THE APPLICATION OF LINEAR PROGRAMMING ON THE SPACE SURVEILLANCE OF HIGH-ALTITUDE OBJECTS

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ABSTRACT

We consider the planning problem of the space surveillance of high-altitude objects with optical telescopes. We present a mathematical model and solve it with linear programming to show the optimization effect.

INTRODUCTION

Optical Telescopes are usually used in the space surveillance of high-altitude objects which are much different from LEO objects. They move slowly and can be visible to the observation station for a very long time. According to numerical simulations, there are over 200 high-altitude objects to be visible to a certain site at the same time. Obviously it is impossible for a telescope to track the whole pass of all objects, which is also unnecessary. To efficiency the observation, we just pick up several parts from the whole pass and track them. Usually the arc length which plays most important roll in orbit determination, is determined only by the start and end point of the measurements, so if the selection is done carefully, the accuracy of the orbit determination will not be affect at all. In this way, much more high-altitude objects can be tracked.

Linear Programming is widely used in many fields[1]，in this paper, we construct a mathematical model for this selection problem and transform the problem to a linear programming case. We give out the complete equations for the problem including the linear objective function and linear constraints. All observation conditions are represented with mathematical constraints and the total arc length of all objects is chosen as the objective function. Benefit from the linear programming solving method, according to the solution to the mathematical model, we achieve the most optimized and efficient selection of observation arcs. Based on this model, we also give out the equations for many different kinds of constraints and objective functions, which represent real occasions in the observation, such as earth shadow and etc. An example based on our real observation is also given in the end of the paper to show the optimization effect of the linear programming on this problem.

MODELING

Problem Analysis and Discrete of the Time Series

According the analysis, for high-altitude objects, we just pick up several parts from the whole pass to efficiency the observation. To optimizing the selection, we should find out the objective function and constraints first. As we know, the main work of space surveillance is to calculate the orbit of the objects form the measurements and keep a catalog of the objects for further tracking. Arc length is most important for the orbit determination, so we choose the arc length as the objective function and the constraints are much more obvious, as the follows: every object must be visible when observed and only one object is being observed at the same time for one device. Based on the objective function and constraints, a model will be construct to transform them to mathematical equations in linear format.

For high-altitude objects, the visible arc length is very long, which is very different with the LEOs. So missing of several minutes of the arc length is not as critical as that for LEOs. To construct the model, we spilt the whole night period into many short periods and the observation period is represented in discrete series. Let us take T as the time length of the whole night and t as the length of every short period, then we get $N = INT(T/t + 0.5)$ periods. In fact, the length of the short period can be choose as any length, dozens of seconds or several minutes, which will not alter the problem at all. For us a period of 3-4 minutes is a good choice. Every short period is numbered for 1 to N as the fig 1.

1	2	3	…	$N-2$	$N-1$	N

Fig. 1 Concrete of the time series

The problem then becomes selection of one object for every time period. For the same object, if the number of the period for the first observation is i and the last period of observation is j, the arc length can be expressed as $j-i+1$.

Model I

If the quantity of the short periods for the whole night is TP and total of the visible objects is TS, optimal variables are defined as:

$$Y_{ij}(i = 1 \ldots TS, j = 1 \ldots TP)$$

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Where Y_{ij} is the type of BINARY which can be as 1 or 0. $Y_{ij}=1$ means observing object No. i at period No. j. As mentioned above, the most important periods are the first and last one, we define them separately：

$$\begin{aligned} &YS_{ij}(i=1\ldots TS, j=1\ldots TP) \\ &YE_{ij}(i=1\ldots TS, j=1\ldots TP) \end{aligned} \quad (1)$$

Which is same as Y_{ij}, but YS_{ij} is for the first period only and YE_{ij} is for the last period only

The constraints can be expressed as the follows:

$$\begin{aligned} &s.t. \quad \sum_{j=1}^{TP} YS_{ij} \leq 1(i=1\ldots TS) \\ &s.t. \quad \sum_{j=1}^{TP} YE_{ij} \leq 1(i=1\ldots TS) \end{aligned} \quad (2)$$

which means for every object only one first period and one last period.

With this constraints, for every i, only one $Y_{ij} \neq 0$, we obtain the number of last period：

$$\sum_{j=1}^{TP} j \times YE_{ij}(i=1\ldots TS) \quad (3)$$

The same, the number of the first period for every object is:

$$\sum_{j=1}^{TP} j \times YS_{ij}(i=1\ldots TS) \quad (4)$$

The first period must be before the last period, constraint is expressed as：

$$s.t. \quad \sum_{j=1}^{TP} j \times (YE_{ij} - YS_{ij}) \geq 0(i=1\ldots TS) \quad (5)$$

For every object, if first period is available, last period is available too：

$$s.t. \quad \sum_{j=1}^{TP} (YE_{ij} - YS_{ij}) = 0(i=1\ldots TS) \quad (6)$$

The most import constraint, for every period only one object can be observed, is expressed as：

$$s.t. \quad \sum_{i=1}^{TS} (YE_{ij} + YS_{ij}) \leq 1(j=1\ldots TP) \quad (7)$$

If the object is invisible at certain periods, we just let the corresponding periods $YE_{ij}+YS_{ij}=0$.

