ABSTRACT

Among the objectives of a future European Space Situational Awareness System (ESSAS), the capacity of maintaining and providing allowed users with a trustable catalogue of orbiting objects and their associated state vectors is of prime importance. The publication of these orbital data can be done by means of different parameterization methods, all of them with advantages and drawbacks. An analysis of the capacity of three of the possible set of parameters for broadcasting orbital information of orbiting objects is addressed in this paper. These methods are: an interpolation scheme as the one proposed in the CCSDS standard [1], the Global Position System (GPS) navigation message and the Two Line Elements (TLE) model.

Regarding the provision of covariance matrix information, none of the analysed ephemeris types allows the reporting of such data. A modification of the interpolation scheme is suggested so that the accuracy information can be provided to users.

The reported analysis for the Ephemeris generation within the ESSAS is performed by means of the Advanced Space Surveillance System simulator (AS4) developed by DEIMOS Space under several ESA contracts.

1. INTRODUCTION

Different models of ephemeris are used for several purposes. Three parameterizations are studied in this paper: a) Interpolation algorithm, based on Lagrange formulation; b) Two Line Elements (TLE) format and c) Global Position system (GPS) parameterization.

The use of simplified models for ephemeris calculation is one of the sources of error in the position accuracy (on top of it, the Orbit determination accuracy has to be also accounted for); therefore, the selection of an adequate model is an important task. For such a selection, the following factors have to be considered:

- **Accuracy and complexity of the model.** The accuracy analysis exposed in this paper is based on the assumption that the best estimated orbit is that obtained by numerical propagation from the best estimation after the cataloguing process. The results of the different estimated orbits from the different proposed ephemeris (by interpolating, GPS or TLE model) are then compared with the best estimated at different control points.

- **The number of required parameters:** 15 for GPS model, 2 lines of parameters for TLE models. The main feature of the interpolation ephemeris is not the number of parameters but the size of the required files.

- **Interval of validity of the ephemeris.** For the case of the interpolation algorithm, the interval of validity is the period when the orbital data are provided. Within this period, the user must interpolate the data given for every orbit at different steps, to obtain the orbital information at the required time. On the contrary, the GPS and TLE ephemeris are obtained by means of a fitting of the best-estimated orbit during a period of time (Time of Validity). This time span is the interval of validity of the ephemeris, but the user can obtain the orbital state vector out of this interval of validity.

- **Degraded accuracy out of the nominal validity interval.** Interpolation algorithms case does not allow the computation out of the interval of validity. On the contrary, for the GPS and TLE parameterization, it would be possible to assess how the accuracy degrades out of the interval of validity of the generated ephemeris.

2. DESCRIPTION OF EPHEMERIS TYPES

2.1. Interpolated Ephemeris (CCSDS standard)

The implemented interpolation algorithm is an 8th order Lagrange polynomial. Additionally, six and four points interpolation algorithms have also been implemented and tested, and their features analyzed against the eight points algorithm. Tab. 1 gives the typical error provided in position and velocity obtained by different Lagrange order interpolation. 8th order was found to be the most
adequate and therefore it was chosen as the interpolation order.

Table 1: RMS at position and velocity for different Lagrange order interpolation

<table>
<thead>
<tr>
<th>LAGRANGE ORDER</th>
<th>RMS-POSITION (km)</th>
<th>RMS-VELOCITY (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>2.66·10⁻⁸</td>
<td>2.41·10⁻¹⁰</td>
</tr>
<tr>
<td>6</td>
<td>9.82·10⁻⁶</td>
<td>1.065·10⁻⁸</td>
</tr>
<tr>
<td>4</td>
<td>6.45·10⁻⁶</td>
<td>6.69·10⁻⁶</td>
</tr>
</tbody>
</table>

A set of recommendations for the interchange of orbit data, [1] was defined by the Consultative Committee for Space Data Systems. These recommendations include three types of orbit data messages: a) OPM: Orbit Parameter Message; b) OCM: Orbit Conjunction Message and c) OEM: Orbit Ephemeris Message. The OEM is the format implemented in the simulator, since it proposes an ephemeris type for interpolation. On the contrary, both OPM and OCM require the use of appropriate propagators. In particular, the OCM can be considered of interest for reporting covariance data information, which may be required for a number of analyses.

**Orbit Parameter Message (OPM):** An OPM specifies the position and velocity of a single object at a specified epoch. This message is suited to inter-agency exchanges that: involve automated interaction and/or human interaction, and do not require high fidelity dynamic modelling. The OPM requires the use of a propagation technique to determine the position and velocity at times different from the specified epoch, leading to a higher level of effort for software implementation than for the OCM.

