RISK ANALYSIS AND DAMAGE PREDICTION OF SMALL SPACECRAFT

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ABSTRACT

The space debris population has increased in the last decades and is still growing. This problem results amongst others in protecting spacecraft against particle impacts, especially for manned missions in low Earth orbits. Depending on the mission, even for smaller spacecraft, it is reasonable to protect the main structure against space debris and meteoroid impacts. For the risk analysis of spacecraft in meteoroid and space debris environment, so called risk analysis and damage prediction codes such as BUMPER or MDPANTO have been developed [1]. They are used in order to quantify the risk for a defined mission resulting from impacts of particles. This analysis can be done based on two idealization concepts of the spacecraft surface. In the first concept, the surface area of the shield is taken for the analysis, in the second concept the surface area of the protected structure is taken. Due to the difference in surface area of the shield and of the protected structure, the analysis results could be described on one hand to be conservative and on the other hand to be optimistic. For large spacecraft this difference in surface area is negligible, but for small spacecraft it becomes more important. In this paper, an approach is developed in order to estimate a surface area, that reflects better the actual impact relations for small spacecraft.

Key words: risk analysis; damage prediction; damage prediction tools; representative surface area.

1. INTRODUCTION

For spacecraft, the meteoroid and space debris fluxes have to be treated as vector quantities and the effects of directionality must be carefully evaluated. This is done by risk analysis and damage prediction codes. A basic relation in assessing the risk for a spacecraft in orbit, is the determination of number of impacts. The number of impacts N increases linearly with the exposed area A, flux F and exposure time T [1]:

$$N = F \cdot A \cdot T \tag{1}$$

F is the cumulative number of impacts (of a particle with given diameter and larger) per unit area and time. Once

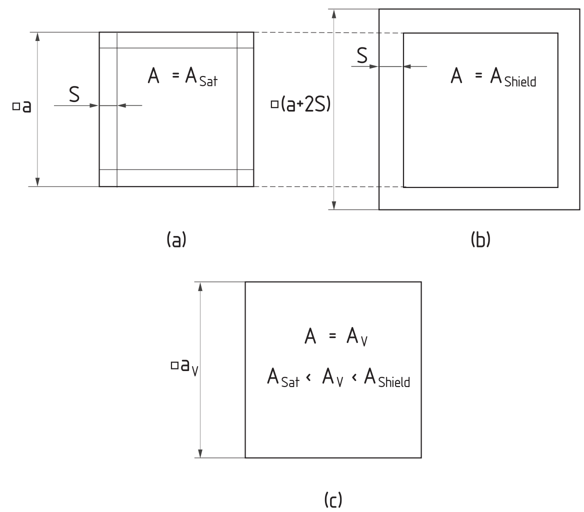

Figure 1. Different Surface Areas can be used for the Risk Analysis and Damage Prediction

N has been determined, the probability of no impacts in the corresponding time interval is given by Poisson statistics:

$$p_0 = e^{-N} \tag{2}$$

The same equations apply if N is the number of failures rather than the number of impacts. The number of failures depends on the failure criterion which is determined by the shielding thickness and effectiveness as expressed by a damage equation. A widely used failure criterion is the complete penetration (that is ”perforation”) of the structural wall. But other failure criteria are possible as well, such as a hole larger than a given critical size or penetration depth that exceeds an allowable amount.

For the risk and damage prediction analysis, a surface area has to be defined for the determination of the number of impacts (equation (1)). For this, two approaches are thinkable:

- Use of the surface area of the satellite itself A_{Sat} or
- the use of the surface area of the shield A_{Shield}.

In figures 1a and 1b, these two possible concepts are sketched for a simple cubic spacecraft.

