STATISTICAL CONJUNCTION ANALYSIS OF LEO SPACE OBJECTS: A MONTE CARLO APPROACH

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ABSTRACT

Conjunction assessments for the LEO space debris objects are essential to identify the risk and to take necessary mitigation measures to avoid possible collisions. The number of space debris objects in the Low Earth Orbits, especially in the Sun Synchronous Orbits, is alarmingly increasing and the recent fragmentation events occurred during the year 2007-2009 added more risk in this region. A statistical conjunction analysis is a good tool for the primary mission design for a spacecraft launch. It will provide vital information on the over all risk in the specified orbit. With the first cut assessment using the statistical conjunction analysis, the orbital definitions can be modified to minimize the collision risks. In an earlier study, a methodology is presented to carry out statistical conjunction analysis of the LEO space objects. In this approach, the numbers of conjunctions in a prescribed altitude bin was estimated. The low Earth orbits, which are affected most by the accumulation of space debris objects, are analyzed using this approach with special emphasis on sun synchronous orbits and is observed that, after the major breakups happened recently, the number of conjunctions in the sun synchronous orbital region is very significant. The study is based on the catalogued objects from the two line element sets. TLEs come with its inherent data inaccuracies. Also the specified orbit of the spacecraft will have some dispersion on the orbital parameters at the design level. This paper utilizes the Monte Carlo approach to account for these uncertainties on the orbital parameters of catalogued objects and on the target object to obtain the expected number of conjunctions during a short term together with the error bands in number of conjunctions.

1. INTRODUCTION

From the time of the first satellite launch in 1957 and the subsequent satellite breakups from 1961 to date, the number of space debris objects has been increasing in an exponential way[1]. Increasing launch activities, the experimental intentional explosions, accidental collisions and fragmentation events and other means of debris creations are real threat to space utilizations in a meaningful way including the human space missions [1]. It is clear that the region, which is more affected by the large sized catalogued debris objects, is the Low Earth Orbit (LEO)[1, 2].

Conjunction may be defined as an event in the future when two objects in Earth orbit could possibly collide or come very close with each other. Conjunction analysis typically provides insight into the possibilities of collisions and also can provide assessment of risk between the space objects for a possible prevention, mitigation or maneuver [2,3]. Conjunctions of the thousands of space debris objects with the operational satellites are primarily very important concerns of space fairing nations. Assessment of number of conjunctions is of importance in mission planning, selection of orbital bands and possible mitigation plans.

In an earlier paper a procedure for estimating the number of possible conjunctions for a target body in a statistical way was presented [4]. In this analysis the risk assessment was based on the gross orbital properties of space objects as a sample space, rather than the individual (discrete) assessment of conjunction among the objects. In discrete analysis the study identifies the secondary space objects come closer to the primary object within a well-defined working or maneuver box [2, 5]. This procedure provides an overall assessment of the conjunction threat in the different altitude bins considering target bodies passing through the altitude bins. The study was based on the catalogued objects from the two line element sets. The present day conjunction analyses and close approach studies are handicapped by many factors including the accuracy of Two Line Elements (TLE), model inaccuracies incur due to the assumptions on atmospheric density models, the orbital perturbing forces and the time constraints required for thorough analysis. Hence the three main factors affecting the soundness of the conjunction analysis are the precision of the data, the accuracy of the propagation models and how early the predictions can be carried out [6]. TLEs come with its inherent data inaccuracies. Also the specified orbit of the spacecraft will have some dispersion on the orbital parameters at the design level. This paper utilizes the Monte Carlo approach to account for these uncertainties on the orbital parameters of catalogued objects and on the target object to obtain the expected number of conjunctions

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during a short term together with the error bands in number of conjunctions.

The analyses are carried out with two TLE sets obtained from space-track website (www.space-track.org), one during Dec 2006 and the other in August 2007, to assess the conjunctions in different altitude bins considering the TLE accuracies.

2. REVIEW OF EXISTING METHODS FOR CONJUNCTION ANALYSIS

With the increased awareness in the risks posed by the space objects on the satellites in orbits, most of the spaces fairing nations carry out discrete conjunction estimates in terms of close proximities. The analysis usually considers the primary objects as the operational satellites and the secondary objects as the large sized catalogued objects in the space. The procedure starts with the collection of the TLEs for all catalogued objects. The trajectories are then propagated forward for a set period of time, usually few days. The resulting ephemerides of the objects are compared with other space objects to identify the objects that come within a critical distance as determined by the warning boxes [2, 3, 4, 5].