**Orbit Conjunction Message (OCM):** An OCM specifies the position and velocity of a single object at a specified epoch. This message is suited to inter-agency exchanges that: involve automated interaction and/or human interaction, and require high fidelity dynamic modelling. The OCM is very similar to the Orbit Parameter Message, and in some sense can be viewed as an extension of the OPM with some additional requirements to accommodate the special needs of orbit conjunction studies. Additionally, some fields are required in the OCM and are less constrained in the OPM. The OCM facilitates the use of a higher fidelity propagation technique than the OPM, thus allowing a better understanding of the position and velocity at times different from the specified epoch. Such needs arise in the effort to determine the probability of orbit conjunctions or radio frequency interference conditions of two (or more) spacecraft. There are some special classes of orbits for which the OCM may be useful (e.g., geostationary, polar, sun-synchronous, LEO, etc., in short, orbits with “traffic problems”). While it is possible to conduct such studies using OPM’s and/or OCM’s as they were introduced in Version 1 of the ODM standard, the OCM contains features specifically included to facilitate a higher fidelity study.

**Orbit Ephemeris Message (OEM):** An OEM specifies the position and velocity of a single object at multiple epochs contained within a specified time range. The OEM is suited to inter-agency exchanges that: involve automated interaction, and require higher fidelity or higher precision dynamic modelling than with the OPM. The OEM allows for dynamic modeling of any number of gravitational and non-gravitational accelerations. The OEM requires the use of an interpolation technique to interpret the position and velocity at times different from the tabular epochs. The OEM is fully self-contained; no additional information is required. Currently, the standard specify a unique file per object, although it is possible that the proposed architecture may support multiple objects per file, and this could be considered in the future.

### 2.2. GPS navigation message

The ephemeris model adopted for GPS (and also used to describe the ephemeris of the Galileo satellites) is based on 15 parameters, 6 Keplerian orbital elements and 9 correction parameters that enhance the overall accuracy for the validity time. The following list summarizes the parameters of the model:

Keplerian parameters:
- \( a^{1/2} \): Square root of the semi-major axis
- \( e \): Eccentricity
- \( i_0 \): Inclination angle at reference epoch
- \( \Omega_0 \): Right ascension of the ascending node at reference epoch
- \( \omega \): Argument of perigee
- \( M_0 \): Mean anomaly at reference epoch

Correction parameters:
- \( \Delta n \): Mean motion difference from computed value
- \( \dot{\Omega} \): Rate of change of right ascension of the ascending node
- \( \dot{i} \): Rate of change of inclination
- \( C_{\sin} \): Amplitude of the sine harmonic correction to the argument of latitude
- \( C_{\cos} \): Amplitude of the cosine harmonic correction to the argument of latitude
- \( C_{\sin}^i \): Amplitude of the sine harmonic correction to the inclination angle
- \( C_{\cos}^i \): Amplitude of the cosine harmonic correction to the inclination angle
- \( C_{\sin}^r \): Amplitude of the sine harmonic correction to the orbit radius
- \( C_{\cos}^r \): Amplitude of the cosine harmonic correction to the orbit radius

A least square method is used for the generation of these parameters once the state vector of each satellite is estimated. From the current estimation of the object
orbital data, it is feasible to compute the satellite state vector by making use of the relationships in Eq.5:

\[ a = \left(\frac{a^{(\frac{1}{2})}}{2}\right)^2 \]

\[ n = \sqrt{\frac{GM}{a^3}} + \Delta n \]

\[ M_k = M_k + n(t - t_0) \]

\[ E_k = M_k + e \cdot \sin E_k \quad \text{(iterative solution)} \]

\[ \cos v = \frac{\cos E - e}{1 - e \cdot \cos E} \]

\[ \sin v = \sqrt{1 - e^2} \sin E \]

\[ \delta u_k = C_w \cos(2v + \omega) + C_w \sin(2v + \omega) \]

\[ \delta r_k = C_w \cos(2v + \omega) + C_w \sin(2v + \omega) \]

\[ \delta i_k = C_w \cos(2v + \omega) + C_w \sin(2v + \omega) \quad (1) \]

\[ u_k = v + \omega + \delta u_k \]

\[ r_k = a \left(1 - e \cdot \cos E\right) + \delta r_k \]

\[ i_k = i_0 + \delta i_k \]

\[ X' = r_k \cos u_k \]

\[ Y' = r_k \sin u_k \]

\[ \Omega_k = \Omega_0 + \left(\dot{\Omega} - \omega_k\right)(t - t_0) - \omega_k t_0 \]

\[ X_k = X' \cos \Omega_k - Y' \sin \Omega_k \cos i_k \]

\[ Y_k = X' \sin \Omega_k + Y' \cos \Omega_k \cos i_k \]

\[ Z_k = Y' \sin i_k \]

The formulation of the least squares method is well known and it is based on the minimisation of the sum of squares of the so-called residuals.

