INCREASING THE ACCURACY OF ORBIT FORECASTING ON THE BASIS OF IMPROVEMENT OF STATISTICAL METHODS FOR PROCESSING MEASUREMENTS

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ABSTRACT.

To overcome the limitations of the traditional approach the author developed the adaptive technique of orbital elements determination taking into account statistical characteristics of disregarded disturbances. This technique, called "Optimum Filtration of Measurements (OFM)", has some common features with the Least Square Technique and the Kalman filter. Special corrections are added to the results of integration of the equations of motion. The experience of application of this technique demonstrated the possibility of increasing the accuracy of estimation and prediction of satellite orbits. This paper presents the review of investigation results obtained recently on the basis of applying the OFM technique, namely:

- Development of the algorithm and software for filtering the measurements with using the considered technique;
- The technique efficiency evaluation from the results of processing the modeled and real information.

1. INTRODUCTION

The development of the techniques of orbital parameters determination from measurements began some hundreds years back. It is associated with names of Kepler (beginning of 17-th century), Gauss and Legendre (beginning of 19-th century), Fisher (beginning of 20-th century) and some other scientists. They developed the least-square technique (LST) and the maximum likelihood technique (MLT), which remain as a basis of modern algorithms of estimation and prediction of orbits [1, 2].

The intensive space exploration began after launching of the first Soviet satellite in 1957. A lot of new applied problems appeared. On this basis, as well as in connection with unique achievements in computer technology, the new era began in the development of orbit estimation and prediction techniques. The most essential methodological achievements consist in *accounting for random disturbances in the satellite motion model on the basis of applying the Kalman filter (KF) [3], as well as in developing the technique of successive processing of measurements [4]*. A lot of publications were devoted to these issues, such as work [5], monographs by V. Mudrov [6] and P. Elyasberg [7]. The most complete review of modern orbit estimation and prediction techniques was given in D. Vallado's monograph [8].

We shall consider the problem of orbit estimation and prediction from measurements in the simplified (linear) formulation. In the majority of cases the solution of nonlinear problems is reduced just to this formulation.

The time variation of satellite's state vector (x) occurs according to the differential equation

$$\frac{dx}{dt} = A(t)\cdot x + B(t)\cdot q(t). \qquad (1)$$

Here A and B are known matrixes, q is the Gaussian random process with known statistical characteristics:

$$M[q(t)]_0 = 0, \quad M\left[q(t)\cdot q^T(\tau)\right]_0 = K_q(t,\tau)_0 . \qquad (2)$$

The measurements, carried out at various time instants (t_i), are the known linear function of the state vector

$$z_i = h_i \cdot x(t_i) + v_i, \; i=1, \ldots, k, \qquad (3)$$

and contain random errors v_i distributed according to the normal law with specified statistical characteristics

$$M[v_i]_0 = 0, \; M\left[v_i \cdot v_j^T\right]_0 = R_{ij}, \; M\left[v_i \cdot q(t)^T\right]_0 = 0 . \qquad (4)$$

It is required to determine the state vector estimate $\hat{x}(t)$ with the minimum variance at any time instant $t \geq t_k$.

Table 1. Techniques of solution of the considered problem under various conditions

Problem solution techniques	Modifications of statistical characteristics			
	Noise q is absent		Noise q is present	
	$R_i \cdot \delta_{ij}$	R_{ij}	$K_q \cdot \delta(t-\tau)$	$K_q(t,\tau)_0$
Joint	LST	MLT	**Optimum filtration of measurements**	
Successive	Recurrent LST	-	KF	KF modifications

Various techniques are applied for solving the considered problem. They differ in these or those simplifications (modifications) of statistical characteristics Eq. 2 and Eq. 4, as well as in application of grouped (joint) or successive processing of measurements. Tab. 1 presents the solution techniques corresponding to various modifications of problem formulation.

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Explanatory.

