FIRST ATTEMPT OF ORBIT DETERMINATION OF SLR SATELLITES AND SPACE DEBRIS USING GENETIC ALGORITHMS

F. Deleflie[(1)], **D. Coulot**[(2)], **R. Decosta**[(3)], **A. Fernier**[(4)], **and P. Richard**[(5)]

[(1)]*Institut de Mécanique Céleste et de Calcul des Ephémérides, Paris, France, Email: Florent.Deleflie@imcce.fr*
[(2)]*IGN/LAREG, Université Paris Diderot, Sorbonne Paris Cité, Paris, France, Email: David.Coulot@ign.fr*
[(3)]*Institut de Mécanique Céleste et de Calcul des Ephémérides, Paris, France, Email: Romain.Decosta@imcce.fr*
[(4)]*ENSTA ParisTech, Palaiseau, France*
[(5)]*Centre National d'Etudes Spatiales, Toulouse, France, Email: Pascal.Richard@cnes.fr*

ABSTRACT

We present an orbit determination method based on genetic algorithms. Contrary to usual estimation methods mainly based on least-squares methods, these algorithms do not require any *a priori* knowledge of the initial state vector to be estimated. These algorithms can be applied when a new satellite is launched or for uncatalogued objects that appear in images obtained from robotic telescopes such as the TAROT ones.

We show in this paper preliminary results obtained from an SLR satellite, for which tracking data acquired by the ILRS network enable to build accurate orbital arcs at a few centimeter level, which can be used as a reference orbit ; in this case, the basic observations are made up of time series of ranges, obtained from various tracking stations. We show as well the results obtained from the observations acquired by the two TAROT telescopes on the Telecom-2D satellite operated by CNES ; in that case, the observations are made up of time series of azimuths and elevations, seen from the two TAROT telescopes.

The method is carried out in several steps: *(i)* an analytical propagation of the equations of motion, *(ii)* an estimation kernel based on genetic algorithms, which follows the usual steps of such approaches: initialization and evolution of a selected population, so as to determine the best parameters. Each parameter to be estimated, namely each initial keplerian element, has to be searched among an interval that is preliminary chosen. The algorithm is supposed to converge towards an optimum over a reasonable computational time.

Key words: orbit determination, analytical propagation, genetic algorithms, space debris.

1. INTRODUCTION

The main goal of this study consists in finding a way, to compute an orbit from tracking data, when no *a priori* information on the trajectory is available at all. In that case, classical methods such as least-squares can not be used any more (since in that case the function to be minimized can not be linearized in the neighborhood of the *a priori* values of the parameters). Moreover, the usual methods may suffer from many drawbacks which can frequently make them be unappropriated: the well-known Gauss, Laplace, Escobal... approaches are not valid for all dynamical configurations in case of singularities due to orbital planes alignments ; they are often merely based on keplerian modelings, and can hence not be applied over time scales longer than a couple of hours, since in that case a propagator has to account for the main perturbations, at least for the secular ones ; and from time to time, these methods can provide results extremely far from the expected results, and can not be used without great manual care, as *"good-enough" a priori*, that is close enough to the expected values so as to algorithms based on iterative approaches can converge.

On the contrary, even if other kinds of difficulties have to be managed, methods based on genetic algorithms are supposed to be valid for all dynamical configurations, since the algorithm itself is independent from the orbit propagator used to compute the cost function. With an efficient dynamical modeling, they can be used over different periods of time, from a couple of minutes (for Too-Short Arcs, TSA) up to a couple of days or weeks.

The starting point is the system of the equations of motion, that can be written in an usual way:

$$\begin{aligned} \frac{d^2\boldsymbol{r}}{dt^2} &= \boldsymbol{F}(\boldsymbol{r}, \dot{\boldsymbol{r}}, t, \sigma) \\ \boldsymbol{r}(t_0) = \boldsymbol{r}_0 \qquad & \dot{\boldsymbol{r}}(t_0) = \dot{\boldsymbol{r}}_0 \end{aligned}$$

and where the initial positions and velocities to be estimated "from scratch" at an epoch t_0 are denoted $\boldsymbol{r}(t_0)$ and $\dot{\boldsymbol{r}}(t_0)$. The right-hand side describes the force model through the vector $\boldsymbol{F}$, that is characterized with a set of parameters σ (namely, for instance, the gravity field parameters). Genetic algorithms allow a way to find satisfying initial conditions $\boldsymbol{r}(t_0)$ and $\dot{\boldsymbol{r}}(t_0)$, without testing

Proc. '6th European Conference on Space Debris'
Darmstadt, Germany, 22–25 April 2013 (ESA SP-723, August 2013)

all the possibilities in a space of dimension 6, once the frame is roughly defined.