The outcome of this particular activity is a set of predicted satellite state vectors valid for the prediction range (nominally four or five hours for current GPS navigation message).

2.3. Two Line Elements format

US Space Command provides data on position of satellite orbiting the Earth by means of the Two Line Elements (TLEs). Each set of elements is made up of two 60-character lines, which supply the position and velocity of the satellite together with additional information such as the element set number, orbit number and drag characteristics. Line 2 of each set consists basically of position and velocity (mean elements) of the satellite (together with satellite number and a checksum field for error checking), while line 1 contains information on object designation, epoch of the orbital data and additional fields.

Two line elements are mean orbital elements sets generated by fitting observations to a trajectory based upon the SGP4/SDP4 orbital model. SGP4 is applied to objects with orbital period less than 225 minutes (near Earth objects in NORAD classification), while SDP4 is used for orbital periods greater than 225 minutes (deep space objects). Thus this model SGP4/SDP4 needs to be used when handling TLEs to obtain good predictions of position and velocity.

SGP4/SDP4 model gives the position and velocity in an Earth-centred inertial reference frame which is true equator, mean equinox of epoch, called TEME. This reference frame has its z-axis aligned with the true (instantaneous) North Pole and the x-axis aligned with the mean direction of the vernal equinox (accounting for precession but not nutation).

Orbital elements representing the exact position and velocity state vectors are known as osculating elements. The mean elements are fictitious elements.

The difference between these two types of elements is the basic perturbations that cause a satellite to deviate from an ideal keplerian orbit. The main two perturbations are the non-spherical mass distribution of the Earth and the atmospheric drag. SGP uses a third order geopotential model to describe the mass distribution of the Earth. This model includes the equatorial bulge (second order) and the greater amount of mass in the southern hemisphere (third order). SGP4/SDP4 uses a fourth order geopotential model.

The geopotential deviations from 'ideal' spherical mass distribution result in predictable changes to the orbit. The primary gravitational perturbation effects are on the orbital plane and the orientation of the orbit apsidal line. These primary effects are secular, representing constant drift for the ascending node and the apogee-perigee line as a function of time. These constant drift rates are a function of the semimajor axis, eccentricity and inclination of the orbit. The secondary effects are short and long-term periodic effects superposed to the secular drift.

Osculating elements can be obtained by adding these perturbations to the corresponding mean elements. Perturbations are easily computed by using the NORAD software. The way to obtain the mean elements once the osculating elements are known is not as straightforward.

A simple iterative process can be performed. The initial guess for the TLE compatible mean orbital elements are set to the osculating elements. With this initial mean guess, the corresponding osculating elements are obtained by using the NORAD routines. The obtained osculating elements are compared to the input one, and the differences between them are used to obtain the new mean elements guess. The mean elements set for each
iteration is obtained by adding to the *mean* elements set of previous iteration the difference between the *osculating* elements of the considered iteration and the input *osculating* elements. Since the two state vectors (*mean* and *osculating*) are not dramatically different, the algorithm converges in a reasonable number of iterations. Equinoctial elements are commonly used in this iterative process.

But, this simple algorithm does not offer good results for some kind of orbits, i.e. geostationary satellites. In order to avoid these wrong solutions, the osculating elements can also be obtained by solving the system of non-linear equations given by the input osculating vector and the related mean osculating vector (also in equinoctial elements).

This equation system can be defined as:

\[ F_i(X) - X_0 = 0, \quad i = 1,6 \]  

being \( X \) is the mean elements, \( F(X) \) the \( i \) component of the osculating vector associated to the mean element \( X \), and \( X_0 \) the input osculating elements.

In order to compute the osculating element associated to a mean element, that is \( F(X) \), the propagation model to be used has to be defined. Then, periodic variations are added to the mean element. These periodic terms are caused by the gravitational field and Sun and moon effects. Complete description of the long and short terms periodic effects to be added are defined in [2] depending on the propagation model used. First secular effects should be added, this part is dependent on time, but, since the transformation form osculating to mean elements is performed for the same epoch, these secular effects do not play a role. First effects to be taken into account are the long period periodics and then, the short-period periodics are also added.

\( F(X) \) is obtained by calling the propagation routines for model SGP, SGP4, SDP4, SGP8, SDP8 with a propagation time equal to 0. Thus, the output osculating element \( F(X) \) corresponds to the same epoch than the input mean element \( X \).

