atmosphere

The main uncertainty in the computation of the drag acceleration partial derivatives comes from the derivatives of the atmospheric density that cannot be derived analytically in most of the case. Then, a finite difference technique has to be adopted.

There is no particular difficulty in the computation of the partial derivatives of solar radiation pressure acceleration.

3.3 Conservative perturbations

For conservative perturbations the Planetary Lagrange equations L and the mean potential $\overline{U}$ defined in Eq. 1 are used to compute the orbital element derivatives:

$$f = \frac{dE}{dt} = L.\frac{\partial \overline{U}}{\partial E} \quad (11)$$

Consequently for conservative perturbations we have:

$$\frac{\partial f}{\partial E} = \frac{\partial L}{\partial E} . \frac{\partial \bar{U}}{\partial E} + L . \frac{\partial^2 \bar{U}}{\partial E^2} \quad (12)$$

The Lagrange equations L and the mean potential $\bar{U}$ have been computed analytically, so we can easily compute their first and second order derivatives with respect to orbital elements as well.

3.4 Implementation in STELA software

STELA uses the set of equinoctial elements E. The corresponding Lagrange and Gauss equations have been written as well as their derivatives. Tab. 3 gives the dynamical model implemented in STELA for variational equations.

Table 3: Dynamical model for variational equations

Perturbation	Variational equations model
Earth's gravity field	Zonal terms up to J7 (Including J2²)
Solar and Lunar gravity	Yes (up to degree 4)
Solar radiation pressure (SRP)	Yes (conservative perturbation, no shadow)
Atmospheric drag	Yes (Finite difference for atmospheric density variations)

Solar radiation pressure is modelled as a potential to save computation time. Further improvement may change this to take into account the eclipse periods, even though the current implementation is accurate enough for our main application (see next section). There is no theoretical difficulty into considering a dissipative solar radiation pressure in the variational equations.

Drag is the only dissipative perturbations taken into account in the current variational equations force model. The MSIS-00 model is used to compute the atmospheric density. MSIS-00 is not an analytical model. As a consequence, a single-sided finite difference technique is used to compute the atmospheric density derivatives with respect to position. The derivatives of drag acceleration (taking into account a rotating atmosphere and the projection in a local orbital frame) are computed with respect to Cartesian coordinates and then converted to derivatives with respect to equinoctial elements using the Jacobian matrix of the Cartesian to equinoctial transformation. It would also have been possible to perform the finite difference technique on the whole drag acceleration computation (and not only on atmospheric density computation) to obtain the drag partial derivatives, but it seemed less accurate since the small deltas to consider, especially for velocity, might be hard to justify.

One can note that all the partial derivatives with respect to the fast variable (ω+Ω+M) will be zero since we are considering perturbations that have been averaged over one orbit (the mean element rates do not contain the fast variable). No partial derivatives of short period perturbations have been considered yet. As a consequence, the actual model allows mapping deviation from one time to another on mean elements only. The osculating state transition matrix cannot be computed here. It is not a problem in our main application (see next section) since the state transition matrix is used in an orbit determination scheme in mean elements. However, further improvements may include the short periodic motion partial derivatives since there is no theoretical difficulty in their computation.

The semi-analytical scheme adopted for the state transition matrix propagation allows a large integration step. Since the variational equations are solved throughout the orbit propagation, the natural choice is the STELA's default integration step: 24H. Tab. 4 is an example of the impact of the dynamical model for state transition matrix propagation on computation time. The dynamical model considered in the transition matrix propagation can be adapted (i.e. simplified) by the user to save computation time. The test case is a GTO propagation over 10 years (Intel Core i3, 2.1Ghz, 3.4Go RAM, Windows XP).

