ORBIT DETERMINATION ISSUES AND RESULTS TO INCORPORATE OPTICAL MEASUREMENTS IN CONJUNCTION OPERATIONS

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ABSTRACT

Operations in geosynchronous orbit are important for many aspects of commerce. Avoiding conjunctions between an ever increasingly crowded geosynchronous environment is therefore becoming more important especially in light of the Iridium 33 – Cosmos 2251 collision. SOCRATES has processed Two-Line Element (TLE) set information for over 5 years. Unfortunately, the TLE information is of limited quality, and obtaining high quality ephemerides is difficult. A next step is to see how we can replace the TLE data for those objects for which we do not get operator data (non-participating SOCRATES-GEO oerational satellites or debris). The International Scientific Observing Network (ISON) is an excellent resource to obtain high-quality observations on satellites. The paper introduces the orbit determination, along with test cases and comparisons with known operator orbits. Finally, we discuss how these observations could be used operationally in the conjunction processing and what considerations should be taken into account.

1. INTRODUCTION

For over 5 years, the SOCRATES effort has processed Two-Line Element (TLE) information for all published satellites in Earth orbit to determine if conjunctions exist. This information is publicly posted at http://www.celestrak.com/SOCRATES/. An important extension has been developed over the last year to focus on the geosynchronous (GEO) population. The fundamental reason for focusing on GEO is the fact that over 25% of the total known population is comprised of operational satellites, where we can obtain data directly from the operators. Each of the owner ephemerides (the data we receive from each owner-operator) includes all maneuvers, whether planned or already executed. The maneuver information is arguably the most important and unique aspect of using this information. Combining the data sources affords us the opportunity to use the owner ephemerides, coupled with the TLE information. Unfortunately, the TLE information is of significantly lower quality, but a large improvement in the conjunction processing is still achieved. With over a third of the operators already providing data, the next step is to see how we can replace the TLE data for those objects for which we do not get operator data (non-participating operational satellites or debris). The International Scientific Observing Network (ISON) is an excellent resource from which we can obtain high quality observations on satellites and can provide the observational data to support non-operational satellites and debris ephemeris generation.

This paper shows how processing of additional observational data can provide better Space Situational Awareness (SSA) for conjunction operations. We compare processing ISON data to Intelsat ephemerides derived from active ranging to give some estimates on how accurate the data is, and what value it would be for conjunction processing. Using Analytical Graphics Inc. Orbit Determination Toolkit (ODTK), we process the ISON observational data as well as Intelsat ranging information. Details are provided to demonstrate the filter results of the orbit determination, and to give confidence in the overall processing. A standard set of test metrics reveals the nature of the processing and the potential accuracy that can be expected for routine operations.

2. BACKGROUND

Beginning in about 2001, cooperation of optical observatories began, and the international Scientific Optical Network (ISON) was created using the Pulkovo telescopes in St Petersburg Russia. Additional observatories have been added to primarily study scientific and applied problems in space, notably in geosynchronous orbits. A great deal of modernization of equipment has taken place, and the Keldysh Institute has been a main financial supporter for the project. By 2009, 32 telescopes at 25 observatories in 9 countries had begun operation around the world.

Since May 2004, CSSI has been providing daily reports of likely conjunctions for the upcoming week for all payloads in earth orbit using the full catalog of unclassified NORAD TLEs available to the public.

For the most part, these data are regularly updated and made available electronically via the Air Force Space Command (AFSPC) Space Track web site. That database is not fully comprehensive, however, because

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it intentionally omits those satellites deemed vital to US national security—about 184 payloads along with the associated rocket bodies and upper stages which delivered them to orbit. Even so, current orbital data is available for 12,765 of 14,058 (91 percent) cataloged by NORAD. Not all of these missing objects are for restricted objects, though. Some are considered 'lost' since they have not been tracked for the past 30 days or longer. It should also be noted that this database does not include those objects too small to be detected or regularly tracked by the US Space Surveillance Network (SSN).

SOCRATES-GEO grew from the initial SOCRATES effort and began operations in December 2007 and focuses on geosynchronous satellites. These satellites represent an interesting area because it's a limited resource, close locations are desired for many satellites, and any debris created from a collision would impact hundreds of additional satellites. At the time of this writing, we are processing 133 of the satellite owner-operator satellites, with 29 more in work. This is about 38% of the active geosynchronous satellite population.

Because the TLE information is of limited quality, we use public data such as GPS almanacs, GLONASS precise ephemerides, and the Intelsat 11-parameter data to supplement the TLE's. These data sources can all be imported into STK directly, or used to generate TLE's.