Formerly, the iterative generation of TLE mean elements from osculating elements has been explained. This procedure for generating the mean elements that minimise the differences in the associated osculating elements at the time when the state vector is provided, leads on large differences between the best estimate (osculating elements propagated with the most suitable propagator) and the TLE propagated vector (and translated to osculating for comparison) when propagating.

In order to avoid such large errors, TLE are generated by fitting the best estimated orbit during an interval, and trying to minimize the residuals (differences between the TLE propagated vectors and the best estimates) at different points within that interval. This process is performed by means of a least square (LSQ) method (similar to that used for the generation of the GPS navigation message). This LSQ process is initiated with the TLE data generated by the direct translation from osculating elements (as explained in former paragraph), and iterated in that way the residuals are minimised along the fitting interval. In such iterations, not only the orbital parameters of the TLE are modified, but also the terms providing information on the variation of those parameters.

### 3. ANALYSIS OF INTERPOLATED EPHEMERIS

Table 2 provides the resulting RMS errors at position for different type of orbits in a 7-days time span. The first column indicates the discretization time of the simulation. Automatic indicates that the user has imposed no discretization time, and relies in the time step recommended by the propagator. This time step varies as a function of the type of orbit, and point in the orbit (less frequent for a GEO than for a LEO orbit; more frequent at perigee than apogee). Figure 1 provides a graphical representation of position accuracy at the different control points. These control points have been defined so that they do not fit the discretization points when the orbital data is provided, so that the required orbital information has to be obtained by means of interpolation.

<table>
<thead>
<tr>
<th>Discretization (s)</th>
<th>LEO</th>
<th>GEO</th>
<th>MEO</th>
<th>GTO</th>
<th>OTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>auto</td>
<td>2.9x10^{-8}</td>
<td>2.8x10^{-7}</td>
<td>6.1x10^{-8}</td>
<td>1.1x10^{-7}</td>
<td>9.9x10^{-8}</td>
</tr>
<tr>
<td>300</td>
<td>8.5x10^{-8}</td>
<td>2.7x10^{-7}</td>
<td>6.2x10^{-8}</td>
<td>5.6x10^{-7}</td>
<td>2.8x10^{-7}</td>
</tr>
<tr>
<td>1000</td>
<td>7.0x10^{-8}</td>
<td>7.4x10^{-8}</td>
<td>6.2x10^{-8}</td>
<td>2.9x10^{-7}</td>
<td>3.7x10^{-7}</td>
</tr>
</tbody>
</table>

![Figure 1- OEM accuracy at position for LEO, and GEO objects at different discretization times](image)

It can be concluded that: a) Lagrange order interpolation of 8th was found to provide a very good accuracy for
interpolation; b) The introduction of time discretization defined by the user significantly reduces the amount of lines and therefore the file text size. Obviously, the time step defined for discretization has to be defined low enough to avoid large errors in the interpolated resulting data; c) Discretization times introduce uncertainties at interpolation, reducing ephemeris accuracy. The results of the performed simulations reveals that good performances are achieved at GEO and MEO type objects for discretizations up to 1000s; however, the rest of orbits requires discretizations below 300s to maintain good accuracies.

4. ANALYSIS OF TWO LINE ELEMENTS FORMAT

TLE ephemeris are generated in two ways:
- **Direct transformation from osculating state vector to TLE**: mean elements are computed by solving a non-linear equation system. This is to minimise the difference between the input osculating element (translated to TLE reference frame) and the state vector computed by SGP4/SDP4 algorithm. Fig. 2 provides the position accuracy of the TLE propagated data during 7 days, when the TLE is generated by direct translation of the osculating state vector. The error at the initial time is null, since the transformation TLE data at this time directly represents the osculating vector at that time. As it can be observed in the Figure, uncertainties rapidly increase with time simulation.

- **By Fitting the estimated orbit during a validity period**: The TLE is obtained by introducing the obtained the TLE directly obtained from the osculating state vector at an epoch into a Least Square Routine that minimise the residuals at difference control points between the best estimated orbit and that obtained by propagating the TLE.

The results obtained by means of the second method (fitting the estimated orbit during a validity interval) are represented in Fig. 3. This plot corresponds to the achieved accuracy in terms of position for different validity periods of 100, 50, 25, 15 and 5 hours (these validity periods are also represented by vertical lines on the figures).

According to the results, if no fit is applied, uncertainties rapidly increase with time (Fig. 2). When a fitted period is applied (Fig. 3), the large uncertainties in the propagated orbits can be diminished; and since, the TLE is obtained by minimising the mean residual during the complete fitting interval, the residual at the initial time is not null (contrary what occurred when implementing the direct transformation). The achievable accuracy, both inside and out of validation time, depends on the type of object and the duration of the validity period.