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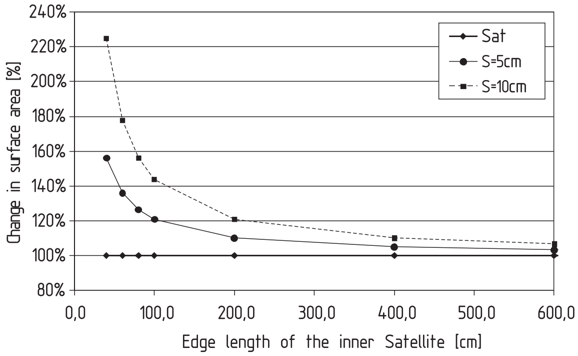

Figure 2. Changes in the Surface Area of S/C due to shielding

Due to the overlapping shield at the satellite egdes (figure 1b), not all particles impacting in the edge region of the shield can be seen as critical for the satellite itself. The debris cloud caused by these impacts hits the satellite partially or even miss it. Keeping this in mind, it is obvious, that the use of the shield surface area leads to conservative results. On the other hand, the use of the satellite surface area would be too optimistic. Hence, in order to obtain more realistic results, which reflect better the actual relations of impacts on such edge regions, it is proposed to use a surface area, which size is between the surface area of the shield and that of the satellite. The increase in surface area by the shielding with certain spacing (S) between the back wall (satellite) and the shield, is shown in figure 2 versus the edge length of a cubic satellite. It can be seen, that for smaller spacecraft, increase in surface area due to the shielding is very high. For larger spacecraft, the increase in surface area is negliable.

The presented approach computes a "virtual" surface area A_V, that can be used for a realistic risk and damage analysis of small spacecraft (figure 1c).

An advantage of the use of a representative surface area is the fact, that the damage prediction codes need not to be changed. Only the geometrical model has to be adopted. This can be done by the scaling factors r_A and r_l, which are used as given in the equations (3) and (4). In equation (4), a is the edge length of the satellite and a_V is the representative edge length of the satellite. The scaling factors r_A and r_l depend not only on the spacecraft geometry but also on the spacing S, and of course on the assumptions of debris cloud characteristics. In this paper, the scaling factors are determined for the example spacecraft at hand, which is described below.

$$A_V = r_A \cdot A_{Sat} \tag{3}$$

$$a_V = r_l \cdot a \tag{4}$$

The modification of the geometrical model is important due to the fact, that common ballistic limit equations, such as the Cour-Palais and the Christiansen equations assume a defined wall setup. The determination of the probability of the back wall damage is based on a wall setup, where the area of shield and back wall have the same size and are parallel to each other. Furthermore it is assumed, that the debris cloud, generated upon penetration of the shield, hits completely the back wall. This is not the case for all impacts if the shield and back wall area are not the same. When applying the virtual surface area, this effect is already considered adequately.

2. DETERMINATION OF A REPRESENTATIVE SURFACE AREA

In this study, two approaches in the determination of the representative surface area have been investigated. They differ in the consideration of the impact angle:

- only normal impacts are considered;
- normal impacts and oblique impacts are considered.

The center-of-mass trajectory and the cone angle of the debris cloud will change according to the impact angle. But not only the impact angle influences the debris cloud characteristic, but also the diameter of the impacting particle d_p, the impact velocity v, the shield thickness t_S and the speed of sound of the shield material C [2]. At least these parameters have to be defined, in order to perform the analyses for the approach considering oblique impacts.

For the first approach, that considers normal impacts only, a simple assumption for the debris cloud expansion has been taken. It is assumed to be independent on the parameters mentioned before. The center-of-mass trajectory and the cone angle of the debris cloud do not change. For a double wall system, the diameter d_i of the debris cloud in the distance S (figure 3) is assumed to be:

$$d_i = \frac{S}{2} \tag{5}$$

2.1. Consideration of Normal Impacts Only

A simple cubic spacecraft with double wall protection system (spacing S=5cm) has been used, in order to investigate the behaviour of particles impacting in regions of the shield, where the wall setup is not anymore conform with the assumption for the use of common ballistic limit equations. The relations are easy to derive and the geometry of the spacecraft is a good starting point in order to transfer the developed method to other spacecraft geometries. Due to symmetry, it is sufficient to use one quadrant of the shield of the cubic spacecraft for further investigations (figure 3). The area of this quadrant can be divided into several surface area elements. Each element describes the different behaviour of the debris cloud hitting the back wall (satellite). Depending on the expansion of the debris cloud of a normal impact, four different regions are assumed close to the shield edge of the cubic spacecraft (figure 3):