During the warning step, the prediction models usually sacrifice some accuracy for speed. The temporal distance between epoch and the point of conjunction can also hamper the precision of the close approach calculations. The objects predicted to enter the warning boxes are reassessed in closer intervals of time using more accurate algorithms and are checked against a maneuver box, which is smaller than the warning box. However the maneuver box is many times larger than the primary object in order to provide a safety margin as the orbital parameters of the tracked space objects are not precise enough [3,6].

Some spacecraft operators already implemented collision avoidance on some key assets [7, 8]. For example, for the space shuttle, NASA uses a defined “warning box” approximately 25 km along the track of the orbit (either leading or trailing), 5 km across the track of the orbit, and 5 km radially from the Earth. The estimated 10 to 30 objects per day that come within the warning box are reassessed using a more accurate algorithm to determine whether any come within a maneuver box of 5 km along track X 2 km in the radial direction. If an object does come within these parameters, the Shuttle may initiate a maneuver to avoid collision [8].

3. PROCEDURE OF STATISTICAL CONJUNCTION ASSESSMENT

To assess the number of objects passing through the particular altitude bin [A km, B km] in a year the following steps [4] are followed.

(i) Perigee filtering: The catalogued objects whose perigee is above B km were filtered out.

(ii) Apogee filtering: The catalogued objects whose apogee is below A km were filtered out

(iii) Other objects, say ‘N’, pass through the altitude bin [A km B km]. Assume that in one orbit, the object crosses the bin 2 times on an average.

(iv) Number of objects per cubic km, Spatial Density S = (2*N) / V, where V is the volume of the spherical shell enclosed by the bin.

(v) Number of objects in a cubic volume in a year is Sy = Sy*N_{orb} where N_{orb} is the number of times an object orbits the earth in a year, considering the repeated orbits of the objects in a year.

(vi) Conjunctions in any warning box are obtained by considering the box’s volume and altitude bin size on the longitudinal distance.

(vii) Number of conjunctions in a year for a target body (Target orbit) is estimated assuming the residential period of the body in different altitude bins and integrating spatial densities in the orbit passing through the altitude bins.

Figs. 1, 2, 3, and 4 gives a sketch of four possibilities of debris objects passage in the spherical shell between A km to B km in altitude. Figure 1, shows a situation where perigee of all the of debris objects lies in [A km, B km] band. Figure 2, shows a possibility where perigee of all those debris is less then A km and apogee greater than B km. Figure 3, represents all those debris objects whose perigee is less than A km and apogee is in the bin [A km, B km]. Figure 4, describes those circular or near circular orbits which completely lie in the bin [A km, B km]. Perigee filtering, apogee filtering and circular orbits within A km, B km bin can also be visualized in these figures.
Fig. 5, provide the number of objects passing through altitude band of 20 km, it also gives a comparison between the two TLE data sets under study, from this it is clear that the number of objects have increased considerably in each of the altitude bands. Figure 6,

gives the number of objects passing through the altitude bands of 20 km in a year taking into account the orbital period of each of the objects, It may be seen that the maximum number of objects are in the band of (820, 840) km. Figure 7, provides the spatial density in the unit of number objects per km^3 volume in a year. It may be observed that the maximum spatial density is in the band of (820, 840) km . From the figures it can be seen that there is a significant change in the risk due to orbiting space debris objects from Dec. 2006 to Aug. 2007. This increased risk can be attributed to the major breakups occurred during 2007.

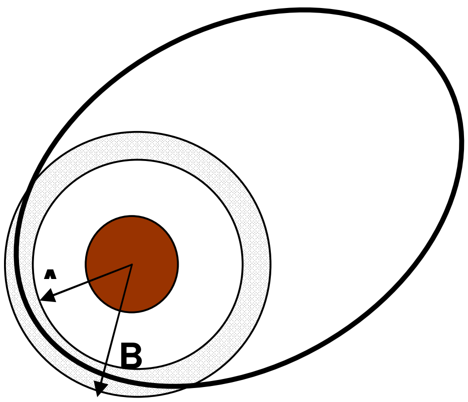

Figure. 1 Perigee is in bin (A km, B km)

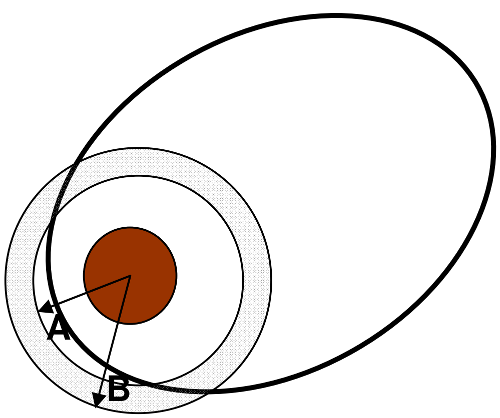

Figure. 2 Perigee is below A km, and Apogee above B km

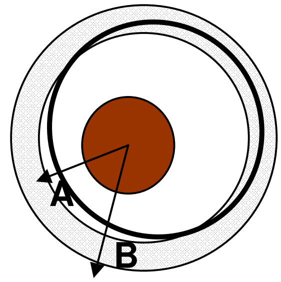

Figure. 3 Perigee is below A km and Apogee is in bin (A km, B km)