We provide here the preliminary results we obtain with two kinds of data: *(i)* range measurement on the Lageos-1 satellite, tracked by the International Laser Ranging Service, the ILRS (Pearlman, 2002) network, *(ii)* azimuth and elevation time series for the geostationary satellite Telecom-2D.

2. ORBITAL MODELING

To keep a reasonable computation time for the propagation step, we use an analytical approach to get orbital element time series. For the same purpose, and to get an orbital modeling close enough to a numerical reference, we account for the main perturbations of the trajectory, but only the J_2 parameter (Other terms can be included if required by the dynamics of the trajectory). Since we intend to test many different dynamical configurations, the modeling is supposed to be valid whatever the values of the eccentricity or the inclination, small (even equal to zero), or large: the model is written in a set of equinoctial elements (Deleflie, 2013), namely: a, $\xi = \Omega + \omega + M$, $e\cos(\Omega+\omega)$, $e\sin(\Omega+\omega)$, $i_x = \sin\frac{i}{2}\cos\Omega$, $i_y = \sin\frac{i}{2}\sin\Omega$, where a, e, i, Ω, ω, M stand for the classical keplerian elements. For the set $\bar{E}$ of equinoctial elements, the general form of the solution we use is the following:

$$\boldsymbol{E}(t) = \bar{\boldsymbol{E}}(t) + \mathcal{L}(\bar{\boldsymbol{E}})\frac{\partial W}{\partial \bar{\boldsymbol{E}}}(\bar{\boldsymbol{E}}(t)) \quad (1)$$

where the notation $\bar{\boldsymbol{E}}$ stands for the averaged part of the motion, governed only by secular or long periodic effects, and where, as a consequence, the quantities $\boldsymbol{E}(t) - \bar{\boldsymbol{E}}(t)$ correspond to the short periodic part of the theory ; this part is governed by the Lagrange Planetary Equations (written though a matrix 6×6 $\mathcal{L}(\bar{\boldsymbol{E}})$), and the additional function W generating the short periodic part of the motion. Under the assumption of a small eccentricity (but the function can as well be written in a close form to be valid for all values of the eccentricity), which is the case for the two examples shown in that paper, this function W reads (when only the terms independent from the eccentricity are kept):

$$\begin{aligned} W &= \mu(-J_2)\frac{R_0^2}{\bar{n}a^3} \times \quad (2) \\ & 3\left(1 - i_x^2 - i_y^2\right)\left((i_x^2 - i_y^2)\frac{1}{2}\sin 2\xi - i_x i_y \cos 2\xi\right) \end{aligned}$$

so that the temporal variation of each parameter reads, after projection into a more common set of keplerian-like orbital elements:

$a(t) = \bar{a} + \frac{3}{2}J_2\frac{R_e}{\bar{a}}\sin^2\bar{i}\cos 2(\bar{\omega}+\bar{M})$

$h = e\cos\omega$, $k = e\sin\omega$, $e(t) = \sqrt{h^2(t)+k^2(t)}$

$h(t) = \frac{R_e^2}{\bar{a}^2}J_2\Big(\frac{3}{4}\sin^2\bar{i}\big(\frac{7}{6}\cos 3(\bar{\omega}+\bar{M})$
$- \frac{5}{2}\cos(\bar{\omega}+\bar{M})\big) + \frac{3}{2}\cos(\bar{\omega}+\bar{M})\Big)$

$k(t) = \frac{R_e^2}{\bar{a}^2}J_2\Big(\frac{3}{4}\sin^2\bar{i}\big(\frac{7}{6}\sin 3(\bar{\omega}+\bar{M})$
$- \frac{5}{2}\sin(\bar{\omega}+\bar{M})\big) + \frac{3}{2}\sin(\bar{\omega}+\bar{M})\Big)$