Table 4: Dynamical model and computation time

Dynamical model for variational equations	Computation time
None: STELA simple orbit propagation (dimension of state vector is 6)	12 s
Earth's gravity field only	19 s
Earth, Moon and Sun gravity only	27 s
Complete without drag	28 s
Complete (same as in Tab. 3)	36 s

One can see that computing the state transition matrix from the variational equations triples the computation time with respect to a simple orbit propagation. It is worth noting that it is much faster than using a finite difference technique. Indeed, for a single-sided finite difference technique, one has to perform a nominal propagation then apply one small delta on each one of the six initial orbital elements and two force model parameter $k_{d,p}$. It represents nine simple orbit propagations. For a double-sided difference technique (plus or minus a small delta for each parameters) it represents 16 simple orbit propagations. The gain of using the variational equations is obvious in computational efficiency, not to mention that the semi-analytical approach yields expressions that are generally

more accurate than the finite-differencing approaches. As expected, drag perturbation is the most time-consuming perturbation since a finite difference scheme is necessary to compute the atmospheric density variation. In this case, Moon and Sun gravity perturbation also took some computational time since the full model (up to the fourth degree in the development) was considered. One can note that solar radiation pressure has no impact on the computation time since it is modelled as a potential in the variational equations.
Validation of the implementation of the variational equations in STELA software has been done mainly through comparison with results coming from finite difference techniques. The use of the state transition matrix in the OPERA software (see next section) also gives us confidence about our model.

4 PRACTICAL EXEMPLES

State transition matrices are commonly used in orbit determination processes or for covariance matrix propagation. The best example of the benefits of using the STELA software, and consequently a semi-analytical scheme for state transition matrix propagation, is given by the OPERA software presented hereafter. Another example is given for covariance matrix propagation to illustrate the possibilities of STELA implementation, although this issue have not been studied extensively yet.

4.1 Covariance matrix propagation

State transition matrices can be used to propagate a covariance matrix P (modelling injection or estimation errors):

$$P(t) = \Phi(t)P(t_0)\Phi(t)^T \qquad (13)$$

For validation purpose we computed the R quantity defined by Rice [7]

$$R(t) = \sqrt{\sum_{i=1}^{3}\sum_{j=1}^{3} {\Phi_{CAR}}_{ij}^{\,2}(t)} \qquad (14)$$

With Φ_{CAR} standing for a state transition matrix in Cartesian elements. Rice interprets R as a measure of “error growth rate”, corresponding to the linear propagation of a covariance matrix whom only non-zero values are on the position components of state (in Cartesian elements):

$$P(t_0) = \begin{pmatrix} \sigma^2 I_3 & 0_3 \\ 0_3 & 0_3 \end{pmatrix} \qquad (15)$$

The test case given by Rice is a low circular equatorial orbit propagated with only the J2 perturbation. Fig 1. is a plot of the R values computed from:

1) Numerical propagation. The orbit is propagated with a Cowell numerical propagator using a small step size (60s). The state transition matrix is computed from single-sided finite differencing technique to evaluate the state transition matrix Φ_{CAR} (as a results, 7 orbit propagations are performed)
2) Semi-analytical propagation of the orbit and state transition matrix Φ using the STELA software. The transition matrix computed by STELA is in equinoctial elements, then it is converted in cartesian elements using the Jacobian J of the cartesian to equinoctial transformation:

$$\Phi_{CAR}(t) = J(t)^T . \Phi(t) . J(t_0) \qquad (16)$$

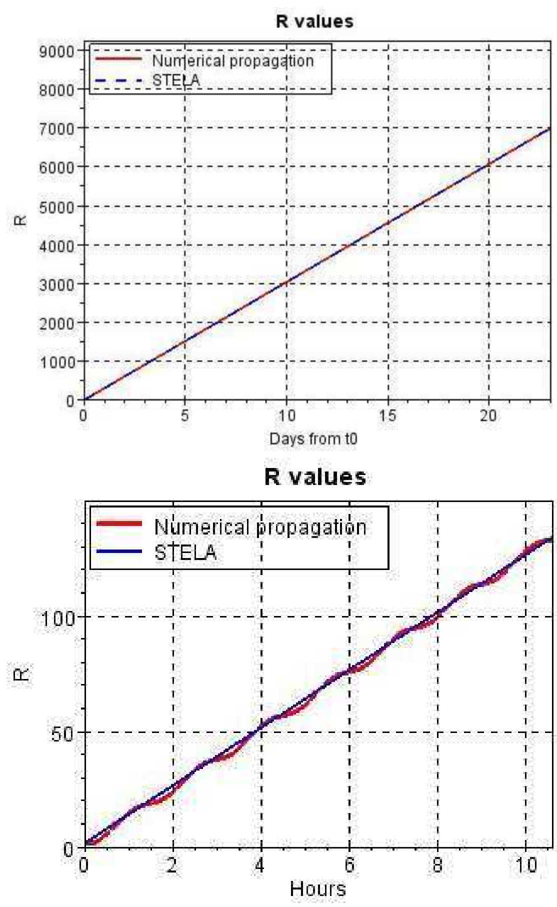

Figure 1. R value time history, Rice test case.

One can see the very good match over several days between R values from numerical and semi-analytical propagation. Computed values are consistent with [7]. The bottom plot illustrates the difference between mean and osculating state transition matrix (let us remind that no short periodic motion is considered in the STELA variational equations).

Even though this issue has not been extensively studied here, covariance matrix propagation using semi-analytically computed state transition matrix in equinoctial elements is promising, since studies show that working natively in equinoctial element space rather than in Cartesian representation has a significant impact on the consistency between propagated covariance and actual state error distribution [8].

4.2 Orbit Determination

As an early warning and crisis management tool, CNES has developed and implemented an algorithm to detect and monitor the short and middle term uncontrolled re-entry of space objects, OPERA. In order to cover a space objects population as large as possible, CNES makes use, in addition to the French space debris catalogue, of the United States Strategic Command (USSTRATCOM) public catalogue (https://www.space-track.org). To forecast the uncontrolled re-entry of a great number of objects, which are orbiting in the lower region of Low Earth Orbit as well as on GTO, it is required to dispose of an eccentricity / inclination singularity-free orbital propagator, with a high degree of computational efficiency. STELA propagator in library mode has been chosen, and is used during the filtering stage of the orbit determination process as well as once the state vector has been estimated to propagate the space object up to re-entry. Note that when the lifetime estimated by OPERA is under a threshold value (a week, for example); the data are transmitted to an operational entity that monitors the re-entry using a precise numerical propagator. The OPERA algorithm is divided into 5 steps:

1) External Data Filtering: from the external catalogue the TLE time series have to be filtered (Outliers detection and suppression, orbital maneuvers to detect controlled spacecraft, etc.)
2) Initial Conditions Estimation: a first guess of reasonable quality is needed in the non-linear orbit determination. The ballistic coefficient initial guess estimation comes from energetic considerations related to the decrease rate of semi-major-axis.
3) State Transition Matrix computation with method of variational equations of section 3: being able to map deviations on the state vector from one time to another is needed in the differential correction estimation problem
4) Orbit determination with differential correction: an orbit determination algorithm in which the orbital elements from the catalogue are directly considered as measurements has been developed (use of mean elements)
5) STELA propagation up to re-entry: once the unknown initial state has been estimated, the state vector is propagated up to re-entry in order to evaluate if the spacecraft is re-entering within the targeted timeline (typically a few months).