### Table 3- RMS position accuracy for the different fitting periods for TLE ephemeris during 7 days.

<table>
<thead>
<tr>
<th>Fitting Period</th>
<th>LEO</th>
<th>GEO</th>
<th>MEO</th>
<th>GTO</th>
<th>OTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>100h</td>
<td>3.15</td>
<td>7.13</td>
<td>3.91</td>
<td>7.13</td>
<td>26.50</td>
</tr>
<tr>
<td>50h</td>
<td>3.86</td>
<td>6.56</td>
<td>4.87</td>
<td>6.56</td>
<td>9.93</td>
</tr>
<tr>
<td>25h</td>
<td>18.40</td>
<td>59.70</td>
<td>5.41</td>
<td>59.70</td>
<td>23.40</td>
</tr>
<tr>
<td>15h</td>
<td>228.00</td>
<td>94.90</td>
<td>4.800</td>
<td>94.90</td>
<td>243.0</td>
</tr>
<tr>
<td>5h</td>
<td>809.00</td>
<td>48.00</td>
<td>12.00</td>
<td>48.00</td>
<td>1340.00</td>
</tr>
</tbody>
</table>

### Table 4- RMS position accuracy during fitting interval for different fitting periods for TLE ephemeris

<table>
<thead>
<tr>
<th>Fitting Period</th>
<th>LEO</th>
<th>GEO</th>
<th>MEO</th>
<th>GTO</th>
<th>OTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>100h</td>
<td>1.34</td>
<td>3.42</td>
<td>1.19</td>
<td>3.42</td>
<td>12.90</td>
</tr>
<tr>
<td>50h</td>
<td>0.771</td>
<td>1.860</td>
<td>0.557</td>
<td>1.860</td>
<td>3.970</td>
</tr>
<tr>
<td>25h</td>
<td>0.607</td>
<td>6.530</td>
<td>0.265</td>
<td>6.530</td>
<td>4.58</td>
</tr>
<tr>
<td>15h</td>
<td>0.741</td>
<td>6.340</td>
<td>0.171</td>
<td>6.340</td>
<td>13.70</td>
</tr>
<tr>
<td>5h</td>
<td>0.0315</td>
<td>0.0688</td>
<td>0.0204</td>
<td>0.0688</td>
<td>1.9600</td>
</tr>
</tbody>
</table>

Table 3 provides the computed position accuracy for the different considered fitted periods during the simulated time span of seven days. Table 4 provides the position accuracy during the corresponding the fitting period. Results are expressed by type of orbits. The first column indicates the duration of validity time in hours. This Table 4 provides the mean accuracy within the interval of validity where the orbit has been fitted, whereas Table 3 provides the mean accuracy during the complete evaluation period (7 days). These two types of data allow to evaluate the accuracy within the interval of validity and how the accuracy degrades out of that interval.

![Fig. 2: Ephemeris position accuracy for a LEO, and GEO objects](image-url)
The conclusions of the performed simulations with TLE ephemeris can be summarised as follows.

TLE uncertainties at position and velocity rapidly increase with time, when obtaining the TLE by direct translation from osculating state vector to TLE parametrization. In order to achieve acceptable accuracies, TLE needs to be fitted so that the residual remains low during a validity interval.

It can be distinguished two cases regarding best accuracy performances: On one hand, inside fitted periods (validity interval): best accuracy is achieved for shortest fitted periods; On the other hand, outside fitted period (7 days in our simulated cases): the larger the fitting interval, the better accuracy is achieved out of the validity interval.

By types of orbits, it can be concluded:
- LEO orbits: good accuracy about 30 m is achieved within a 5 h.-fitting interval, but it degrades up to 800 km in 7 days (RMS during the 7 days). On the contrary for a 100 h.-fitting interval, accuracy is about 1.3 km but during 7 days it remains about 3 km.
- GEO orbits: accuracies of the order of 70 are achieved within the 5h-fitting interval. It grows up to 48 km during 7 days, while for 100 h. interval fit accuracy are about 3.5 km inside and remains about 7 km outside the fitting interval. Larger fitting intervals seem to be more appropriated than for LEO objects as the orbit dynamics is slower.
- MEO orbits: provide the best accuracy performances. It remains below 4 km during 7 days at 100 h-fitting intervals (with an accuracy of 1 km inside the interval), and about 12 km when fitting 5 h. periods (with accuracy of 20 m inside such a fitted period).
- GTO orbits provide worse results than previous analysed groups: They show the better accuracy when fitting 5 h. intervals, providing an accuracy of about 2 km but degradation up to 1300 km in 7 days. For the case of 100 h.-fitting interval, 13 km can be achieved inside the fitted period, but it grows up to 25 km during 7 days.
- OTHER orbits: as in the case of GTO, it does not provide good accuracy results. The best accuracy is achieved inside the 5h-fitted period with RMS of the order of 1.5 km; however, it grows up to 2300 km in 7 days. The largest fitted period of 100 h.

provides an accuracy about 6 km inside the fitted period, growing up to more than 25 km in 7 days.

