SPACE DEBRIS ORBIT DETERMINATION METHOD WITH THE USE OF ONBOARD OPTICAL SENSORS OF SPACE VEHICLES

Principal Author Sokolov N.,(1) Co-Author Ivanov V.,(1) Co-Author Nosova K.,(2) Co-Author Selezneva I.(3)

(1,1,2,3)Mission Control Centre, Federal State Unitary Enterprise "Central Research Institute of Machine Building"
4, Pionerskaya St., Korolev, 141070, Moscow region, RUSSIAN FEDERATION
E-mail: snl@mcc.rsa.ru

ABSTRACT

The analytical method is proposed for determining the parameters of space debris orbits avoiding the iterative calculation process. The initial information is the measurements of absolute magnitude and orientation of the vector connecting the center of mass of operated space vehicle and the position of space debris fragment. The calculation errors are estimated depending on the initial data.

1 INTRODUCTION

Collisions with space debris pose a serious threat to flight safety of operated space vehicles as well as with its small and medium-sized pieces in the space debris environment characterized by high levels of space debris density. Practice shows that detection of space debris up to 10 cm by ground-based facilities is rather difficult. That is why there is a necessity of searching other ways for detection of hazardous approaches of space vehicles with different space objects.

One of the prospective methods to solve this problem is forecasting of dangerous close approaches of space vehicles with space debris using onboard facilities. Its implementation efficiency is determined by the possibility of accurate and prompt definition of space debris motion parameters using onboard optical sensors.

Orbit determination of space objects, that fulfill the given boundary conditions, is one of the main problems in the flight mechanics. There are certain different methods of orbit determination in the two-body problem [1-5]. The use of these methods is connected with the conducting of iterative computational processes. The analytical method is proposed below for calculation the values of orbital elements by the end formulae, which reduces the duration of calculations in ~ 5-8 times.

2 PROBLEM DEFINITION

It is supposed that on conditions that space debris fragments are in the visibility zone of optical sensors installed on board of a space vehicle, it is possible in time points t_i to measure distances to a piece of space debris Δr and slope angles of vector $\tilde{r}$ to the local horizon ϑ and motion plane of space vehicle β.

By the known values of radius-vector of space vehicle r_{sv} and measurements of Δr and ϑ we calculate the radius-vector of space debris r_{sd} and angular distance δr between the current positions of space vehicle and space debris:

$$r_{sd} = \sqrt{r_{sv}^2 + \Delta r^2 - 2r_{sv}\Delta r\, cos\left(\frac{\pi}{2} + \vartheta\right)},$$

$$\delta r = arcsin\left[\frac{\Delta r}{r_{sd}} sin\left(\frac{\pi}{2} + \vartheta\right)\right].$$

We calculate angular distance δr_{sv} between the vectors $r_{sv}(t_i)$ and $r_{sv}(t_{i+1})$ for two following one after another sampling instants. Considering the spherical triangle formed by the arcs, connecting space vehicle positions in the moments t_i and t_{i+1} (δr_{sv}), space vehicle and space debris positions with $t_i(\delta r)$, and also space vehicle positions at the moment t_{i+1} and space debris positions at the moment $t_i(\sigma)$, we define the arc σ by the formulae of spherical trigonometry:

$$\sigma = \arccos[\cos\delta r(t_i) cos\,\delta r_{sv} -$$

$$- \sin\delta\, r(t_i) \sin\delta\, r_{sv} \cos\beta\,(t_i)]$$

and the angle δ between the arcs δr_{sv} and σ:

$$\delta = \ arcsin\left[\sin\delta\, r(t_i) \frac{\sin\beta(t_i)}{\sin\sigma}\right].$$

Then considering the spherical triangle formed by the arc σ, and also the arcs connecting the positions of space vehicle and space debris at the moment $t_{i+1}(\delta r)$ and space debris positions at the moments t_i and $t_{i+1}(\Delta\nu)$, we define the angle α between the arcs δr and σ:

$$\alpha = \pi - \beta(t_{i+1}) - \delta$$

and angular distance between the positions of space debris fragment $\Delta\nu$:

$$\Delta\nu = arccos[cos\delta\ cos\delta\, r(t_{i+1}) -$$

$$- \sin\sigma\, sin\,\delta\, r(t_{i+1})\, cos\,\alpha]$$

Thus, we define the variational problem: by the known values of angular distance $\Delta\nu$ and flight time $\Delta t = t_{i+1} - t_i$ between two points of orbit, characterized by the vectors $r_{sd}(t_i)$ and $r_{sd}(t_{i+1})$ determine the motion parameters of space debris: focal parameter, major semi-axis, eccentricity, inclination, true anomaly.