A precise description of OPERA, its methodology and assumptions as well as accuracy results are given in [3] and won't be discuss here. Let us remind that once initial conditions of a reasonable quality have been estimated, the non-linear estimation problem can be linearized using a Taylor's series expansion about the reference trajectory and becomes a least-squares estimation problem. It is worth noting that OPERA algorithm is optimized from a computation time point of view by the implementation of the differential correction algorithm in mean elements, both for the observations as for the unknown state vector. As a consequence we need to be able to translate the observations (TLEs) from TLE mean elements to STELA mean elements. For orbit with small eccentricities the best way to do so is to use SGP-SDP theory to convert TLE mean elements to osculating, and then to use STELA short periods model to convert osculating elements to STELA mean elements. However, the SGP-SDP conversion to osculating parameters is not valid for high eccentricities due to limitations in the SGP4-SDP4 short period model. Then, for such orbits, it is better to assume that STELA mean elements are equals to TLE mean elements without using osculating conversion. In addition to this, we have to be able to compute the state transition matrix to relate the mean observation made at different times to the unknown initial state that we are willing to estimate. In the first OPERA version the state transition matrix was computed numerically by performing several propagations applying small deltas to the initial state. What we are interested in this paper is the gain brought by the use of variational equations, implemented in the last STELA release, for the state transition matrix computation. Note that the state transition matrix is compute on a fixed-step (typically 24H) integration time grid; a Lagrange interpolation is then performed to compute the state transition matrix at measurements (TLE) epoch. The orbit determination process stops when the weighted root mean square of the residual state vector does not change enough from one iteration to another or is below a threshold value.

As an example, we consider the object n°29518 in the USSTRATCOM public catalogue. It is a rocket part in GTO that encountered an uncontrolled re-entry on November 7th, 2012. Fig. 2 shows the semi major axis evolution from May 15th 2012 up to re-entry (TLE history).

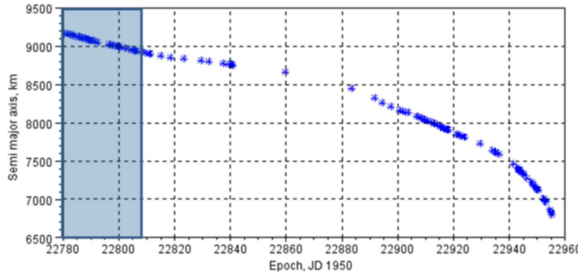

Figure 2: Semi major axis evolution from object 29518

The blue area in Fig.2 shows the one-month screening period used hereafter. We performed several re-entry predictions using the OPERA software: the impacts of the method to compute the state transition matrix (finite difference or semi-analytical) as well as the force model (full or simplified) are presented. Only the force model for state transition matrix propagation changes: we always consider a full model (described in Tab. 1) for orbit propagation. Tab. 5 gives the test case initial conditions and assumptions.

Table 5: OPERA test case

Object	**n°29518 (GTO)**
Re-Entry effective date	07/11/12
Screening period	15/05/12 to 14/06/12 (5 months before re-entry)
Orbit data	Observations (i.e. TLE)
Solar activity	Real data (a-posteriori measured values)
Atmospheric model	MSIS-00
Drag coefficient	STELA default file i.e. drag coefficient as a function of altitude

Tab. 6 gives the information brought by the first TLE (at the 15/05/12 epoch) as well as the result of the orbit determination process, considering a state transition matrix computed from finite difference technique and semi-analytical method. OPERA orbit determination output is the state vector and force model parameters at the first TLE date. Further improvements such as the ability to perform backwards in time propagation would allow changing the epoch of the estimated parameters. The orbit parameters are mean parameters in TEME frame at first TLE epoch.

Table 6: Results of orbit determination

	TLE at epoch	**Finite difference**	**Semi-analytical**
Semi-major-axis, km	9166.659	9161.689	9161.864
Eccentricity	0.27649	0.27672	0.27681
Inclination, degrees	88.3984	88.4015	88.3974
Area to mass ratio m²/kg	-	0.284	0.260
Number of iterations before convergence	-	4	5

Tab. 7 gives the impact of the method and force model for state transition matrix computation on the estimated re-entry date and computation time (Intel Core i3, 2.1 GHz, 3.4Go RAM, Windows XP).