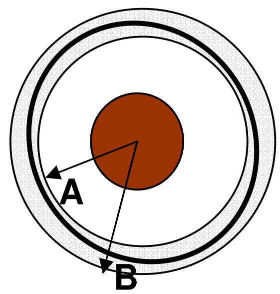

Figure. 4 Orbit lies completely Bin (A km, B km)

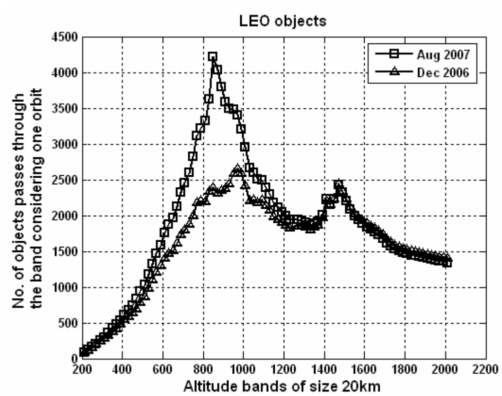

Figure. 5 Comparison of No. of objects in the altitude band of 20 km considering one orbit for TLEs Aug2007, Dec2006.

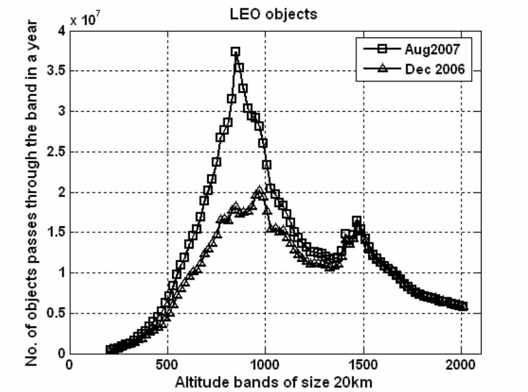

Figure. 6 Comparison of No. of objects in the altitude band of 20 km in a year for TLEs Aug2007, Dec2006.

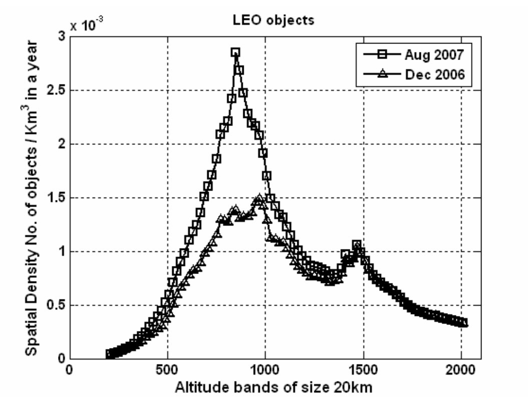

Figure. 7 Comparison of No. of objects per cubic km in a year in the altitude band of 20 km in a year for TLEs Aug2007, Dec2006.

4. ACCURACIES OF TLE ELEMENTS

There are many literatures on the TLE accuracies [9, 10, 11, 12, 13, 14]. For discrete conjunction analysis the TLE accuracies plays important role. The close approaches of the space objects with the primary objects will identify the threats of space objects coming close to the target body. If the TLE accuracies are above the close approach distance, the prediction cannot be considered as a feasible one. One has to keep sufficient margin in warning/maneuver box for the close approach assessments. When the box considered is more, it can lead to unwanted maneuver and studies. So the requirement for the knowledge of TLE accuracy is very important. TLE accuracies will affect both the target object as well as secondary objects. TLE accuracies depend on altitude of the object considered and also other orbital parameters such as inclination.

The present study assumes a unified number, which is sufficiently large to cover the variations with respect to the altitude and other orbital parameters. 3 sigma values used are on position component errors are +/- 2 km and on velocity components are +/- 5 m/s.

5. MONTE CARLO APPROACH

Monte Carlo (MC) approach is the simulation technique used to assess the deviations in the performances of a system when there are variations in the input conditions. In this procedure, a large number of simulations are carried out with randomly generating the input conditions within the constraints of allowable dispersions (3 sigma level) on the inputs. Here the TLE inaccuracies are randomly selected based on normal distributions within the 3 sigma values and the conjunction assessments are carried out. The procedure is repeated with many simulations and the statistical characteristics of the conjunction are generated.