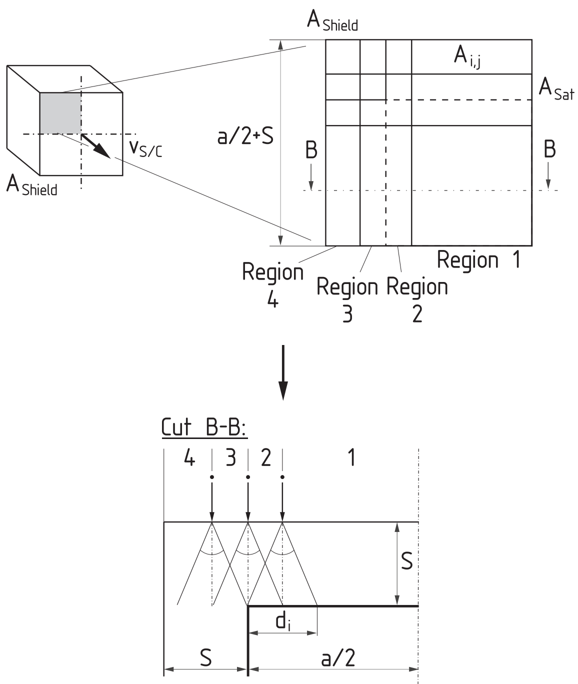

Figure 3. Approach Concept

- **Region 1** - Debris cloud completely impacts the back wall (satellite)
- **Region 2** - More than 50% of the debris cloud impact the back wall
- **Region 3** - Up to 50% of the debris cloud impact the back wall (front and side wall)
- **Region 4** - Up to 50% of the debris cloud impact the back wall (only side wall)

The concept of computing a virtual surface area is to reduce the shield surface area according to the impact consideration of the debris cloud on the back wall of the spacecraft. Obviously, only in the regions 2 to 4, the surface area of the shield is going to be reduced partially. Instead of the surface area of the shield, the problem can be reduced to one dimension only. Thus, the relevant length, is the edge length of the shield (figure 3).

The edge length of the shield is divided into n elements. The length of each element is different for the two models used:

- Sector Model - n = 4 (corresponding to the four regions, that are defined above)
- Element Model - n is arbitrary

At the center of each element, one impact is simulated. In the simple approach, the impact angle θ_p is zero, so normal to the shield surface. Depending on the impact

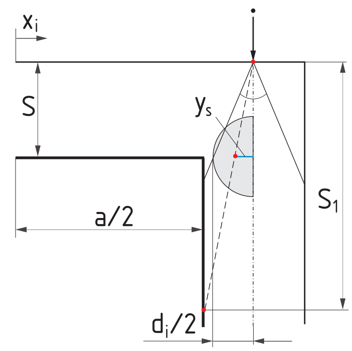

Figure 4. Spacing Factor (SF)

position x_i, at which the impact along the edge length of the shield occurs, the edge length can be classified into the regions defined above. For each region, a so called reduction function g_i is calculated, that expresses the impact behaviour of the debris cloud on the back wall (satellite):

$$g_i = \frac{a_i}{d_i} \cdot SF \tag{6}$$

In equation (6), a_i is the covered length on the back wall by the debris cloud with the diameter d_i in the distance S (equation (5)). The spacing factor SF in equation (6) is applied, in order to consider the increased spacing when impacting on the side surface of the satellite (figure 4). It is defined by equation (7). The point on the side surface for the distance S_1 is determined by extending the line from the impact position through the center-of-mass of the half debris cloud circle to the side wall. If S_1 becomes larger than $S + a$, which means, the extended line passes the inner satellite, the reduction function is set to zero for this element.