An advance is possible through the supplemental TLE's. Here, we process an external ephemeris (usually from a numerical orbit determination and subsequent propagation), and fit a TLE to this information. Because of the larger observation density, SGP4 is able to better model the orbit, and predictions from the resulting TLE's can be 10-20 times better than a comparable TLE developed from AFSPC processing of SSN observations (Kelso 2007).

The process of converting the ephemerides to TLE's can improve the accuracy over a standard TLE (Kelso 2007), but this is still not as accurate as simply using the external ephemerides. Where ephemerides do not cover the interval of interest, the best option is to perform a numerical orbit determination on the ephemeris and then propagate through the desired interval of time. This effort is currently in work.

The various options of data are shown in Fig. 1. SOCRATES uses the TLE on TLE approach. SOCRATES-GEO takes advantage of each of the various options depending on what data is available. The status of the second satellite (active or not) influences the choices available for ephemeris information. There are many permutations when combining each of these data sources. The operator ephemeris vs operator ephemeris derived from observations results in the most accurate processing, and the TLE vs TLE is the least accurate.

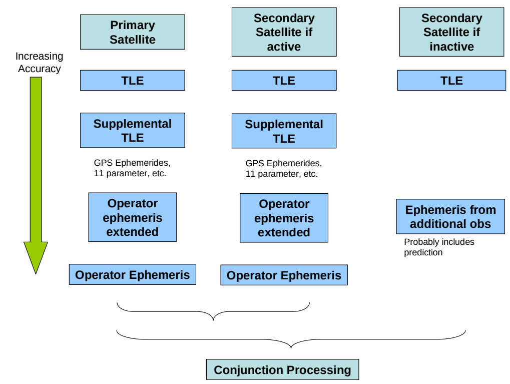

Figure 1. Conjunction Processing Options

The SOCRATES-GEO effort looks at all objects that pass within 250 km of GEO. The reports are similar to the regular SOCRATES reports, but also include the ability to directly use improved data sources (supplemental TLE's or ephemerides), getting standard reports, allowing for restricted access, and customizable user notification.

Our experiences from SOCRATES-GEO shows us many more things than just the original intent of refining the conjunction processing for GEO satellites. The data center concept conclusively demonstrates improvements to orbital accuracy, the ability to reduce search volumes for sensors, the reduction of false alarm rates for conjunctions, and even shows how SSA tracking requirement could be reduced (trust but verify).

3. ORBIT DETERMINATION PROCESSING

We process additional observations to replace the TLE information with the Kalman filter in Analytical Graphic Inc's Orbit Determination Toolkit (ODTK). Because the filter processes data differently than traditional batch least squares, we briefly introduce the types of reports we'll use to give us confidence in the results. Later, we'll compare to independent reference orbits to assess the general accuracy of the independent observations.

We'll use an example (NORAD satellite 8832) to illustrate the processing of each of these steps. The number of measurements = 724 and ranged from 19 Jul 2007 15:28:57.900 to 08 Nov 2007 13:35:32.460 UTC. The satellite is in a geosynchronous orbit, tracked from ISON optical sensors. We used the initial setup for the sensors provided by ISON.

The initial step consisted of obtaining the TLE's for satellite 8832 and estimating an initial state with the ODTK/InitialStateTool. The results are done at the time of the first observation (19 Jul 2007 15:28:57.9 UTC) and the TLE had an epoch of 19 Jul 2007 06:02:51.866 UTC. The orbital elements were: a = 42738.6, e = 0.01441189, i = 9.90743°, Ω = 177.34°, ω = 224.777°.

We then look at when the measurements are taken, shown in Fig. 2. The intervals are shown for each 10-

day period. Notice that many more observations were taken during the first 2 weeks, so we could focus on that interval to better represent the operations expected to incorporate ISON data for conjunction processing.

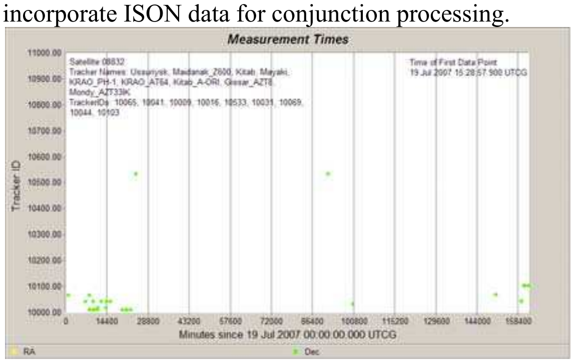

Figure 2. Measurement Times

Running the filter over the entire interval, we can examine the filter position uncertainty (Fig. 3). Because the results are not too good at this point, we will try to obtain a better initial estimate.