5. ANALYSIS OF GPS EPHEMERIS

Simulations were performed at the validity times of 100h, 50h, 25h, 15h and 5h, similarly to the case of TLE parametrization. Resulting data are reported in Table 5 (position accuracy during the simulated time span of seven days) and Table 6 (position accuracy during the fitting period).

Table 5- RMS at position at the different validation periods for GPS ephemeris during 7 days.

<table>
<thead>
<tr>
<th>Fitting Period</th>
<th>LEO</th>
<th>GEO</th>
<th>MEO</th>
<th>GTO</th>
<th>OTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>100h</td>
<td>5760.00</td>
<td>5.33</td>
<td>2.27</td>
<td>110.00</td>
<td>148.00</td>
</tr>
<tr>
<td>50h</td>
<td>1450.00</td>
<td>9.08</td>
<td>4.060</td>
<td>138.0</td>
<td>292.0</td>
</tr>
<tr>
<td>25h</td>
<td>3.280</td>
<td>11.60</td>
<td>4.95</td>
<td>229.0</td>
<td>331.0</td>
</tr>
<tr>
<td>15h</td>
<td>4.66</td>
<td>15.30</td>
<td>5.170</td>
<td>385.00</td>
<td>347.00</td>
</tr>
<tr>
<td>5h</td>
<td>10.40</td>
<td>7.370</td>
<td>6.360</td>
<td>11000.00</td>
<td>379.00</td>
</tr>
</tbody>
</table>

Table 6- RMS at position during fitting interval for the different validation periods for GPS ephemeris.

<table>
<thead>
<tr>
<th>Fitting Period</th>
<th>LEO</th>
<th>GEO</th>
<th>MEO</th>
<th>GTO</th>
<th>OTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>100h</td>
<td>5760.00</td>
<td>1.36</td>
<td>0.446</td>
<td>61.00</td>
<td>70.30</td>
</tr>
<tr>
<td>50h</td>
<td>1440.00</td>
<td>0.628</td>
<td>0.144</td>
<td>35.90</td>
<td>55.3</td>
</tr>
<tr>
<td>25h</td>
<td>0.579</td>
<td>0.163</td>
<td>0.0527</td>
<td>32.20</td>
<td>28.10</td>
</tr>
<tr>
<td>15h</td>
<td>0.507</td>
<td>0.0236</td>
<td>0.029</td>
<td>26.80</td>
<td>16.6</td>
</tr>
<tr>
<td>5h</td>
<td>0.2630</td>
<td>0.0005</td>
<td>0.0010</td>
<td>44.60</td>
<td>9.3800</td>
</tr>
</tbody>
</table>

The conclusions of the performed simulations for GPS ephemeris can be summarised as follows. As in the case of TLE ephemeris, it can be distinguished two cases regarding best accuracy performances: Inside fitted periods, the best accuracy is achieved at shortest fitted period, whereas outside fitted period, the larger the fitting interval, the better accuracy is achieved.

By types of orbits, it can be concluded that:
- LEO orbits: good accuracy about 30 m is achieved within a 5 h.-fitting interval, but it degrades up to 800 km in 7 days (RMS during the 7 days). On the contrary for a 100 h.-fitting interval, accuracy is about 1.3 km but during 7 days it remains about 3 km.
- GEO orbits: accuracies of the order of 70 are achieved within the 5h-fitting interval. It grows up to 48 km during 7 days, while for 100 h. interval fit accuracy are about 3.5 km inside and remains about 7 km outside the fitting interval. Larger fitting intervals seem to be more appropriated than for LEO objects as the orbit dynamics is slower.
- MEO orbits: provide the best accuracy performances. It remains below 4 km during 7 days at 100 h-fitting intervals (with an accuracy of 1 km inside the interval), and about 12 km when fitting 5 h. periods (with accuracy of 20 m inside such a fitted period).
- GTO orbits provide worse results than previous analysed groups: They show the better accuracy when fitting 5 h. intervals, providing an accuracy of about 2 km but degradation up to 1300 km in 7 days. For the case of 100 h.-fitting interval, 13 km can be achieved inside the fitted period, but it grows up to 25 km during 7 days.
- OTHER orbits: as in the case of GTO, it does not provide good accuracy results. The best accuracy is achieved inside the 5h-fitted period with RMS of the order of 1.5 km; however, it grows up to 2300 km in 7 days. The largest fitted period of 100 h.
LEO orbits: best accuracy of 260 m is achieved inside validation period of 5 h., growing up to 10 km in 7 days. Very bad results are obtained for large fitting periods of 100 h. with accuracies of the order of 5700 km both inside and out validation period. It can be concluded that the GPS parameterization does not allow to mimic the large perturbations affecting the LEO dynamics. These perturbations are really observables during large propagations, thus estimated orbit cannot be fitted during a large period. It can be concluded that GPS parameterization is not suitable for largely perturbed orbits for long validity times, as this navigation message has been defined for the GPS type orbit, which is poorly perturbed when compared with the LEO orbits.