Table 7: OPERA test case results

State transition matrix computation method and force model	**Re-entry date (Error)**	**Computation time**
Finite differences (Numerical)	15/10/12 (15%)	630 s
Semi-Analytical Full model (as in Tab. 3)	21/10/12 (11%)	217 s
Semi-Analytical J2, J3, Drag, SRP, Sun	21/10/12 (11%)	190 s
Semi-Analytical J2, Drag, SRP	21/10/12 (11%)	162 s

We can see that using a semi-analytical method to propagate the state transition matrix makes OPERA run about three times faster than using a finite difference scheme (even though one more iteration is needed for orbit determination convergence). There is no loss in the accuracy of the re-entry date, proving that this method fits well in OPERA algorithm. The error percentage on the re-entry date is consistent with previous OPERA results [3]. It is worth noting that the dynamical model for state transition matrix propagation can be significantly reduced up to the simplest one: J2, drag and SRP that are necessary to estimate the force model parameters. In this case there is no impact on the re-entry date whereas the computation time is greatly

reduced. More generally, one can adopt a very simple model for state transition matrix propagation in an orbit determination process: it is a compromise between the number of iterations needed for convergence and the computation time for a single iteration. More complete investigations are still on-going to determine what force model offers the best compromise between precision and computational efficiency.
The implementation of variational equations in the STELA software allowed OPERA to significantly reduce its computation time to monitor the short and middle term uncontrolled re-entry of objects from French space debris and public catalogue.

5 CONCLUSION AND FUTURE WORK

The implementation of the variational equations in the STELA software allows the propagation of the orbital elements and their sensitivity at the same time. A complete force model is available for the partial derivatives computation and has been presented in this paper. The use of a semi-analytical theory makes the propagation very efficient from a computation time point of view.

The STELA propagator is used in the OPERA software to monitor the short and middle term uncontrolled re-entry of space objects from public space debris catalogue. The state transition matrix, needed in the orbit determination process, is now computed from the semi-analytical variational equations. It allows the orbit determination process to run much faster without losing precision on the estimated parameters. More complete investigations are still on-going to determine what force model, considered for the state transition matrix propagation, offers the best compromise between precision and computational efficiency.

Some possible improvements have been identified, with future work which could be: improvement of the force model available in the variational equations (short periodic motion, tesseral terms, eclipse period for the solar radiation pressure, etc.), backwards in time propagation, implementation of the semi-analytical theory in a Kalman filter estimation process, etc.

More generally, the benefits of using semi-analytical method for state transition matrix propagation still need to be further analysed. In particular, covariance matrix propagation or orbital resonance detections are to be investigated.

6 REFERENCES

1. Fraysse, H et al. (2012). STELA A TOOL FOR LONG TERM ORBIT PROPAGATION. *5th International Conference on Astrodynamics Tools and Techniques*
2. Le Fèvre et al. (2012). COMPLIANCE OF DISPOSAL ORBITS WITH THE FRENCH SPACE ACT: THE GOOD PRACTICES AND THE STELA TOOL. *63rd International Astronautical Congress , Naples*.
3. Dolado-Perez et al. (2012.) Short and Middle Term Uncontrolled Re-entry Monitoring Using Public Space Debris Catalogues. *AIAA/AAS Astrodynamics Specialist Conference*
4. Dolado-Perez et al. (2013) Introducing MEDEE – A new orbital debris evolutionary model. *6th ESA conference on Space Debris*.
5. Metris, G., Exertier, P., (1995) Semi--analytical Theory of the Mean Orbital Motion, *Astron. Astrophys., 294, 278-286,1995*
6. Deleflie et al. (2010) SAT-light: un intégrateur semi-analytique comme contribution de de l'IMCCE à la Loi sur les Opérations Spatiales ("L.O.S."), *Comptes rendus des Journées Scientifiques de l'IMCCE*
7. Rice (1967). An investigation into the effects of using simplified force models in the computation of state transition matrices. *AIAA 5th Aerospace meeting*.
8. Sabol (1996). Linearized orbit covariance generation and propagation analysis via simple monte carlo simulations. *AAS/AIAA Spaceflight Mechanics Meeting*.