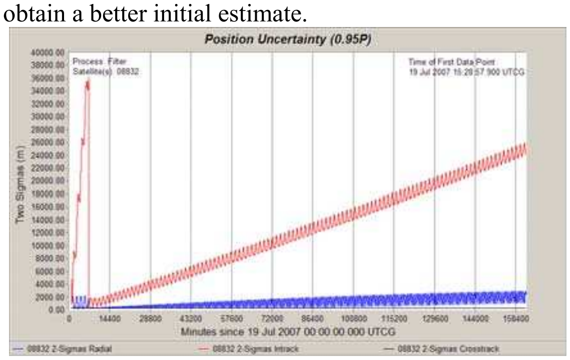

Figure 3. Filter Position Uncertainty

The first step consisted of using an Initial Orbit Determination (IOD) method to obtain an estimate of the orbit. Because the data is optical angles-only, the Gooding angles-only method is used to process the data. Beware that different results are produced by each combination of observations, thus, several trials were conducted. It "seems" that observations that are about 3 or 4 minutes apart will work acceptably. Also, it's important not to have the data too far from the first observational data point for later processing. However, consistent results didn't appear, so we tried the Least Squares approach.

With the LS approach, it's important to setup the stages properly. Because there was more frequent data during the initial weeks, we chose an interval from 19 Jul 2007 15:28:57.900 to 5 Aug 2007 17:38:22.000 UTC. This resulted in 506 observations being processed. This is more observations than we usually use in the LS phase, but because the data span covers several months, we wanted to get a better averaged orbit to initiate the filter. A difference from traditional filter approaches is that in ODTK, the LS solution is only used to initiate the state for the filter. The LS covariance is not used because the process noise is mathematically modeled depending on the force model settings and orbit type. This eliminates the need for "tuning" and is quite useful when working difficult problems because the initial state does not have to rely on a LS solution as long as an initial state can be found that is accurate "enough". We generally do not estimate ballistic coefficient (BC) or solar radiation pressure coefficient (SRP) unless we're reasonably confident of the data and satellite. The MaxIterations was set to 20. The LS run ran well – 8 iterations, 465 residuals accepted, only 41 rejected. There were no large condition numbers or other warnings, so the state was transferred to the satellite. The orbital elements were as follows: a = 42739 km, e = 0.0141292, i = 9.90656°, Ω = 177.3°, ω = 225.265°.

After running the filter, we can look at several reports.

We can look at the residuals divided by the sigmas and the position uncertainty. These are essentially "normalized" and should fall within a limit of +-3 (Fig. 4.).

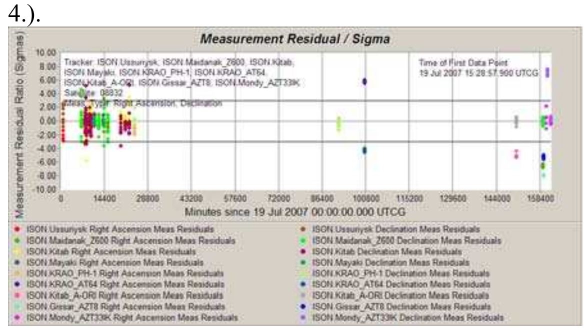

Figure 4. Filter Residual Ratios

Next, we can look at the smoother position uncertainty results. We usually expect a "bathtub" like figure where the ends exhibit a little more uncertainty, and the middle shows the least variation. We didn't quite get that here, although the values are somewhat constant throughout the interval. Remember that there were not many observations in the middle of the span, so we would expect a slight increase in uncertainty during those times.

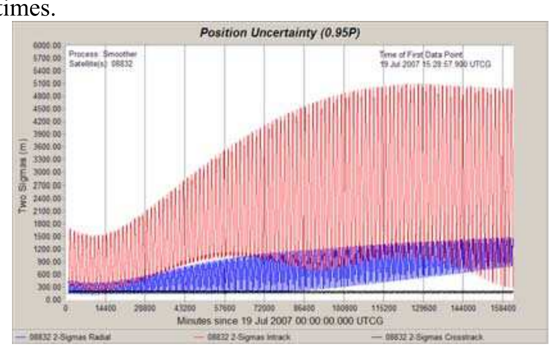

Figure 5. Smoother Position Uncertainty

A unique report in ODTK is the filter-smoother consistency test. The report is found using the filter and smoother runs. Essentially, the report is an instantiation of the McReynolds filter-smoother consistency test. It states that the difference of the filter and smoother runs is a zero mean Gaussian vector with a covariance given by the difference of the two covariances. The report is not a vector per se. It's the difference in the filter and smoother estimates divided by the difference between the filter and smoother sigmas. Thus, it's a test statistic that can be applied to any parameter being estimated, and the results should fall within a ±3 limit. We often examine the position and velocity consistency, as well as SRP and others.