$i(t) = \bar{i} + \dfrac{R_e^2}{\bar{a}^2}J_2\frac{3}{8}\sin 2\bar{i}\cos 2(\bar{\omega}+\bar{M})$

$\Omega(t) = \bar{\Omega} + \dfrac{R_e^2}{\bar{a}^2}J_2\frac{3}{4}\cos\bar{i}\sin 2(\bar{\omega}+\bar{M})$

$\omega(t) + M(t) = \bar{\omega} + \bar{M} + \dfrac{R_e^2}{\bar{a}^2}J_2\frac{9}{4}\sin^2\bar{i}\sin 2(\bar{\omega}+\bar{M})$

with the secular part $\bar{\boldsymbol{E}}(t)$ which is governed by the traditional secular variations induced on each angular elements (and mainly by the J_2 parameter), as a function of initial mean semi-major axis, eccentricity and inclination: $\dot{\Omega} = -\frac{3}{2}\left(\frac{R_e}{a}\right)^2\bar{n}J_2\frac{\cos i}{(1-e^2)^2}$
$\dot{\omega} = -\frac{3}{4}\left(\frac{R_e}{a}\right)^2\bar{n}J_2\frac{1-5\cos^2 i}{(1-e^2)^2}$,
$\dot{M} = -\frac{3}{4}\left(\frac{R_e}{a}\right)^2\bar{n}J_2\frac{1-3\cos^2 i}{(1-e^2)^{3/2}}$
and where $\bar{n}$ stands for the mean motion determined through the third Kepler law.

Hence, the whole analytical modeling is governed by the set of mean initial conditions $\bar{\boldsymbol{E}}(t_0)$, whereas it is the corresponding osculating initial conditions $\boldsymbol{E}(t_0)$ that are adjusted by the genetic algorithm, and that can be directly compared to the reference orbits of Lageos-1 and Telecom-2D. The relation between these two sets is merely obtained by setting the time t to the initial epoch t_0 in Eq. (1) so that $\boldsymbol{E}(t_0) = \bar{\boldsymbol{E}}(t_0) + \mathcal{L}(\bar{\boldsymbol{E}})\frac{\partial W}{\partial \bar{\boldsymbol{E}}}(\bar{\boldsymbol{E}}(t_0))$.

3. MULTI-OBJECTIVE GENETIC ALGORITHM (MOGA) USED

We give here further information about the way the algorithm is designed and parameterized.

3.1. Description

The main goal consists in estimating the best set of osculating initial conditions, "best" being defined as a couple of criteria (see also hereafter) to be minimized or maximized. These criteria are defined as functions of the initial conditions, and they are optimized through a large number of iterations that make the process converge to a set of optima.

The Multi-Objective Genetic Algorithm (MOGA) used here is the ϵ-MOEA (Deb et al., 2003). Between two successive iterations, some vectors of initial conditions are replaced by other ones and the best ones are archived. The evolution through the iterations of the set of initial conditions is governed by mutations (random small changes in vectors of possible initial conditions) and by crossover (mix two vectors of possible initial conditions)

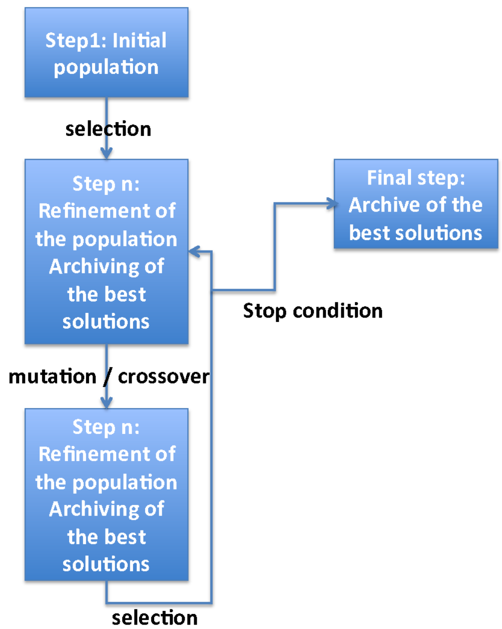

Figure 1. General scheme of the iteration process used to optimize the set of initial conditions with the MOGA.

; the probabilities of changes are usual parameters of the approach. At the end of the iteration procedure, a set of solutions is supplied (Coello Coello et al., 2007).