GEO orbits: accuracy of the order of 50 cm within the 5 h.-validity period is achieved, and it, remains about 7 km in 7 days. Good results are also obtained for larger periods; accuracy of 1.5 km inside fitted interval, remaining below 5.5 km during 7 days.

MEO orbits: the best accuracy results are for MEO objects. Accuracy remains below 6.5 km at 7 days in the 5 h fitted period (with accuracy of the order of 1 m inside such interval). Very good results are also obtained for larger fitted period of 100 h. This case provides accuracies remaining below 2.5 km in 7 days. It has to be remarked that the GPS navigation message is defined for GPS orbits, which are included in this objects group. As expected, the GPS navigation message is most suitable for this kind of orbits than for others.

GTO and OTHER type orbits provide worse results than even the poor TLE ephemeris type. GTO results at shorter fitted periods do not follow the previously seen behaviour, as its accuracy is much worse than longer fitted periods. Further analysis needs to be carried out for GTO and OTH cases. Probably, as it was explained for the case of LEO objects, the GPS parameterization is not suitable for highly eccentric orbits, since it has been defined for almost circular GPS orbits.

### 6. COVARIANCE INFORMATION

The accuracy knowledge of the estimated state vector is as important as the knowledge of the state vector itself. This accuracy information is commonly saved by means of the covariance matrix. The knowledge covariance matrix provides the relationship between the real and estimated state vector, and contains the variance of the accuracy of every element in its diagonal, and the cross-correlation between the accuracy of the elements in the non-diagonal elements.

The achievable accuracy with a Space Surveillance System depends on the type or orbit, processed measurements and determination process. For the analyzed case of the European Space Surveillance system, based on radar and on ground-based optical telescopes (as defined in AS4 project, [3]), measurement processing during seven days led on the following results for orbit determination accuracy.

<table>
<thead>
<tr>
<th>Orbit</th>
<th>Total position (m)</th>
<th>Total velocity (mm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEO</td>
<td>10-10</td>
<td>5-10</td>
</tr>
<tr>
<td>MEO</td>
<td>10-1000</td>
<td>20-100</td>
</tr>
<tr>
<td>GEO</td>
<td>10-100</td>
<td>2-50</td>
</tr>
<tr>
<td>GTO</td>
<td>20-100</td>
<td>1-10</td>
</tr>
<tr>
<td>OTHER</td>
<td>10-20</td>
<td>10-20</td>
</tr>
</tbody>
</table>

The results listed on the table show different accuracy levels depending on the object classification based on their visibility criteria (radar or telescopes). LEO objects state vector is determined with highest accuracy due to the uniformity and precision of their radar measurements, while objects observables only by telescopes (MEO and GEO) show a “worse” orbit determination. Combined measurements obtained for GTO and Other objects, imply a higher accuracy level than only telescope observables, but lower than radar ones. Related differences between both of them are due to the percentage of objects observables by radar and telescopes.

### 6.1. Proposed Model for Covariance Matrix

TLE, GPS format and interpolation ephemeris, as the one proposed in the CCSDS standard [1], lack of information on the covariance matrix data. Thus, none of the analysed ephemeris types would allow providing the exporting of these covariance data for users. In order to make use of the knowledge covariance information, it would be required to provide the user with this matrix. Two options are envisaged:

- Provide the complete state vector and covariance matrix set at an instance of time, to be used by an adequate propagator of both state vector and covariance matrix. This option would imply the use of some ephemeris type as the OCM proposed by the CCSDS standard [1], and force the user to have an appropriate propagator for the state vector and covariance matrix.