The differences let you examine the transponder, station biases, etc. to determine if the modeling is proper. The position consistency check is shown in Fig. 6. If the modeling is off in any parameter, the test will exceed the ±3 limit. Sometimes uncertainty in the satellite model (like tumbling) can cause periodic variations in the consistency results).

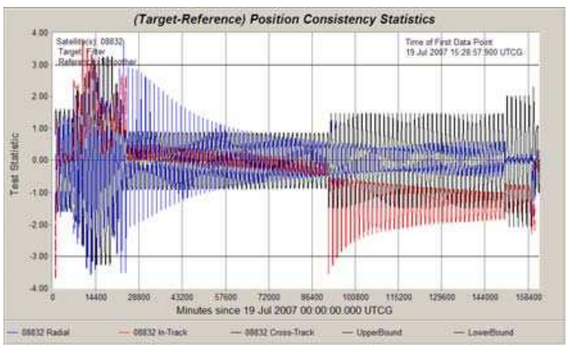

Figure 6. Position Consistency Test

We could do additional analysis, to include looking at the observations that were omitted in the original scenario. However, these results appear to be reasonable for an initial analysis. Given the span of time and the orbit type, an average uncertainty of a few km is significantly better than comparable TLE's. Although the TLE was accurate enough to initiate the process, the position uncertainty can be misleading because the error growth will show simply the covariance propagation as many or all of the observations are being rejected. The consistency test provides additional confidence that the modeling is properly set, and that the results should be adequate to use in analysis. Note that in this case, there was a TLE within a few hours of the first observation time. This is not generally the case, and if a maneuver has occurred, there may be no TLE within a few days of the first observation. In these cases, additional work may be performed with the IOD and LS techniques coupled together.

To obtain an idea of the improved accuracy of the entire ephemeris generated from the smoother, we took individual TLE's for satellite 8832 and created an ephemeris propagating from each TLE epoch to the next TLE epoch. We then compared this to the smoother ephemeris because it is the best estimate of the ephemeris from the filter run. Figure 7 shows the results. Note that if we assume the smoother ephemeris is "close" to the true orbital position the TLE's are off by an average of about 10-15 km at epoch.

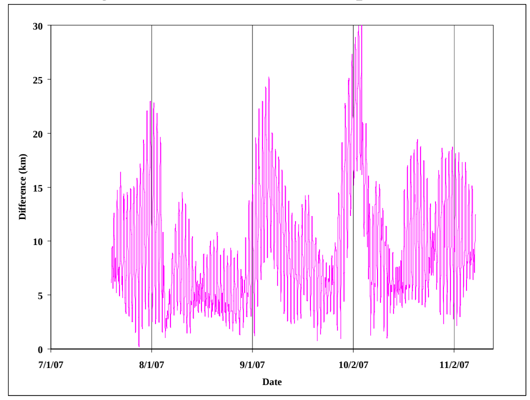

Figure 7. Smoother vs TLE Ephemeris Differences

4. INTELSAT TESTING

To evaluate the accuracy of the ISON derived orbits, we processed observations on some Intelsat satellites, for which we had independent observations and maneuver information. The first phase was to create a reference orbit from the observations.

The first test case was for Intelsat 904, and there were 13166 measurements from 30 Jul 2008 00:41:34.846 UTC to 15 Nov 2008 23:46:44.844 UTC. There were 5 sensors that took data – PRE15T, FOT10L, IEU15A, PET-1T, 64. The filter state is initialized from the Orbit Parameter Message (OPM). Running the filter for the entire interval, we get the following residual ratios. While the test statistic fails at some points, it's likely an improperly inserted maneuver in late August or early September.

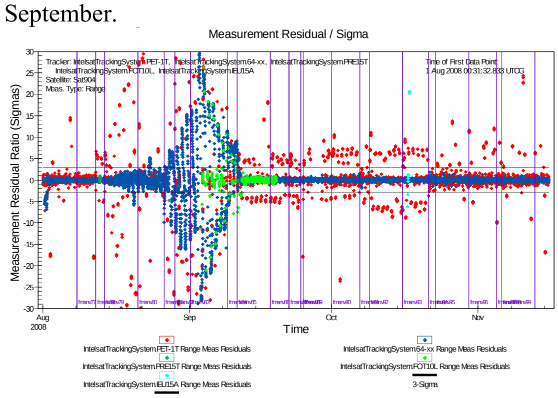

Figure 8. Residual Ratios, Intelsat 904

Nevertheless, the filter processes through the data. If we examine the smoother position uncertainty, we can see where the maneuvers are, and where likely small corrections could be made to the maneuver values. Note that the general uncertainty is about 5 – 20 km.