- The assumption on the absence of noise is some idealization, because the unknown disturbances *(q)* are always present under real conditions. Therefore, the estimates corresponding to the «noise is absent» condition, namely, the estimates based on LST and MLT application do not provide obtaining state vector estimates, which are optimum in accuracy. They approach to optimum ones only in the case of rational choice of a fit span (the number of measurements). Under these conditions, the application of the recurrent LST, though provides lowering labor consumptions, results in the negative consequences when the fit span excessively increases. The allowance for correlation of measurement errors (R_{ij}) on the MLT application basis results in essential increasing labor consumption at calculations (growing computer time expenses), that has especially great effect with growing number of measurements.
- The allowance for statistical characteristics of noise makes it possible to avoid the LST limitations and to increase the accuracy. The Kalman filter (KF) and its modifications, based on successive (recurrent) processing of measurements, have been widely spread. Such a technology occurred to be especially useful under the conditions of massive calculations and provided essential computer time saving as compared to LST application. The allowance for color noise requires knowledge of noise's statistical characteristics and is implemented in KF modifications. One of such modifications was developed by the author and was used at processing the real information [9 - 11]. The limitation of KF and its modifications consists in possible divergence of updating process at excessive growth of a fit span (the number of measurements).
- The orbit estimation and prediction theory, based on accounting for color noise and Optimum filtration of measurements (*OFM*), is outlined in detail in paper [9]. 36 years have passed after this publication. However, this technique has not been widely applied for a number of reasons: a) insufficient computer technology characteristics (speed, word length, memory) have complicated its application; b) statistical characteristics of disturbances have not been studied well enough; c) strict demands to accuracy have not been placed on the results of massive calculations. The role of listed reasons is not so essential nowadays. So, the application of the considered technique of the *optimum filtration of measurements* became quite topical.

The above statements are illustrated by the data of Fig. 1.

2. COMPARISON OF VARIOUS APPROACHES TO SO'S STATE VECTOR ESTIMATION

The considered problem of estimating the state vector x (n×1) from Z (k×1) measurements is given below in the classical formulation. In this case the noise effect is expressed in the form of some nuisance (noise) parameters q (m×1). The basic initial relation is as follows:

$$Z = X \cdot x + B \cdot q + V . \qquad (5)$$

Here X (k×n) and B (k×m) are known matrices, V (k×1) is the vector of measurement errors, which are accepted to be of equal accuracy and statistically independent, i. e.

$$M\left(V \cdot V^T\right) = \sigma_z^2 \cdot E . \qquad (6)$$

The correlation matrix $M\left(q \cdot q^T\right) = \sigma_q^2 \cdot K_q$ of nuisance parameters is supposed to be known. We shall consider three approaches to state vector estimation, which differ in the way of accounting for nuisance parameters:

I. *Without accounting for nuisance parameters.* In the process of state vector estimation the influence of nuisance parameters is not taken into account. In this case the classical least-square technique (LST) is applied for estimation:

$$\hat{x} = \left(X^T \cdot X\right)^{-1} \cdot X^T \cdot Z . \qquad (7)$$

It can easily be shown that the correlation matrix of state vector errors x is expressed as follows:

$$K_x = \sigma_z^2 \cdot \left(X^T \cdot X\right)^{-1} + \left(X^T \cdot X\right)^{-1} \cdot X^T \cdot \left(B \cdot K_q \cdot B^T\right) \cdot X \cdot \left(X^T \cdot X\right)^{-1} . \qquad (8)$$

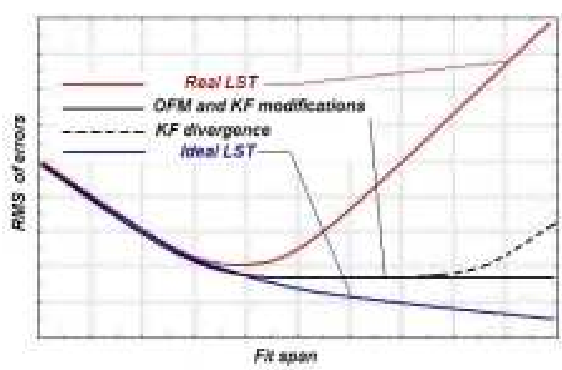

Figure 1. Accuracy of various techniques depending on the fit span value

II. *Parameterization.* The state vector of nuisance (disturbing) parameters is introduced into the structure of an *extended* state vector $y^T = \|x \quad q\|^T$, and then the LST is applied. In this case the required estimate and its correlation matrix are expressed as follows:

$$\hat{y} = \begin{Vmatrix} \hat{x} \\ \hat{q} \end{Vmatrix} = \left(\begin{Vmatrix} X^T \\ B^T \end{Vmatrix} \cdot \begin{Vmatrix} X & B \end{Vmatrix} \right)^{-1} \cdot \begin{Vmatrix} X^T \\ B^T \end{Vmatrix} \cdot Z \,, \tag{9}$$

$$K_y = \begin{Vmatrix} K_{11} & K_{12} \\ K_{12}^T & K_{22} \end{Vmatrix} = \sigma_z^2 \cdot \left(\begin{Vmatrix} X^T \\ B^T \end{Vmatrix} \cdot \begin{Vmatrix} X & B \end{Vmatrix} \right)^{-1} = \\ = \sigma_z^2 \cdot \left(\begin{Vmatrix} X^T \cdot X & X^T \cdot B \\ B^T \cdot X & B^T \cdot B \end{Vmatrix} \right)^{-1} \tag{10}$$

II. *Without parameterization (the optimum filtration of measurements).* The a priori correlation matrix of nuisance parameters is used for “weighing” the measurements without extension of a state vector. The influence of nuisance parameters is taken into account by combining them with measurement errors ($V_\Sigma = B \cdot q + V$), and then the MLT is applied. In this case the required estimate and its correlation matrix are expressed as follows:

$$\hat{x} = \left(X^T \cdot P \cdot X \right)^{-1} \cdot X^T \cdot P \cdot Z \,, \tag{11}$$

$$P = \left(\frac{\sigma_q^2}{\sigma_z^2} \cdot B \cdot K_q \cdot B^T + E \right)^{-1} = \left(S_n^2 \cdot B \cdot K_q \cdot B^T + E \right)^{-1} . \tag{12}$$

$$K_x = \sigma_z^2 \cdot \left(X^T \cdot P \cdot X \right)^{-1} . \tag{13}$$

Here parameter S_n can be treated as the signal-to-noise merit.

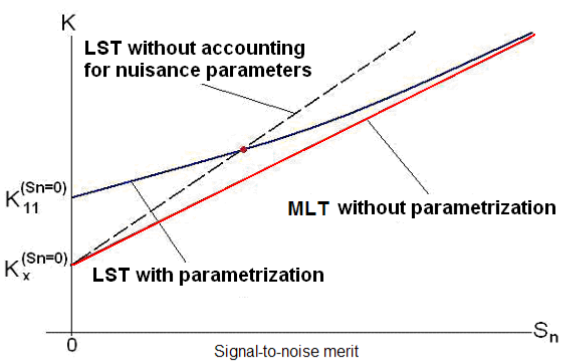

Figure 2. Dependence of errors on the used approach

As a result of performed analysis, the comparative relationships were established between state vector errors with using the techniques listed above.

The results of analysis are presented in Fig 2. It is seen that there exists the level of (small) disturbances, for which it is more profitably to apply the LST without state vector extension. However, even in this case the errors are greater, than in case of using the non-parametric approach, which is realized on the basis of application of the technique *of optimum filtration of measurements*.

3. FEATURES OF ALGORITHM

The traditional formula is applied for determining the state vector at the arbitrary time instant (t_j)

$$\hat{x}_j = \left(X_j^T \cdot P_j \cdot X_j \right)^{-1} \cdot X_j^T \cdot P_j \cdot Z_k \,, \tag{14}$$

where $X_j = \partial Z_k / \partial x_j$ is the matrix of partial derivatives, P_j is the weighting matrix, which is calculated with regard to noise and measurement errors:

$$P_j = \left(H \cdot K_{wj} \cdot H^T + R_\Sigma \right)^{-1} . \tag{15}$$

Here

$$H = \begin{Vmatrix} h_1 & 0 & \dots & 0 \\ 0 & h_2 & .. & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & h_k \end{Vmatrix}, \quad R_\Sigma = \begin{Vmatrix} R_{11} & R_{12} & \dots & R_{1k} \\ R_{21} & R_{22} & .. & R_{2k} \\ \dots & \dots & \dots & \dots \\ R_{k1} & R_{k2} & \dots & R_{kk} \end{Vmatrix}, \tag{16}$$