$$SF = \sqrt{\frac{S}{S_1}} \tag{7}$$

$$S_1 = \frac{S}{y_s} \cdot (x_i - \frac{a}{2}) \tag{8}$$

$$y_s = \frac{2}{3} \cdot \frac{d_i}{\pi} \tag{9}$$

The reduction funtcion g_i is calculated for each element and multiplied by the element length l_i. The sum over all element lengths results in the virtual shield edge length l_V (equation (10)). From this representative length l_V, the virtual surface area A_V can be easily derived (equation (11)):

$$l_V = \sum_{i=1}^{n} g_i \cdot l_i \tag{10}$$

$$A_V = 24 \cdot l_V^2 = 24 \cdot (\sum_{i=1}^{n} g_i \cdot l_i)^2 \tag{11}$$

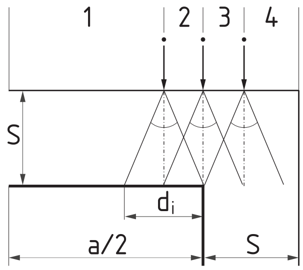

Figure 5. Sector Model (SM)

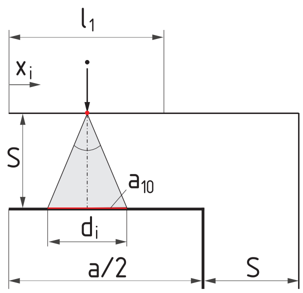

Figure 6. Simple Model - Region 1

2.1.1. Sector Model

The sector model (SM) reflects directly the regions defined along the edge length of the shield (figure 5). The element length is equal to the length of the region. So the number of elements, that are used in the sector model is n=4.

Region 1
Impacts, which occur in region 1 (figure 6), cause a debris cloud, that impacts completely on the back wall (front side of the satellite). The region length is characterized by:

$$0 < x_i \leq \frac{a}{2} - \frac{d_i}{2} \tag{12}$$

$$\Rightarrow l_1 = \frac{a}{2} - \frac{d_i}{2} \tag{13}$$

As mentioned above, the length of the element is equal to the region length. The impact position is the center of the element. The reduction function for this region according to equation (6) is:

$$g_1 = \frac{a_{10}}{d_i} \cdot SF_1 = 1.0 \tag{14}$$

Since the covered length a_{10} is equal to d_i and the spacing factor in region 1 is equal 1, the reduction function is 1.0. This means, that the shield edge length of region 1 is fully considered in the determination of the representative shield edge length.

Region 2
Particles impacting in region 2 (figure 7), cause a debris cloud, that impacts partially on the back wall (front side of the satellite). Between 50% and 100% of the debris cloud impact on the back wall. The region length is characterized by:

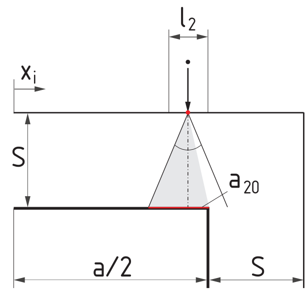

Figure 7. Simple Model - Region 2

$$\frac{a}{2} - \frac{d_i}{2} < x_i \leq \frac{a}{2} \tag{15}$$

$$\Rightarrow l_2 = \frac{d_i}{2} \tag{16}$$

The reduction function for this region according to equation (6) is:

$$g_2 = \frac{a_{20}}{d_i} \cdot SF_2 \tag{17}$$

with

$$a_{20} = \frac{d_i}{2} + (\frac{a}{2} - x_i) \tag{18}$$

The spacing factor in region 2 is SF=1. The shield edge length of region 2 is considered between 50% and 100% in the determination of the representative shield edge length.