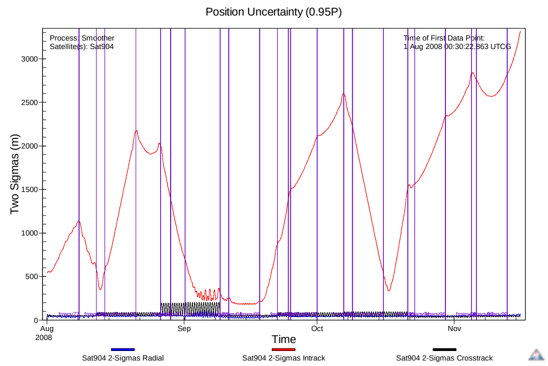

Figure 9. Smoother Position Uncertainty, Intelsat 904

We had ISON Measurements from this same interval with 550 total measurements from 02 Aug 2008 19:38:49.300 to 09 Nov 2008 22:41:57.600 UTC. Because there were numerous maneuvers in the data, we first processed the data with the known maneuvers included, but just during the first two weeks when there were additional observations. Although not realistic, this will give us an indication of the accuracy we can expect from the ISON data. Fig 10 shows the residual ratios for the first few weeks.

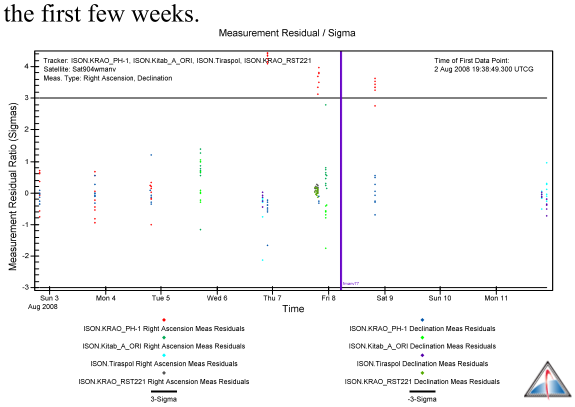

Figure 10. Residual Ratios, Intelsat 904, ISON

The position uncertainty is shown in Fig 11 and the position consistency is in Fig 12.

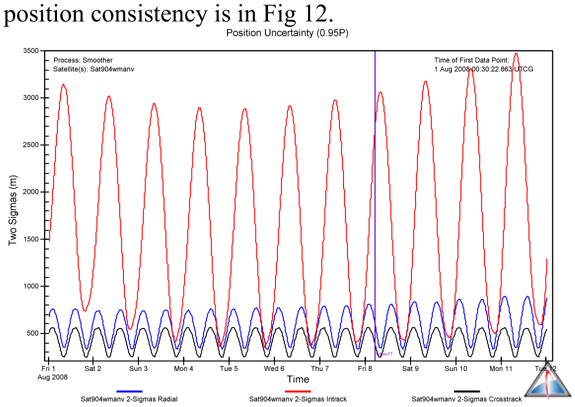

Figure 11. Position Uncertainty, Intelsat 904, ISON

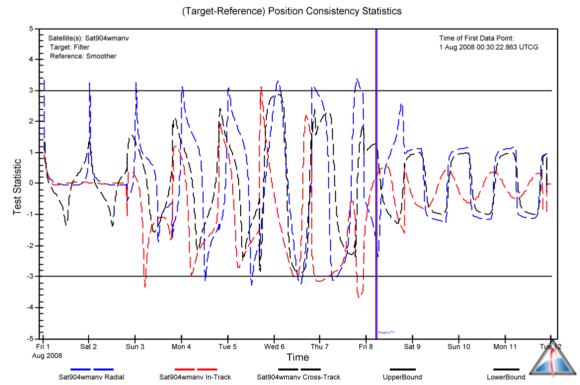

Figure 12. Position Consistency, Intelsat 904, ISON

We then compared the reference ephemeris created from the Intelsat data to the ephemeris created by the ISON data (RSS position magnitude). The results are in Fig 13. There is similarity to Fig 11. This is some of the confidence we sought to compare the ISON created ephemeris to the Intelsat reference orbits.

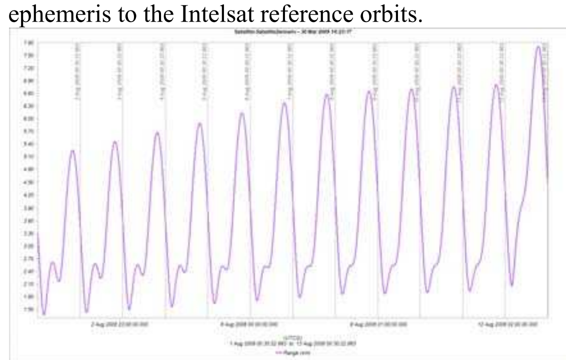

Figure 13. Intelsat vs ISON Ephemeris Differences

We then tried a test by taking all the maneuvers out. As expected, the results quickly showed the first deleted maneuver. However, given sufficient observations (as we had early in the interval), the filter was able to process through the maneuvers. Fig 14 shows the residual ratios for comparison to Fig 12.