Proc. '6th European Conference on Space Debris'
Darmstadt, Germany, 22–25 April 2013 (ESA SP-723, August 2013)

3 METHODOLOGICAL NOVELTY

The novelty of the suggested method is elaboration of simplified formulae for determination of space debris motion parameters by the certain values of angular distances $\Delta\nu$ and flight time Δt between two orbit positions characterized by the radius-vectors $r_{i+1} = r_{sd}(t_{i+1})$ and $r_i = r_{sd}(t_i)$.

As is known the orbital motion of space objects can be described by the system of differential equations in the velocity coordinate system [6, 7]:

$$\frac{dV}{dt} = -\frac{\mu}{r^2} sin\,\theta, \qquad \frac{d\theta}{dt} = \left(\frac{V}{r} - \frac{\mu}{r^2 V}\right) cos\,\theta,$$
$$\frac{dr}{dt} = V\, sin\,\theta, \qquad \frac{d\nu}{dt} = \frac{V}{r} cos\,\theta.$$

Here V – flight speed, θ – slope angle of velocity vector to the local horizon, r - radius-vector, connecting Earth's centre of mass with a space object, ν – true anomaly, t - time, μ – product of gravitational constant by the Earth's mass.

Dividing the first system equation by the third one, we write the following correlation:

$$\frac{dV}{dr} = -\frac{\mu}{r^2 V}.$$

After its integration we get the dependence, connecting the radius-vector of space vehicle with its flight speed, and representing the energy integral $C_1 = 2\mu/r - V^2$ [6].

We transform the second and the third system equations into:

$$tg\,\theta d\theta = \left(\frac{1}{r} - \frac{\mu}{r^2 V^2}\right) dr.$$

Integrating this equation, we define the dependence of trajectory angle θ on the radius-vector r:

$$\theta = arccos\sqrt{\frac{C_2}{r(2\mu - C_1 r)}}\,,$$

where $C_2 = r^2 V^2 cos^2\theta$ – is the area integral [6].

The transformation of the third and the fourth system equations gives opportunity to write the dependencies, connecting the values $\Delta\nu$ and $\Delta t = t_{i+1} - t_i$ with the Kepler's energy integrals C_1 and area integrals C_2:

$$\Delta\nu = arcsin\left\{\frac{\sqrt{C_2}}{(\mu^2 - C_1 C_2)\, r_{i+1}\, r_i}[(\mu r_{i+1} - C_2)\, A(r_i) - (\mu r_i - C_2) A(r_{i+1})]\right\}, \qquad (1)$$

$$\Delta t = A(r_i) - A(r_{i+1}) - \frac{\mu}{C_1^{3/2}} arcsin \cdot \left\{\frac{\sqrt{C_2}}{\mu^2 - C_1 C_2}[(\mu - C_1 r_{i+1}) A(r_i) - (\mu r_i - C_1 r_i) A(r_{i+1})]\right\}, \qquad (2)$$

$$A(r) = \frac{\sqrt{2\mu r - C_1 r^2 - C_2}}{C_1}.$$

It's easy to notice that the determination of integrals C_1 and C_2, and consequently, orbit parameters by the values $\Delta\nu$, Δt and radius-vectors of space debris is possible only conducting the iterative computational process which will lead to reduce in computation efficiency on board the space vehicle.

In order to get the finite calculation dependences we introduce the infinitesimal assumption of function «$arcsin$», composing the equations (1) and (2), i.e. $arcsin\, x \approx x$. The preliminary analysis of these equations shows that in order to provide high calculation accuracy (with calculation errors not more than 0,1%), the differences between two consequent measurements must differ less than 0,5-1% from the absolute value of space object radius –vector. Such conditions can be reached by means of selecting the rational counting slots which first of all depend on the eccentricities of space debris determined orbits.

4 CALCULATION DEPENDENCIES

After the introduction of the mentioned assumption and transformation of equations (1) and (2) we will get quite a simple formula for calculation of integral C_2:

$$C_2 = \frac{r_{sd}^2(t_{i+1}) r_{sd}^2(t_i) \Delta V_i^2}{\Delta t_i^2}.$$

Considering this formula we determine the value of focal parameter of space debris orbit:

$$P = \frac{r_{sd}^2(t_{i+1}) r_{sd}^2(t_i) \Delta\vartheta^2}{\mu \Delta t^2}. \qquad (3)$$

Solving the system of two equations:

$$r_{sd}(t_i) = \frac{P}{1 + e\, cos\,\nu_0} \qquad \text{and}$$

$$r_{sd}(t_{i+1}) = \frac{P}{1 + e\, cos(\nu_0 + \Delta\nu)}, \qquad (4)$$

determine the true anomaly of space debris orbit (ν_0), corresponding to the radius-vector $r_{sd}(t_i)$:

$$\nu_0 = arctg\left\{ctg\Delta\nu - \frac{r_{sd}(t_i)[P - r_{sd}(t_{i+1})]}{r_{sd}(t_{i+1})[P - r_{sd}(t_i)]\, sin\,\Delta\nu}\right\}.$$