As in many orbit determination algorithms, an evaluation is made up of several steps:

- ϵ-MOEA provides a vector of initial conditions, randomly chosen among a large set of possible (osculating) initial values ;
- These initial conditions are used to propagate an analytical orbit over the period when tracking or astrometric data are available ;
- The analytical orbit, as time series of orbital elements, is used to compute predicted measurements, that can be compared to the available data sets, at the same epochs of the observations ;
- These predicted measurements are compared to the true data ;
- The values of the cost functions are computed.

The process is then iterated until optimal values of the set of initial conditions are found (Figure 1).

3.2. Parameterization

The chromosomes represent the initial state of the solution: each one is then made up of six initial orbital keplerian elements, and each chromosome determines a unique orbit. Let us note that the MOGA runs on a population of constant size, but also with an archive of variable size, to keep knowledge of the best solutions found so far. At each iteration, two children may be generated (depending upon the probability of crossover) from two parents randomly taken from both the population and the archive, and mutations may also occur (depending on the probability of mutation).

For this first attempt, we chose a population of 400 chromosomes, with fixed intervals for each keplerian initial element:

- semi–major axis $a \in [12200; 15600]$ km for Lageos, $a \in [40000; 45000]$ km for Telecom-2D
- eccentricity $e \in [0; 0.1]$
- inclination $i \in [0; 180°[$
- angles Ω, ω $M \in [0; 360°[$

Let us note that to reduce computation time, the search for the initial eccentricity has been reduced to an interval with a wideness of 0.1, and the search of the initial semi major axis to intervals large of a few thousands of kilometers. But, the results that are shown hereafter would not have been worse if we have kept all the possibilities ($e \in [0; 1]$, and $a \in [6\,500; 45\,000]$ km for instance) for these two elements as well. But the computation time would have been significantly larger.

The crossover probability has been set up to $p_C = 0.9$, and the mutation probability to $p_m = 1/6 \simeq 0.16667$. The stop condition is the total number of iterations which is here set up to 500 000 (this corresponds to a total CPU time of the order of 30 hours).

4. TWO EXAMPLES

We applied our method (MOGA associated to an analytical orbit propagator) to the preliminary orbit determination of two satellites for which the orbits are usually perfectly known and are thus considered as references to be retrieved by our original approach.

4.1. Genetic algorithm handled with range data: the SLR satellite Lageos-1

We use a set of Satellite Laser Ranging (SLR) data acquired on Lageos-1 by 29 stations of the tracking network of the ILRS, and including 2034 measurements, over eight days in April 2012 (from MJD 56 024 to 56 031 included). We consider here two objectives to be optimized: we search not only for the best vector of initial conditions (the RMS of differences between predicted measurements and the real data has to be minimized); but also, we search for an optimal sub-network of SLR stations so that the number of SLR stations involved in the computation has to be maximized. Without this second objective, the MOGA would probably tend to use a minimal set of stations to get better results regarding the initial conditions.

The RMS of differences between the tracking data and their theoretical equivalent computed with the reference orbit (obtained from post-fit adjustment of a numerical integration with the CNES Gins software) is at the level of 2.15 cm. The adjusted initial conditions, seen as reference ones, are the following:

- $a_0^{\text{ref}} = 12270.009$ km
- $e_0^{\text{ref}} = 0.004261$
- $I_0^{\text{ref}} = 109.801°$
- $\Omega_0^{\text{ref}} = 203.323°$
- $(\omega_0 + M_0)^{\text{ref}} = 76.616°$

The best results found by the MOGA are the following:

- $a_0 = 12274.840$ km: $\Delta a = 4.831$km
- $e_0 = 0.004408$: $\Delta e = 0.000147$
- $I_0 = 109.839°$: $\Delta I = 0.038°$
- $\Omega_0 = 203.306°$:$\Delta\Omega = 0.017°$
- $\omega_0 + M_0 = 76.538°$: $\Delta(\omega + M) = 0.078°$