Region 3
Impacts, which occur in region 3 (figure 8), cause a debris cloud, that impacts partially on the back wall (front wall and side wall of the satellite). It is assumed, that 50% of the debris cloud is a possible threat for the satellite. The region length is characterized by:

$$\frac{a}{2} < x_i \leq \frac{d_i}{2} + \frac{a}{2} \tag{19}$$

$$\Rightarrow l_3 = \frac{d_i}{2} \tag{20}$$

The reduction function for this region according to equation (6) is:

$$g_3 = \frac{a_{31}}{d_i} \cdot SF_{31} + \frac{a_{32}}{d_i} \cdot SF_{32} \tag{21}$$

with (22)

$$a_{31} = \frac{d_i}{2} - (x_i - \frac{a}{2})$$

$$a_{32} = x_i - \frac{a}{2} \tag{23}$$

Since the covered length is equal to $0.5 \cdot d_i$, the shield edge length of region 3 is considered as 50% . If a spacing factor is applied, the reduction function is less than

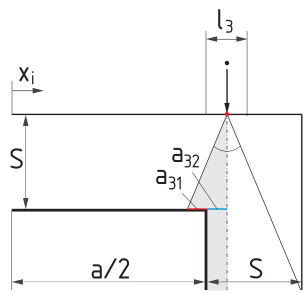

Figure 8. Simple Model - Region 3

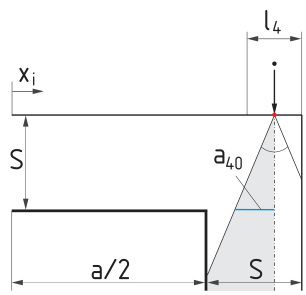

Figure 9. Simple Model - Region 4

50% due to the increased spacing when impacting on the side wall of the satellite (spacing factor $SF_{32} < 1$ and SF_{31}=1).

Region 4
Impacts, which occur in this region (figure 9), cause a debris cloud, that impacts partially on the back wall (side wall of the satellite). It is assumed, that 50% of the debris cloud is a possible threat for the satellite. The region length is characterized by:

$$\frac{d_i}{2} + \frac{a}{2} \quad < \quad x_i \leq \frac{a}{2} + S \tag{24}$$

$$\Rightarrow l_4 \quad = \quad S - \frac{d_i}{2} \tag{25}$$

The reduction function for this region according to equation (6) is:

$$g_4 \quad = \quad \frac{a_{40}}{d_i} \cdot SF_4 \tag{26}$$

with

$$a_{40} \quad = \quad \frac{d_i}{2} \tag{27}$$

Since the covered length a_{40} is equal to $0.5 \cdot d_i$, the shield edge length of region 4 is considered as 50% . If a spacing factor is applied, the reduction function is less than 50% due to the increased spacing when impacting on the side wall of the satellite.

The graph of the reduction function for the sector model with and without spacing factor applied (satellite edge length a=40cm, spacing S=5cm), is given in figure 10. The horizontal axis reflects the x-position along the shield

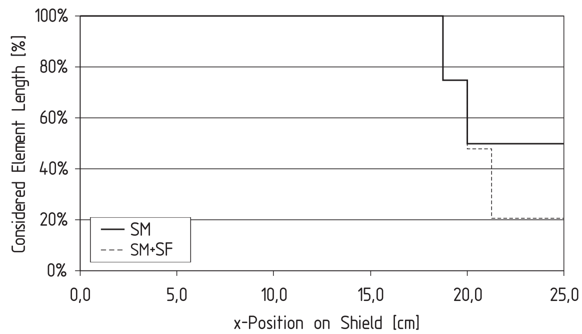

Figure 10. Reduction Function of the Sector Model with/without Spacing Factor (a=40cm; S=5cm)