- Extend the OEM template, to allow the provision of additional terms with information of the required covariance matrix elements. These elements may allow the interpolation of the data at several steps in time for the acquisition of the matrix at other times required by the user. The accuracy of the covariance matrix obtained by this method would be similar to that obtained for the interpolation of the state vector itself. Additional advantage is that the user would not need to have a propagator for state vector nor for the covariance matrix. But this option would
7. EPHEMERIS FILE SIZE

Table 8 gives an estimation of the ephemeris file sizes per object and per total catalogue (considering 15000 catalogued objects). In terms of catalogue size, OEM ephemeris model is not recommended due to the large requirements on catalogue sizing. On the other hand, it provides the better accuracy for the time when the ephemeris are provided, giving accurate enough estimations to make use of the good orbit determination capabilities of the Space Surveillance System. The full catalogue size can be reduced by increasing the time of the reported ephemeris data discretization. Large discretization times can be used for some orbits as GEO and MEO, but not for others.

Table 8: Estimated total catalogue size of each proposed ephemeris type

<table>
<thead>
<tr>
<th>Ephemeris Type</th>
<th>Estimated Total Catalogue</th>
<th>Accuracy for LEO (position, km)</th>
<th>Accuracy for GEO (position, km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OEM (automatic)</td>
<td>16.8 GB</td>
<td>3·10-8</td>
<td>3·10-9</td>
</tr>
<tr>
<td>OEM (∆t=300s)</td>
<td>4.661 GB</td>
<td>8·10-4</td>
<td>3·10-9</td>
</tr>
<tr>
<td>OEM (∆t=1000s)</td>
<td>1.41 GB</td>
<td>7</td>
<td>7·10-8</td>
</tr>
<tr>
<td>TLE</td>
<td>2.16 MB</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(TVAL= 100h)</td>
<td>(TVAL= 100h)</td>
</tr>
<tr>
<td>GPS</td>
<td>4.20 MB</td>
<td>5·103</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(TVAL= 100h)</td>
<td>(TVAL= 100h)</td>
</tr>
<tr>
<td>TLE</td>
<td>75.6 MB</td>
<td>3·10-2</td>
<td>7·10-2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(TVAL= 5h)</td>
<td>(TVAL= 5h)</td>
</tr>
<tr>
<td>GPS</td>
<td>147 MB</td>
<td>0.2</td>
<td>5·10-4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(TVAL= 5h)</td>
<td>(TVAL= 5h)</td>
</tr>
</tbody>
</table>

8. SUMMARY AND CONCLUSIONS

Ephemeris generation of type OEM, TLE and GPS are evaluated within the AS4 software. Results of simulations are exposed in this document along with an analysis of the ephemeris accuracy in terms of the difference between the best estimated orbit and the ephemeris reported orbit, both within the interval of validity of the ephemeris (when applicable) and within a 7 days period.

The analysis is focused on the following factors:
- the accuracy and complexity of the model, based in the assumption that the best estimated orbit is that obtained by numerical propagation from the best estimation after the cataloguing process.
- the number of required parameters, or file size for reporting the complete catalogue
- the interval of validity of ephemeris and
- the degraded accuracy out of such validity period.

Comparing TLE and GPS results, it can be concluded that, although the best accuracies are achieved for GPS model (MEO/GEO orbits inside 5 h validation period), overall evaluation of both parameterizations leads on a recommendation for the use of TLE ephemeris model, since it gives better performances. This is because the GPS model is less reliable for LEO, GTO and OTHER orbits, and it provides also larger uncertainties out of validity periods.

The duration of the fitted period has to be selected as a trade-off between accuracy to be achieved inside and out of such validity period. Highest accuracies will be achieved inside the shortest fitted period but on the contrary, it will increase the uncertainties out of it.

Regarding the provision of covariance matrix information, none of the analysed ephemeris type allows the reporting of such data. It can be proposed to modify the OEM data to report these orbit determination information for the purpose of allow interpolating the required data. This would increase the already large files to be generated for the OEM case. For the reporting of covariance matrix to be used by the user of the Space Surveillance System catalogue data, OCM standardization can also be used, but the this type of data requires the user to have suitable propagators for the state vector and covariance matrix.

In terms of catalogue size, OEM ephemeris model is not recommended due to the large requirements on catalogue sizing. On the other hand, it provides the better accuracy for the time when the ephemeris are provided, giving accurate enough estimations to make use of the good orbit determination capabilities of the Space Surveillance System. The full catalogue size can be reduced by increasing the reported ephemeris data discretization time. Large discretization times can be used for some orbits as GEO and MEO, but not for others.

9. REFERENCES

[1] CCSDS Recommendation for orbit data messages, CCSDS 502.0-P-1.1, 01/10/2006