$$K_{wj} = \begin{Vmatrix} Q_{11}^{(j)} & Q_{12}^{(j)} & \dots & Q_{1k}^{(j)} \\ Q_{21}^{(j)} & Q_{21}^{(j)} & .. & Q_{2k}^{(j)} \\ \dots & \dots & \dots & \dots \\ Q_{k1}^{(j)} & Q_{k2}^{(j)} & \dots & Q_{kk}^{(j)} \end{Vmatrix} = \left\{ Q_{il}^{(j)} \right\} \quad i,l = 1,\dots,k \,, \tag{17}$$

$$Q_{il}^{(j)} = \int_{t_j}^{t_i} \int_{t_j}^{t_l} U(t_i,\xi) \cdot B(\xi) \cdot K_q(\xi,\eta)_0 \cdot B^T(\eta) \cdot U^T(t_l,\eta) \cdot d\eta \cdot d\xi \,, \tag{18}$$

U(…) – is the fundamental matrix of solutions of Eq. 1.

A typical feature of estimate Eq. 14 is the fact, that it is optimum for any time instant (both for the updating instant, and at forecasting). The shortage of its application at forecasting the motion for various time instants consists in the necessity of multiple reversal of a weighting matrix Eq. 15. This operation is rather labor consuming with a great number of measurements. Formula Eq.14 does not contain explicitly the relation of obtained estimates with the results of integration of Eq. 1 with known initial conditions and with regard to the noise estimates:

$$\hat{x}(t) = U(t, t_0) \cdot \hat{x}(t_0) + \int_{t_0}^{t} U(t,\xi) \cdot B(\xi) \cdot \hat{q}(\xi) \cdot d\xi \,. \tag{19}$$

A typical feature of the OFM technique is the possibility of calculating forecasted noise estimates from the results of measurement processing on a fit span. These estimates are calculated by form

$$\hat{q}(t)_k = W_k^T(t) \cdot H^T \cdot P_k \left(Z_k - X_k \cdot \hat{x}_k\right), \tag{20}$$

where

$$W_k(t) = \begin{Vmatrix} K_{wq}(t_1, t) \\ K_{wq}(t_2, t) \\ \dots \\ K_{wq}(t_k, t) \end{Vmatrix}, \tag{21}$$

$$K_{wq}(t_i, t) = \int_t^{t_i} U(t_i, \xi) \cdot B(\xi) \cdot K_q(\xi, t)_0 \cdot d\xi . \tag{22}$$

Estimates Eq. 19 provide obtaining the optimum-in-accuracy forecasted values of a state vector. Their application is more convenient as compared to estimates Eq. 14, since it does not require multiple reversal of a high-dimension matrix. Another advantage consists in the possibility of replacing the term $\hat{x}(t) = U(t, t_0) \cdot \hat{x}(t_0)$ by the results of integration of the initial (nonlinear) differential equations of satellite motion.

4. TECHNIQUE OF ESTIMATION OF FORECASTING ERRORS ON THE MODEL

The simplified equations of motion have been successfully applied for evaluating the effect of disturbances on time parameters of the orbit (the errors along the trajectory) in a number of works [12 - 15]. In this approach the state vector includes only those orbital elements, which characterize the motion in the plane of a near-circular orbit. Following this approach, we shall consider the satellite motion in the orbital plane with a number of revolutions as an argument. State vector components are the following three parameters: the equator crossing time t_i , satellite's period of revolution T_i, and the period change under an effect of disturbances per revolution ΔT_i. The evolution of these parameters is described by the following equations:

$$\begin{aligned} t_{i+1} &= t_i + T_{i+1}, \\ T_{i+1} &= T_i + \Delta T_i, \ . \\ \Delta T_i &= \Delta T_m + q_i . \end{aligned} \tag{23}$$

Quantity ΔT_i is associated with the effect of atmospheric disturbances only. In this case, with constant spacecraft's ballistic factor, the value of ΔT_i is proportional to the current density of the atmosphere. Quantity ΔT_m is the mean value of parameter ΔT_i , and q_i is the Gaussian random process with the known correlation function $K_q(t, \tau)_0$.