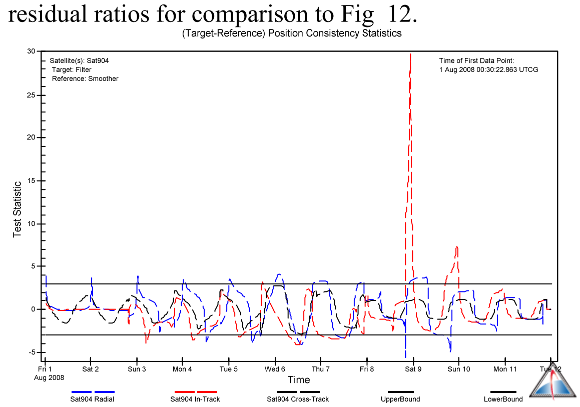

Figure 14. Position Consistency, Intelsat 904, ISON

Notice that when the maneuvers are unknown, the results are significantly worse.

5. INTELSAT F3 (NORAD 4376) TESTING

This satellite is interesting because although it has been dead for several years, AFSPC last tracked it in 1971![1] ISON regularly tracks this object. Aside from the obvious conjunction implications, the ability of external sensor networks to provide observations where holes in coverage may exist. Our ISON data included 876 measurements from 31 Dec 2007 01:18:32.780 to 28 Jan 2009 22:02:44.000 UTC. We found some initial estimates for the mass and area (293 kg and 25m^2 respectively) from the internet (http://www.astronautix.com/project/intelsat.htm).

Because we had no other information to obtain initial estimates with, we broke the data into two portions to take advantage of additional observations in August. This enabled us to get better IOD results.

The first section started in December 2007. IOD results gave orbital parameters of a = 43655.569, e = 0.025511, i = 10.97155°, Ω = 323.687°, and ω = 182.506°.

These parameters let us perform a short LS with the additional observations for about a 2-week period. The new orbital elements were a = 42184.5, e = 0.000499267, i = 10.9499°, Ω = 324.068°, ω = 263.236°. The LS solution converged without comment.

Running the filter, the residual ratios show good performance throughout the first interval, except at the end where we have only a few data points. This will typically show up as a larger uncertainty because there is no additional data past that time.

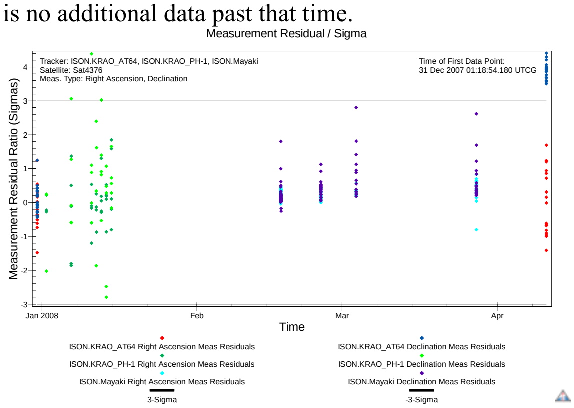

Figure 15. Residual Ratios, 4376, ISON

The position uncertainty for the approximately 4 month interval shows about a 20-25 km uncertainty, but could be considerable improved for shorter intervals with additional data. This is better than most geosynchronous TLE's. We can also be reasonably confident that these results are a good indicator of the true performance based on the previous Intelsat comparisons. Maneuvers are not a problem because the satellite is no longer operational.

[1] We have done additional research to determine that this is indeed Intelsat F3 (4376) and plan a future paper to discuss our findings.

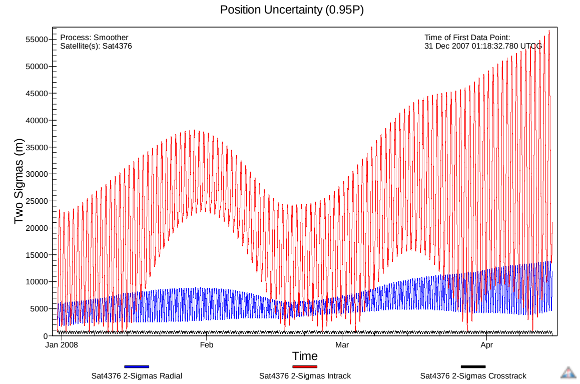

Figure 16. Smoother Position Uncertainty, 4376, ISON

Finally, we look at the consistency test and although some points miss the test 3 statistic, the results are acceptable and we could use this ephemeris for conjunction planning.