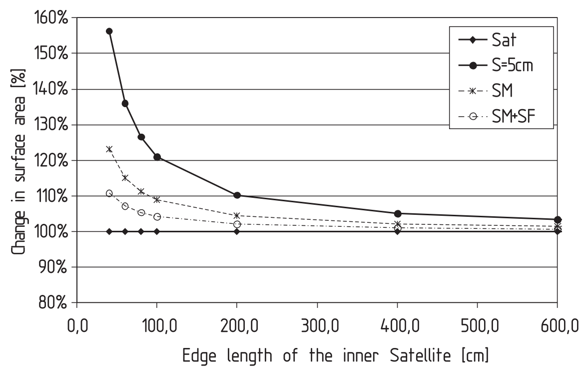

Figure 11. Representative Surface Area of the Sector Model

edge length starting from the symmetry axis of the satellite. The reduction function is decreased near the edge of the shield as expected. That means in this case, that the shield edge length of region 1 is considered 100%, in region 75% and in regions 3 and 4 each 50%. When applying the spacing factor (SF), there is a further decrease of the reduction function in regions 3 and 4. The spacing factor does not effect the reduction function in regions 1 and 2.

When the reduction function for each element and region respectively is calculated, the representative edge length of the shield can be obtained by applying equation (10). In a further step, the representative surface area can be determined with equation (11). The result of the representative area is given in figure 11 for several satellites with different edge lengths a. Applying the presented approach, it can be seen, that the representative surface area is in between the surface area of the satellite and that of the shield. The increase in surface area is reduced by about 58% using the sector model (SM) compared to the increase in surface area due to a shield with a spacing of S=5cm. Considering the spacing factor SF in the sector model, the increase in surface area is lowered even by 80% . This means a decrease in the number of impacts in the same order, because of its linear dependency on the surface area (equation (1)).

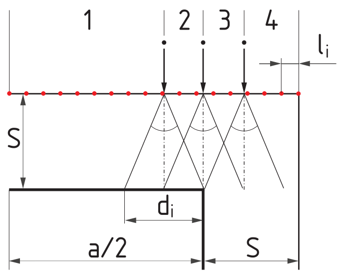

Figure 12. Element Model (EM)

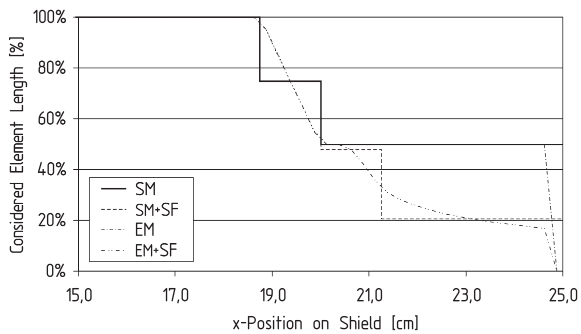

Figure 13. Reduction Function of the Element Model (n=100) with/without Spacing Factor (a=40cm; S=5cm)

2.1.2. Element Model

An alternative model for the approach considering normal impacts only, is the element model (EM). The principle is still the same as with the sector model. The difference is, that the length of the element l_i depends on the number of elements n and it is equal for each element. Thus it is not identical with the length of the defined regions (section 2.1.1). The element length is given by equation (28).

$$l_i = \frac{1}{n} \cdot (\frac{a}{2} + S) \tag{28}$$

The example case is calculated with 100 elements, which leads to a better resolution of the reduction function. Especially the influence of the spacing factor can be seen cleary at curve 'EM+SF' (figure 13). The more the impact occurs towards the edge of the shield, the more decreases the reduction function due to the spacing factor. The result obtained for the representative area applying the element model, is nearly the same compared to the sector model.

The scaling factors for the cubic spacecraft can be obtained easily by applying a scaling function (equation (29)), which represents the scaling factors dependent on the satellite edge length a.

$$r = 1 + x \cdot a^y \tag{29}$$

The scaling factors can be applied as given in equations (3) and (4). The parameters x and y are listed in table 1 for the element model.