Knowing the values of P and ν_0 by the known formulae it is possible to determine other orbit elements of space debris fragment: eccentricity e, pericenter radius r_π, apocenter radius r_α, major semiaxis a:

$$e = \frac{P - r_{sd}(t_i)}{r_{sd}(t_i)\cos v_0}, \qquad r_\pi = \frac{P}{1+e},$$
$$r_\alpha = \frac{P}{1-e}, \qquad a = \frac{r_\pi + r_\alpha}{2}. \qquad (5)$$

Then we determine the inclination of space debris (*i*). First of all we calculate the value of the course angle between the projection of velocity vector of space vehicle on the local horizon and the local parallel at the moment t_i:

$$\varepsilon_{sv}(t_i) = arccos\left[\frac{cos\, i_{sv}}{cos\, \varphi_{sv}(t_i)}\right].$$

Considering the measured value $\beta(t_i)$, we determine the similar course angle for the plane of conditional orbit going through the latitudes of subsatellite points of space vehicle and space debris in the moment t_i

$$\varepsilon = \varepsilon_{sv}(t_i) - \beta(t_i).$$

We determine the inclination of this orbit (*j*), the arc, lying in its plane and connecting the subsatellite point of space vehicle with the equatorial plane (δz) and flight latitude with t_i:

$$j = arccos[cos\,\varepsilon\, cos\,\varphi_{sv}(t_i)],$$
$$\delta z = arcsin\left[\frac{sin\,\varphi_{sv}(t_i)}{sin\, j}\right],$$
$$\varphi_{sd}(t_i) = arcsin[sin\, j\, sin(\delta z + \delta r)].$$

Then consider two spherical triangles formed by the plane of motion of space debris, equatorial plane and also two polar planes going through the latitude $\varphi_{sv}(t_i)$ and the latitude $\varphi_{sv}(t_{i+1})$. According to the law of sines we write the following equations:

$$\frac{sin\,\varphi_{sd}(t_i)}{sin\, i} = sin\, s, \qquad \frac{sin\,\varphi_{sd}(t_{i+1})}{sin\, i} = sin(s + \Delta v),$$

where s – the arc lying in the motion plane of space debris and connecting its position in the moment t_i with the equatorial plane.

The joint solution of these equations allows determining the arc s and inclination of space debris orbit *i*:

$$s = arctg\left[\frac{sin\,\Delta v\, sin\,\varphi_{sd}(t_i)}{sin\,\varphi_{sd}(t_{i+1}) - cos\,\Delta v\, sin\,\varphi_{sd}(t_i)}\right],$$
$$i = arcsin\left[\frac{sin\,\varphi_{sd}(t_i)}{sin\, s}\right]. \qquad (6)$$

Thus, by the formulae (1-6) it is possible to calculate the orbital elements of space debris by the measurement of its parameters using onboard optical sensors of the space vehicle.

5 `CONCLUSION

The conducted research works allowed estimating the calculation errors, generated by the introduction of the above-mentioned assumptions. They also allowed defining the range of initial conditions in which the proposed method provides the required accuracy of calculations for orbit parameters of space debris. It is shown that for the low circular orbits ($e \le 0{,}04$) with the measurement intervals Δt not more than ~100 sec, the calculation errors do not exceed 0,1%. The comparable level of computational accuracy may be provided for parameter determination of high-elliptical orbits ($e \le 0{,}7$), but upon condition of intervals reducing Δt to 20-25 sec. At that, considering the reiterated measurement of space debris orbit parameters and application of the known algorithms of data filtering and data smoothing, the estimated errors can be reduced.

Thus, the proposed method allows providing the orbit parameter calculation of space debris fragments in the wide variation range of initial conditions under the required calculation accuracy. At the same time, for final assessment of calculation accuracy it is necessary to take into account the perturbing factors affecting the dynamics of space objects motion: errors of performing of control inputs, measurements, changes of the ballistic coefficient.

6 REFERENCIES

1. Battin R. Space guidance. M.: Mashinostroyenie, 1966.
2. Baker R. M. L., Jr., Makemson M.W. An introduction to Astrodynamics. N. Y.: Acad. Press. Inc., 1960.
3. Godal Th. Method for determining the initial velocity corresponding to a given time of free flight transfer between given points in a simple gravitational field // *Astronautics.* February. 1961.
4. Plummer M. C. An introductory theatise on dynamical astronomy. N. Y.: Dover publ. Inc., 1960.
5. Sun F. T. A new treatment of Lambertian mechanics //Acta Astronautica. 1981. V. 8. № 2. P. 105-122.
6. Elyasberg P.E. Introduction to the theory of flight of artificial satellites. M.: Nauka. 1965.
7. Barinov K.N., Burdaev M.N., Mamon G.A. Dynamics and design principles of orbital systems of spacecraft. M.: Mashinostroyenie, 1975.