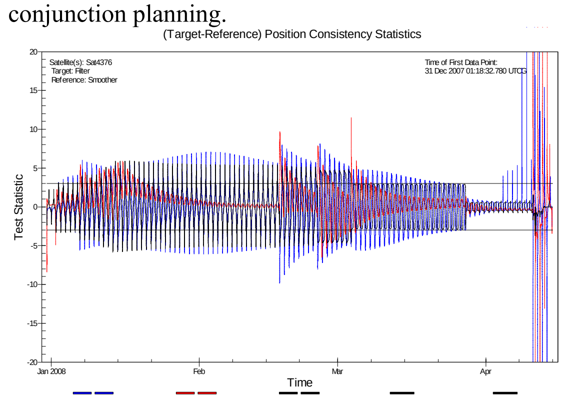

Figure 17. Position Consistency, 4376, ISON

The second section of data begins in August 2008. The IOD yielded a = 38407.6, e = 0.0712387, i = 10.5502°, Ω = 342.961°, ω = 163.68°. There were not a lot of observations to work with, and the various combinations didn't yield any better results than this. Knowing that the orbit must be geosynchronous, we used the semimajor axis and eccentricity from the previous case.

Then doing the LS from 1 Aug 2008 to 8 Aug 08, we find a = 42169.8, e = 0.00120507, i = 10.6178°, Ω = 322.755°, ω = 248.966°. There was a large condition number, but the results seemed to be reasonable.

We processed the ISON data using this initial estimate. We would expect the results to be somewhat similar to the previous section.

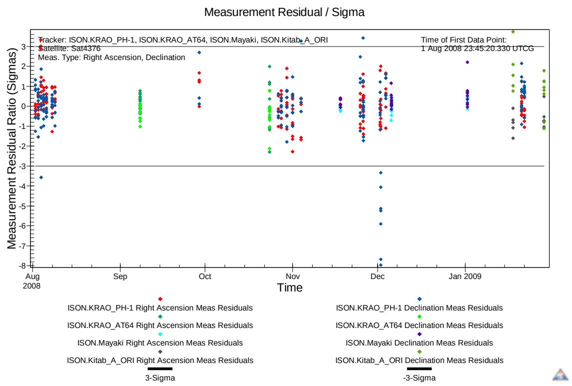

Figure 18, Residual Ratios, 4376, ISON

The position uncertainty is shown in Fig 19. Note how similar it looks to Fig 16.

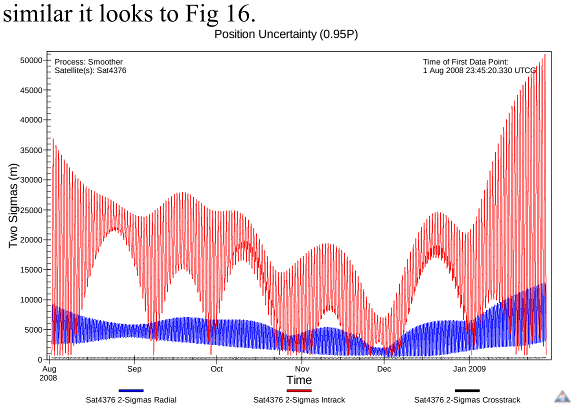

Figure 19. Smoother Position Uncertainty, 4376, ISON

The position consistency is also very similar to Fig 17.

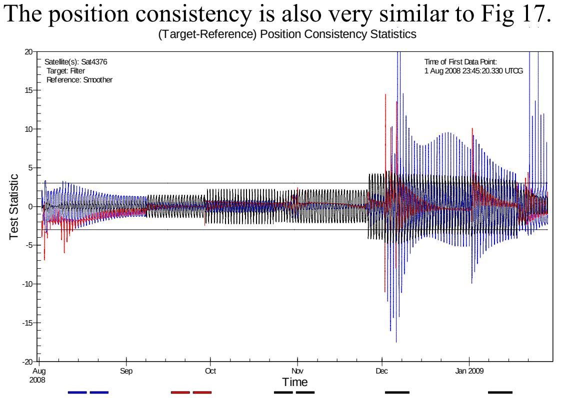

Figure 20. Position Consistency, 4376, ISON

The results are about the same as the first phase. In regions where the observations were sparse, plus a long way from the original epoch, the solution degraded.

6. PROCESSING ISSUES

The previous examples (and others), required several other issues to consider when processing additional observational data. The first considerations involved the coordinate frames, initial orbital estimates, and type of sensors.

The coordinate frame for optical measurements is generally topocentric, but some centers perform conversions to other coordinate systems. We also found that not all systems with the same names are the same as well. Initial test cases between organizations are essential to fully understand exactly what data is being transferred.