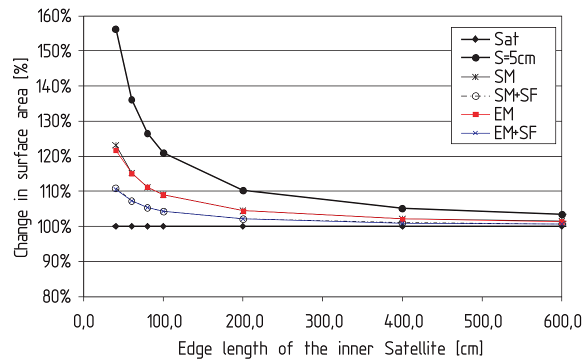

Figure 14. Representative Surface Area of the Element Model

Table 1. Scaling Factors for the Element Model

EM	r_l		r_A	
(n=100)	x	y	x	y
with SF	1.7532	−0.9607	3.6625	−0.9674

2.2. Consideration of Oblique Impacts

The approach presented above has been improved by considering not only normal impacts but also oblique impacts. Five impact angles have been defined as representatives for all impact angles. Thus, instead of one impact on each element, now five impacts are analysed with a different impact angle θ_p. The determination of the representative shield edge length is given by equation (30) instead of equation (10). Due to the fact, that the regions as defined in the approach above depend on the debris cloud deflection and cone angle, it is not possible to use the simple model in combination with the consideration of oblique impacts. The determination of the region lengths and impact positions is too complicated and does not allow the use of equation (30). Thus, the element model has been used for this purpose.

$$l_V = \sum_{i=1}^{n} (\sum_{k=1}^{5} \frac{1}{5} \cdot g_{k,i}) \cdot l_i \tag{30}$$

The following impact angles are used:

θ_p = 60°, 30°, 0°, -30°, -60°

In order to simulate the debris cloud behaviour due to oblique impacts, the empirical relations for the debris cloud center-of-mass trajectory and cone angle have been used according to Schonberg [2, 3] (figure 15). The empirical function for the in-line debris cloud is valid for the threshold of impact angles, that should be analysed. Thus, this approach regards the behaviour of the in-line debris cloud only. Further investigations considering the normal and ricochet debris cloud should be performed.

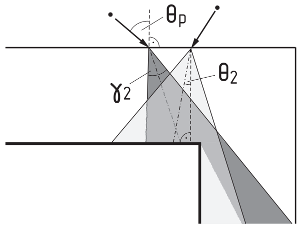

Figure 15. Improved Approach - Considering Oblique Impacts

However, the in-line debris cloud has a larger cone angle than the normal debris cloud (for the set of parameters used), which leads to a higher coverrage of the satellite. Thus, compared to the normal debris cloud, a conservative estimation should be done.

According to Schonberg [2], the empirical relations for the in-line debris cloud are given by:

$$\frac{\theta_2}{\theta_p} = 0.490(\frac{v}{C})^{-0.056}\cos^{0.909}\theta_p(\frac{t_S}{d_p})^{-0.626} \quad (31)$$

$$\frac{\gamma_2}{\theta_p} = 2.539(\frac{v}{C})^{1.217}\cos^{2.972}\theta_p(\frac{t_S}{d_p})^{0.296} \quad (32)$$

The analyses has been done by using the following set of parameters reflecting common values:

- v = 7 km/s
- d_p = 0.1 cm
- t_S = 0.2 cm
- C = 5.1 km/s

Figures 16 and 17 show the results of the consideration of impact angles. Curves 'EM-IA' and 'EM-IA+SF' in figure 16 represent the graph of the improved approach with and without spacing factor. It is obvious, that the region, regarding 100% of the shield edge length is shifted towards the center of the cubic spacecraft. This is due to the deflection of the debris cloud. Applying the spacing factor, there is also a decrease of the reduction function (curve 'EM-IA+SF') in regions 3 and 4, but not as much as with the simple model. This is also because of the deflection of the debris cloud, which reduces the influence of the spacing factor.

It can be seen in figure 17, that the curves of the simple and the improved approach applying the spacing factor (curves 'SM+SF' and 'EM-IA+SF'), have only minor differences. Thus, there is no need for complex calculations as the approach considering normal impacts only leads to satisfying results, at least for a cubic spacecraft with the parameters investigated here.

The scaling factors for the approach considering oblique impacts can be determined by using equation (29) with the parameters x and y given in table 2.