The total arc length, which is the objective function, is：

$$\sum_{i=1}^{TS}\left(\sum_{j=1}^{TP} j \times YE_{ij} - \sum_{j=1}^{TP}(j-1) \times YS_{ij}\right) \quad (8)$$

And can be rewritten as:

$$\text{maximize} \quad \sum_{i=1}^{TS}\sum_{j=1}^{TP}(j \times YE_{ij} - (j-1) \times YS_{ij}) \quad (9)$$

Total number of observing objects is：

$$\sum_{i=1}^{TS}\sum_{j=1}^{TP} YE_{ij} = \sum_{i=1}^{TS}\sum_{j=1}^{TP} YS_{ij} \quad (10)$$

Utilizing these equations, we can easily transform then to any type of language for the Linear Programming Solver as GLPK or CPL.

The model can be expressed in fig 2 as the following:

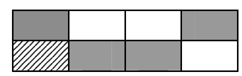

Fig 2 Model I

Each row means one object, while each column means a time period. If the object is invisible in certain period, we mark the cell with cross lines. If selected, the cell is in grey. For every row, 0 or 2 cells can be grey and for every column only one cell is grey.

Model II

Similar with model I， which is described in last section, with TP periods and TS objects，define:

$$Y_{ijk}(i=1\ldots TS, j=1\ldots TP, k=1..TP)$$

Where Y_{ijk} is binary type. $Y_{ijk}=1$ means period No. j is the first and period No. k is the last observing period for Object No. i. From the geometric point of view, this model is expanded model I with one dimension, as showed in fig 3 , one object is expressed in a plane rather than a row like in model I:

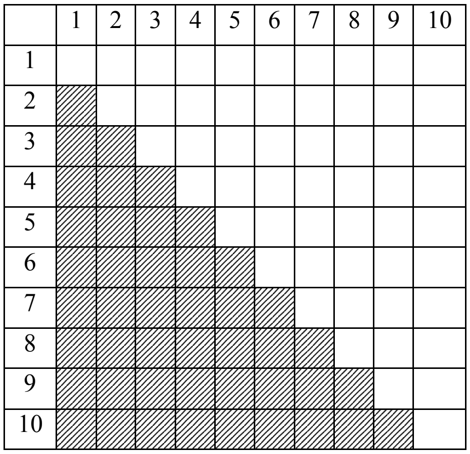

Fig 3 Model II

This figure is much similar with the fare table of sub-way, which tells you the fare fee while the start and end stations are specified. In our problem, while the first and last periods is specified, the arc length can be obtained. For every plane, we can choose one cell at most, as the following equation:

$$s.t. \quad \sum_{j=1}^{TP}\sum_{k=1}^{TP} Y_{ijk} \le 1 \quad (i = 1..TS) \tag{11}$$

Obviously $j \le k$：

$$Y_{ijk} = 0, k < j, i = 1..TS \tag{12}$$

Expressed in fig is the cells with cross lines. In linear format:

$$s.t. \quad \sum_{j=2}^{TP}\sum_{k=1}^{j-1} Y_{ijk} = 0 \quad (i = 1..TS) \tag{13}$$

Rewritten as:

$$s.t. \quad \sum_{i=1}^{TS}\sum_{j=2}^{TP}\sum_{k=1}^{j-1} Y_{ijk} = 0 \tag{14}$$

One object at the same time：

$$s.t. \quad \sum_{i=1}^{TS}\left(\sum_{k=1}^{j} Y_{ikj} + \sum_{k=j+1}^{TP} Y_{ijk}\right) \le 1 \quad (j = 1..TP) \tag{15}$$

The total arc length is:

$$\text{maximize} \quad \sum_{i=1}^{TS}\sum_{j=1}^{TP}\sum_{k=1}^{TP} (j - i + 1) Y_{ijk} \tag{16}$$

Till now, we can replace model I with model II, but the purpose of constructing model II is not just constructing a replacement. We will use model II to achieve more efficient optimization result. For the same total arc length, we prefer the result of lower variance. For model I, $YS_{ij} \cdot YE_{ij}$ must be used to express variance which is not allowed with linear programming. With this new model,variance is:

$$\sum_{i=1}^{TS}\sum_{j=1}^{TP}\sum_{k=1}^{TP} (j - i + 1 - MEAN)^2 Y_{ijk} \tag{17}$$

So while we get the result of best arc length, we change the objective function to variance and get the optimize result for best variance.