Figure 2 shows the time series of each orbital element, over the given period of time: semi–major axis, eccentricity, inclination, longitude of the node, perigee, mean anomaly. The black curves correspond to the reference orbit obtained with the Gins s/w. The red curves correspond to the analytical propagation of the model presented above, but with the reference initial conditions. It appears that at this scale, the analytical simplified model that we use is suitable to handle the dynamics of the trajectory (at the level of the results). Finally, the green curves show the best trajectory that is found by the MOGA. The differences between the two sets of initial conditions (Δa, Δe, Δi, $\Delta\Omega$, $\Delta\omega$, ΔM above) are quite small with respect to each value to be determined ($3\ 10^{-4}$ on a, 3% on e, $3\ 10^{-4}$ on i, relatively), but induce as well a difference that is not compensated during the propagation. We should keep in mind that *(i)* this level of precision is good enough to use these initial values as *a priori* values in a least-squares adjustment ; *(ii)* genetic algorithms have anyway a good capability over the global scale, but locally they can be less accurate than other approaches ; *(iii)* better results are likely to be obtained when changing the parameterization of the MOGA (in a forthcoming paper).

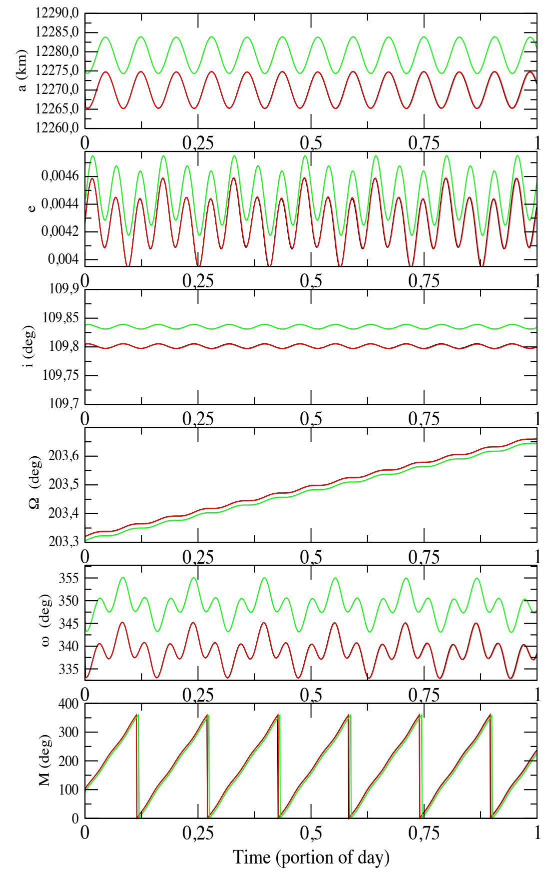

Figure 2. Time series of each keplerian orbital elements $(a, e, i, \Omega, \omega, M)$, computed for the Lageos-1 satellite, in the inertial frame, where the equations of motion are propagated. X-axis: time (portion of day of 7th April 2012). Black curves correspond to the reference orbit (computed with the Gins CNES s/w) ; the red ones correspond to the trajectories obtained with our analytical modeling and with the reference initial conditions provided by Gins ; the green ones correspond to the trajectories obtained with our approach (with initial conditions estimated with the genetic algorithm).

4.2. angular data: the geostationary satellite Telecom-2D

The other example that we tested is based on the assimilation of classical data obtained after astrometric reductions from images acquired by the two TAROT telescopes, respectively located in France and Chile, on the geostationary satellite Telecom-2D. We used data provided by CNES, which has an agreement to benefit from 15% of the available time each night, for space debris activities. Thanks to an upper reachable magnitude of the order of 15 within the GEO region, and the measurement accuracy of the order of 700 m in GEO, the data acquired by the TAROT telescopes enables to build orbits of geostationary satellites operated by CNES. They have an aperture of 25 cm and a field of view of $1.86° \times 1.86°$. The efficiency of the telescopes is very high (except for weather considerations: close to 100% at Calern, France, 85% at La Silla, Chile).

The data set is made up of time series of azimuth and elevation, and includes nine days of angular data (MJD 56 147 to 56 156 included, in Aug. 2012) from the two TAROT-telescopes. The total number of measurements is 86 (27 for Chile and 59 for France).

The reference orbit was computed with the CNES s/w Romance. In terms of latitude and longitude in the geocentric terrestrial frame, this reference orbit is shown Figure 3 (black curves). The adjusted initial conditions, seen as reference ones, are the following:

- $a_0^{\text{ref}} = 42165.980$ km
- $e_0^{\text{ref}} = 0.0001906$
- $I_0^{\text{ref}} = 5.583°$
- $\Omega_0^{\text{ref}} = 61.480°$
- $(\omega_0 + M_0)^{\text{ref}} = 256.934°$

These parameters correspond to a reference mean longitude of $-7.75°$.