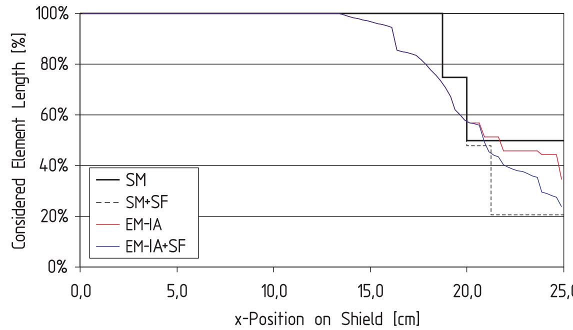

Figure 16. Reduction Function of the Improved Approach considering Oblique Impacts with/without Spacing Factor (a=40cm; S=5cm)

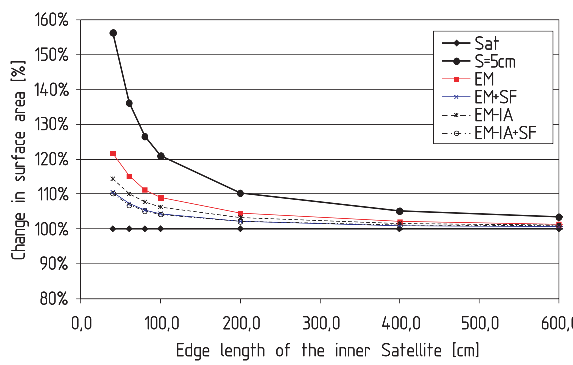

Figure 17. Representative Surface Area of the Improved Approach considering Oblique Impacts

Table 2. Scaling Factors for the Improved Approach (Element Model)

Impr. Appr.	r_l		r_A	
(EM, n=100)	x	y	x	y
with SF	1.5876	-0.9442	3.3117	-0.9505

3. CONCLUSION AND OUTLOOK

This paper describes a conceptual problem of risk and damage prediction analyses concerning the used surface area of small spacecraft. Depending on the analysis concept, the surface area of the shield or the surface area of the satellite itself could be taken. When the surface area of the shield is used for the risk assessment, the analysis results can be described as too pessimistic. On the other side, the use of the satellite surface area is too optimistic. The approach presented in the paper at hand, gives an opportunity to estimate a representative surface area, that reflects better the atual relations when particles impact satellites with shielding (double wall protection system). The representative surface area can be used instead of the shield or satellite surface area. Scaling factors have been determined, in order to scale the geometry model of a cubic spacecraft. This scaling considers the presented approach and leads quickly to the representative area and edge length respectively.

The estimation was applied exemplarily on a cubic spacecraft. Two different approaches have been presented and discussed. An approach which considers only normal impacts, and an improved approach, which considers not only normal but also oblique impacts. In both approaches, the developed estimation is based on reduction of the shield surface and shield edge length respectively. Depending on the expansion of the debris cloud, the shield surface area is either considered full or only partially. Four typical regions could be defined, that reflect this behaviour. Additionally, a spacing factor was introduced, in order to regard the increased spacing, when impacting on the side wall.

It is shown, that the use of the representative surface area instead of the shield area promises realistic results for the risk and damage prediction analyses of small spacecraft without being too pessimistic. Applying the presented approaches with spacing factor, it can be seen, that the representative surface area is in between the surface area of the satellite and that of the shield. The increase in surface area is reduced by about 80% compared to the increase in surface area due to a shield with a spacing of S=5cm. This results as well in a reduction in number of impacts and risk respectively, in the same order. The use of the spacing factor leads in the simple approach (consideration of normal impacts only) and in the improved approach to nearly identical changes in surface area. Hence, the use of an approach under consideration of only normal impacts with spacing factor leads to satisfying results without complex computations.

Due to the fact, that both approaches are applied on cubic spacecraft only, an investigation of other spacecraft geometries is necessary. Furthermore, a generalization of the developed method should be done, in order to apply it to arbitrary spacecraft geometries.

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