The advantage of model I is fast, because for model I, there are $TS \times TP \times 2$ variables, while $TS \times (TP^2 / 2 + TP)$ for model II. In our practice, we use model I to solve the best arc length and use model II to solve the best variance.

EXAMPLE

For 13 June 2007 (MJD=54264), we took the optimization for the real observation for Kunming station which is located in the south of China. Every period of observation is 4 minute and during the night there were 135 periods and 1422 objects can be visible. After optimization, 67 objects were observed and total arc length is 4623 periods. The result is in Table 1:

Table 1. Reulst of Model I

Object #	First Period #	Last Period #	Object #	First Period #	Last Period #
026	014	112	1034	030	110
093	037	102	1045	007	129
117	055	085	1090	016	121
127	020	118	1116	011	125
265	041	098	1132	061	077
298	025	115	1137	044	095
309	023	084	1149	052	088
414	017	079	1153	034	105
445	049	091	1156	058	081
512	005	131	1164	040	099
567	064	074	1193	029	111
578	022	116	1199	063	075
588	046	093	1210	010	126
592	012	124	1218	021	117
597	019	119	1220	038	101
601	043	096	1234	062	076
647	026	114	1245	031	109
663	051	089	1249	045	094
719	047	070	1253	033	107
768	002	134	1263	015	122
793	035	104	1265	054	086
821	013	123	1266	065	073
840	042	097	1282	018	120
841	027	071	1290	004	132
851	057	082	1333	060	078
931	003	133	1344	036	103

949	053	087	1347	039	100
955	008	128	1363	009	127
956	067	069	1365	056	083
957	028	113	1395	048	092
967	050	090	1398	066	072
988	032	108	1403	059	080
999	024	106	1421	001	135
1000	006	130	***1005***	***068***	***068***

For 135 periods and 2 periods per object , the total of objects is $INT(135/2) = 67$. If every object is visible all the time, without any constraints, the best theoretical arc length is $(135-66)*67 = 4623$. The optimization result achieved the best result. Then we used model II to get more optimized result as the following table:

Table 2. Reulst of Model II

Object #	First Period #	Last Period #	Object #	First Period #	Last Period #
26	14	81	1034	30	96
93	47	123	1045	7	73
117	55	128	1090	24	90
127	20	86	1116	11	77
265	41	117	1132	61	131
298	25	91	1137	44	106
309	18	84	1149	52	127
414	17	79	1153	34	103
445	49	125	1156	58	129
512	5	71	1164	40	115
567	64	133	1193	29	95
578	22	88	1199	63	132
588	46	119	1210	10	76
592	12	78	1218	21	87
597	23	89	1220	38	109
601	43	111	1234	62	112
647	26	92	1245	37	108
663	51	114	1249	45	113
719	19	85	1253	33	102
768	2	124	1263	15	82
793	35	105	1265	54	121
821	13	80	1266	65	122
840	42	118	1282	27	99
841	16	83	1290	4	70
851	57	97	1333	60	104
931	3	93	1344	36	107
949	53	116	1347	39	110
955	8	74	1363	9	75
956	67	135	1365	56	98
957	28	94	1395	48	120
967	50	126	1398	66	134
988	32	101	1403	59	130
999	31	100	1421	1	69
1000	6	72	***1005***	***68***	***68***

The variance is 10 times less. Fig 4 which gave the arc length for every object, shows the effect of variance optimization obviously.

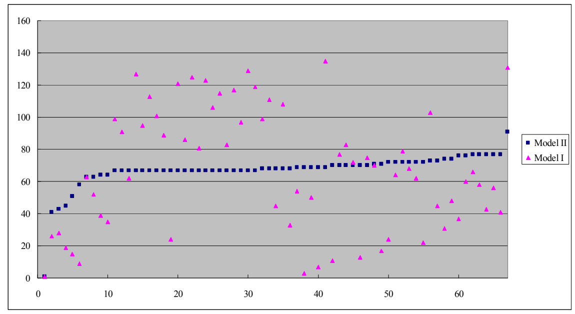

Fig. 4 Effect of variance optimization

With two round optimization, we achieve a very efficiency plan, with maximum arc length and minimum variance, for the space surveillance of high-altitude objects. Though the model is very simple, but it is quiet useful. The real problem can benefit from the model and the model can be easily expand to multi-device or multi-sites cases.

REFERENCES

[1] Huang H., Han J., Mathematical Programming, Tsinghua University Press,2006