The values of the equator crossing time at the beginning of revolution N_j are used as measurements:

$$z_j = t_j + v_j \ , \ (j=1,2,...), \tag{24}$$

where the measurement errors v_j (of discrete white noise type) are distributed according to the normal law with zero mathematic expectation and variance σ_z^2 . The time interval between the measurements is assumed to be constant and equal to ΔN (in revolutions).

The aforementioned Eq. 23, the measurements Eq. 24 and statistical characteristics of random quantities q_i and v_j allow one to determine statistical characteristics of errors of estimation and forecasting of time parameters. As the techniques of estimation of orbital parameters we shall consider the classical least square technique (LST) and its generalization (the optimum filtration of measurements), which takes into account the effect of disturbances as a color noise. In the process of investigations we shall use the following correlation function of the random process q_i :

$$K_q(t, \tau)_0 = \begin{cases} \sigma_q^2 \cdot \left(1 - \dfrac{|t - \tau|}{\Delta}\right), & by \quad |t - \tau| < \Delta, \\ 0, \quad by \quad |t - \tau| \ge \Delta. \end{cases} \tag{25}$$

Then, based on the results of modeling the random sequence q_i , some initial conditions t_0 , T_0 , ΔT_m and Eq. 23, we calculate the sequence of state vector values $x_i^T = \| t_i \quad T_i \quad \Delta T_i \|$ (with a step of one revolution). The next operation of modeling the time parameters of orbit is calculation of the sequence of modeled values of measurements z_j by Eq. 24. In so doing, the random errors of measurements are determined by means of the random-number generator, and the constant time interval between measurements ΔN (in revolutions) is taken into account.

Fig. 3 presents the scheme of successive calculations at processing the measurements Eq. 24 during modeling.

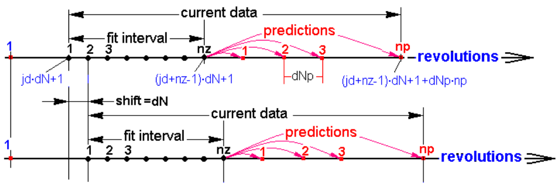

Figure 3. Scheme of successive processing of measurements

Two time intervals of estimation and forecasting of orbital parameters are shown in the figure: the current one (for the *jd*-th updating) and the subsequent one, constructed by shifting all data by *dN* revolutions. The following designations are applied here: *nz* – the number of measurements used at updating, *np* – the number of forecasts, *dNp* – the time interval (in revolutions) between successive forecasts. The black font marks the numbers of measurements, and the red font – the numbers of forecasts. The maximum forecasting interval equals *dNp* × *np* revolutions. The blue font marks serial numbers of revolutions. Performing updating and forecasts according to the given scheme of their cyclic organization allows one to obtain a rather great number of realizations for acceptable time. In the analysis of modeling results it is convenient to use the ratio

$$S_n = \frac{\sigma_q}{\sigma_z}, \tag{26}$$

which can be treated as the "signal-to-noise" merit, i.e. the level of an estimated signal in relation to measurement errors.

5. RESULTS OF INVESTIGATION ON THE MODEL

The results of application of the considered model for studying the LST are outlined in detail in papers [16, 17]. Below the main attention will be given to studying the OFM technique and to comparing it with the results of application of LST. Three modifications of the OFM technique will be considered:

– «OFM 3-1» – application of the 3-dimensional state vector with using ΔT estimates under initial conditions.
– «OFM 3-2» – application of the 3-dimensional state vector with using the forecast of ΔT estimates' deviations from the mean value.
– «OFM 2» – application of the 2-dimensional state vector and the mean value of parameter ΔT_m.

In addition, 4 more versions of calculations were considered:

– «LST 1» – LST application with the number of measurements equal to nz=6.
– «LST 2» – LST application with nz=9.
– «LST 3» – LST application with nz=12.
– «A priori» – aprioristic RMS estimates calculated by the analytical formula.