The MOGA searches for the best vector of initial conditions, and two objectives are considered (both to be minimized): the RMS of differences between predicted measurements and the real data for elevation and azimuth. For the best solution found, the RMS of differences are respectively $0.0485°$ for elevation, and $0.0742°$ for azimuth. The corresponding adjusted best initial elements are:

- $a_0 = 42171.560$ km: $\Delta a = 5.580$km
- $e_0 = 0.0000923$: $\Delta e = 0.0000983$
- $I_0 = 5.578°$: $\Delta I = 0.005°$
- $\Omega_0 = 62.897°$:$\Delta\Omega = 1.417°$

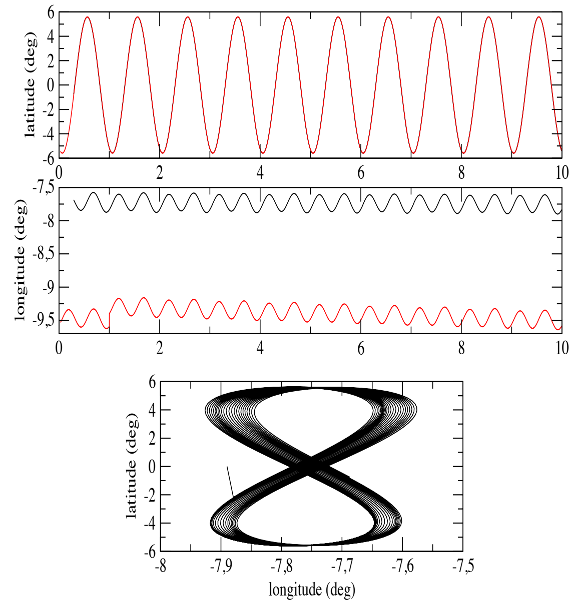

Figure 3. Time series of latitude and longitude for the Telecom-2D satellite (top) ; the X-axis is time (number of days from 8th August 2012, and covering a period w/o any maneuver). The third plot is the ground track trajectory. Black curves correspond to the reference orbit (computed with the Romance CNES s/w), an the red ones to the trajectories obtained with our approach.

- $\omega_0 + M_0 = 257.180°$: $\Delta(\omega + M) = 0.246°$

It seems (Figure 3, red curves) that the analytical modeling is also suitable to describe geostationary orbits (even if adding the effect of resonant tesseral parameters would be helpful for the time evolution of the longitude).

5. CONCLUSIONS

In this paper, we combined a MOGA and an analytical satellite motion theory to roughly adjust an orbit on tracking data, without any *a priori* knowledge of the values of the initial conditions to be retrieved. We tested the method on two kinds of data. Some further developments will be enhanced in the future *(i)* the analytical modeling will be improved by adding some significant terms in the model *(ii)* the parameterization of the MOGA will be refined, with a reduced set of chromosomes, and by empirically decreasing the mutation probability throughout the iterations; we will implement as well a better stopping condition to reduce the CPU required time. We will then test the capabilities of the algorithm in downgraded conditions (data sparse in time, very few number of data).

6. ACKNOWLEDGMENTS

We thank CNES for financial support, in the framework of a GRGS project (Groupe de Recherche de Géodésie Spatiale), and providing data from the Telecom-2D satellite.

REFERENCES

1. Coello Coello, C.A., G.B. Lomont, D.A. Von Veldhuizen, *Evolutionary Algorithms for Solving Multi-Objective Problems*, Second Edition, Springer, 2007.

2. Deb, K., M. Mohan, S. Mishra (2003) *A Fast Multi-objective Evolutionary Algorithm for Finding Well-Spread Pareto-Optimal Solutions*, KanGAL Report Number 2003002.

3. Pearlman, M.R., Degnan, J.J.,, and Bosworth, J.M., *The International Laser Ranging Service*, Adv. In Sp. R., Vol 30, No 2, pp 135-143, 2002.

4. Deleflie, F., *Finalized Analytical Satellite Theory*, CNES Internal Report, 2013.