In this Monte Carlo approach Guassian Distribution is assumed for all parameters and the number of runs considered is 400

Steps Involved in simulations:

- Propagating TLE epoch to a common epoch.
- Converting the parameters to position and velocity components
- Adding random perturbations on the parameters as per the specified dispersion levels and distributions
- Convert back to Apogee, Perigee and Inclination
- Applying the Statistical conjunction algorithm as provided in section II.
- Store the outputs such as the number of possible conjunctions in a bin
- Statistical analysis.

Figure 8 shows the mean and 3 sigma levels of number objects per cubic km in a year in the altitude bins obtained based on the MC simulations. It can be seen that the inaccuracies in the TLEs do not affect much the number of objects in a bin.

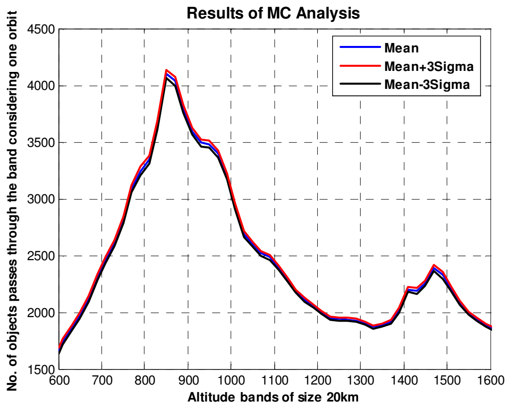

Figure. 8 Results of MC analysis on the number of objects passing through the altitude bands considering one orbit

6. MODELING OF THE SPATIAL DENSITY

The spatial density is modeled using modified Laplace distribution and mixtures of them [16,17, 18, 19]. The Laplace distribution was observed as the best suited distribution for modeling the spatial density distribution, and accordingly the parameters for mixture of Laplace distributions are obtained by random search method. The modeling of spatial density is useful for fast prediction of number of conjunctions in a prescribed altitude band.

6.1 Modified Laplace Distribution Functions

A modified version of the Laplace distribution introducing one more parameter called *area parameter* is considered for the modeling. The modified modeling function with this area parameter '*a*' is of the form

$$f(x) = \frac{a}{2s} \exp\left(\frac{-|x-m|}{s}\right),$$

where 'm' is the location parameter and 's' is the scale parameter.

For binary mixture ,

$$f(x) = a\left[\frac{p}{2s_1}\exp\left(\frac{-|x-m_1|}{s_1}\right) + \frac{(1-p)}{2s_2}\exp\left(\frac{-|x-m_2|}{s_2}\right)\right],$$

where 'p' stands for weight parameter

For tertiary mixture ,

$$f(x) = a.\left(p_2\left(\frac{p_1}{2s_1} exp\left(\frac{-|x-m_1|}{s_1}\right) + \frac{(1-p_1)}{2s_2} exp\left(\frac{-|x-m_2|}{s_2}\right)\right) + \frac{(1-p_2)}{2s_3} exp\left(\frac{-|x-m_3|}{s_3}\right)\right)$$

Here 'a' is the area parameter, p stands for weight, m stands for location and s stands for scale parameter. Monte Carlo analysis is carried out to estimate the model parameters. The number objects in cubic km box was modeled using the binary mixture of the Laplace model. The Table 1 provides the mean and 3 sigma values of the model parameters namely location parameters, scale parameters and weight. It can be noted that the effect of inaccuracies in the TLE sets are not much affecting the estimated modified Laplace model parameters.

7. CONCLUSIONS

A procedure for assessing the number of conjunctions, in a statistical sense, in different orbital bands characterized by altitude bins is presented. Monte Carlo Simulations are carried out to assess the dispersions in number of conjunctions considering the Errors in TLE elements. In statistical sense the TLE inaccuracies do not affect significantly the conjunction estimates.

Table – 1 Variation in model parameters due to inaccuracies in TLEs – results of MC Analysis

TLE Sets	Variation in Parameters From MC					
	a (No of Obj./km^2)	m1(km)	s1 (km)	m2 (km)	s2 (km)	p
Aug, 2007	1.8706 (1.863, 1.881)	850 (849, 852)	200.61 (199.91, 201.34)	1470 (1468, 1471)	394.71 (394.23, 395.43)	0.6
Dec, 2006	1.4121 (1.4041, 1.4210)	900 (898, 902)	291.08 (290.71, 291.64)	1470 (1467, 1472)	395.5 (394.88, 395.93)	0.6

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