The initial estimate is crucial to processing the data. Often, a TLE is not sufficient to initiate an OD process. We saw an example of this earlier with Satellite 8832. The type of data can also affect our ability to arrive at an accurate initial estimate. Radar data generally produces a much more accurate initial estimate, while optical data generally requires additional processing (IOD and LS) to get the initial estimate.

The accuracy we obtain from observational data is a function of many variables including the number, type, and quality of the observations in a pass over a sensor, the location of the data within a pass, the total number of tracks, number of sensors, location of the sensors, the processing technique, etc. To obtain the best accuracy, we trade-off the merits of each of these aspects versus their costs.

Sensor observations will not always be available when needed. Maintenance, downtime, and tasking priority can affect the quantity of observations, and personnel actions can even influence quality and, in some cases, prevent the observations themselves from reaching the user. Unfortunately, we can't reliably predict these effects. One solution is trying to gather as many types of data as possible, so we'll have a backup whenever normal procedures fail. This is particularly challenging for space surveillance operations.

Closely related is the amount of data available for a satellite. Continuous data does not exist for most satellites (except for satellites with GPS receivers). If we observe only a small arc of a satellite's orbit, it's much more difficult to determine an accurate answer. This difficulty can lead to mismodeling of the orbit. Too little data can also result in our inability to estimate additional state (solve-for) parameters. Vallado and Carter (1997) show we need more data if we want to solve for station biases in addition to the position and velocity vectors. This is especially true for eccentric, deep space, and drag perturbed orbits. Remember the solve-for parameters "soak up" any mismodeling in the force models, so we want to have sufficient data to accurately determine the effect of perturbations, and not just the dynamic mismodeling.

In general, more distinct tracking data should give better accuracy (Central Limit Theorem). However, this assumes that the biases are identified and removed, and that the relative accuracy (weighting) of each observation is known and used in the estimation process. If the orbit solution quality degrades with additional data, the model and the calibration are probably the cause. Additionally, suppose a sensor reports only two or three data points. Obviously most modern sensors receive much more data—often hundreds or thousands of points per pass. One approach to obtain more data could be to task more sensors to observe the satellite and report additional sparse sets of observations. Although that would give us a slight improvement, we could incorrectly conclude that sparse data from many sites permits highly accurate orbit determination. This is simply false because there is a trade-off between many variables, as mentioned at the beginning of this section. Quite often, denser data (even from single sites) can actually improve the quality of the orbits and reduce the overall sensor tasking when combined with accurate biases and proper numerical processing. Fonte (1995) showed that dense, real-world observations from a single station could produce orbits accurate to less than 10 m for a 12-hour prediction on a satellite at about 800 km altitude. Sparse data (less than five observations per pass) even from multiple sensors can actually increase this error to over 400 m for the same satellites (Phillips Laboratory, 1995). This suggests that dense observations (perhaps on the order of 50-100 per pass) can produce precise orbits (10 m to 100 m for many satellites).

The location of the measurements in a pass also affect the OD result. When the satellite is very low to the horizon, a small vertical (elevation) error can result in a large uncertainty about where the satellite is in its orbit—the along-track component. As the satellite travels over the site, a small horizontal (azimuth) error will become a large plane change or a cross-track error. This effect is magnified because the satellite is usually closest to the site at its maximum elevation, or culmination. If we combine these results, we get an error ellipsoid about the satellite. In general, along-track errors are greatest because of a lack of precise timing information and the uncertain nature of the local satellite environment (the non-conservative forces such as drag and solar radiation pressure tend to retard the satellite's motion). Cross-track errors are usually smaller, typically resulting from a sensor's misalignment. Radial errors are usually the smallest

A more practical concern is the format of the observational data. Most formats are densely packed files with little to no documentation. ISON uses a .geosc format that is read directly by ODTK. Unfortunately, this format truncates the precision of the measurements, as most formats do. Although simple scripts can be written to convert the data to a useable form for input into ODTK, it may be better to adopt a new format that is a simple ASCII/XML form that permits various data types, precision, and includes enough information to specify what fields are being transmitted.

7. CONCLUSIONS

We have examined the option of processing additional observations for satellites under consideration in conjunction screening to improve the quality of the existing TLE information. In particular, we examined geosynchronous satellites and the addition of data from ISON. The data was shown to be more accurate than the TLE information. The improved accuracy ultimately reduces the number of unnecessary avoidance maneuvers by better modelling the predicted conjunction. Examples of data processing for operational and non-functioning satellite orbits were made with functioning and non-functioning satellites. For the operational orbits, we were able to com-pare against reference orbits generated from external telemetry. We discussed many of the data and formatting issues required to effectively add optical measurements into an OD process.

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