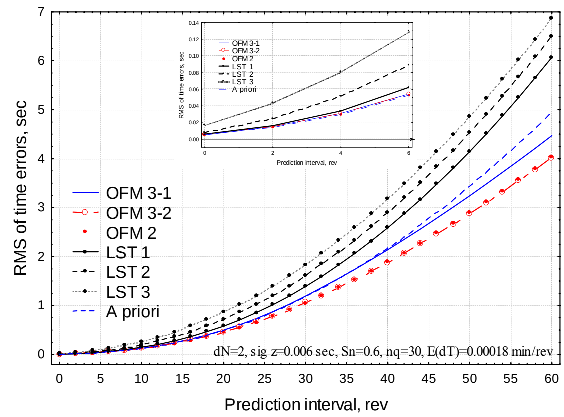

Figure 4. Forecasting errors under various conditions

The tests were organized in such a manner, that identical measurements were used in all cases. The number of realizations for each of techniques was 10000. Fig. 4 presents RMS errors of the forecast for all considered versions of calculation. They were obtained for the following values of the initial data:

- Mean value of the drag parameter $\Delta T_m = E(\Delta T) = 0.000180$ min / revolution;
- Interval between measurements $dN = 2$ revolutions;
- RMS of the drag parameter from the mean value $\sigma_q = 0.00006$ min / revolution;
- RMS of measurement errors $\sigma_z = 0.0001$ min = 0.006 sec;
- "Signal-to-noise" merit *Sn=0.6*;
- Interval of correlation of the atmospheric color noise $\Delta = 30$ revolutions;
- Number of measurements on a fit span with using OFM *nz=30*;
- Number of measurements on a fit span with using LST *nz=6*, 9 and 12.

The upper part of the figure presents, in the enlarged scale, the fragment of a plot related to forecasting intervals up to 6 revolutions. The following conclusions can be drawn from the modeling results:

a) As it should be expected on the basis of materials of Section 2, the minimum errors of estimation and forecasting of orbits are achieved as a result of application of calculation versions «OFM 3-2» and «OFM 2».
b) The results of application of the 3-dimensional state vector with using ΔT estimates under initial conditions (version «OFM 3-1») are rather well correlated with aprioristic estimates RMS and have slightly worse accuracy as compared to versions «OFM 3-2» and «OFM 2».

c) In all cases the application of LST results in increasing the errors of estimation and forecasting of orbital parameters. Even for the optimum fit span (*nz=6*) and at forecasting for 60 revolutions the errors are 1.5 times greater, than the corresponding results for «OFM 3-2» and «OFM 2». With 2-fold increase of a fit span (*nz=12*) the LST application errors grow. This is especially highly revealed for small forecasting intervals (up to 6 revolutions), where the RMS increase 2-3 times.

Consider now the combined data on RMS of residual discrepancies on a fit span and on the RMS errors of the forecast. The corresponding results are presented in Fig. 5. We remind that 30 measurements have been processed on a fit span when using the OFM technique, and 6 measurements – when using the LST. So, the corresponding fit spans were 58 and 10 revolutions, respectively.

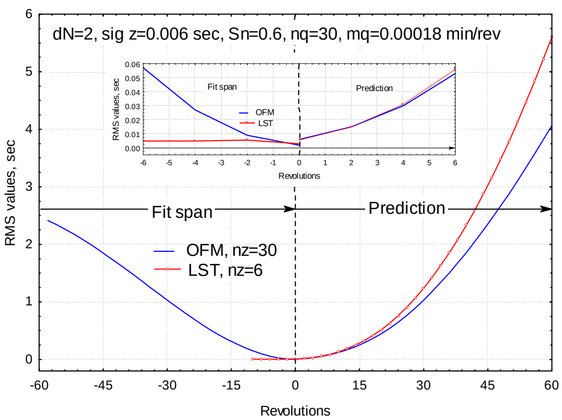

Figure 5. RMS of residual discrepancies and forecasting errors

The data of Fig. 5 (namely, the data on the change of RMS of residual discrepancies on a fit span) clearly show the important distinction between considered techniques. The residual discrepancies very highly change in case of using the OFM. At the beginning of a fit span their RMS equals ≈ 2.5 sec, and at the end of a fit span – 0.002 sec. When using LST all residual discrepancies have RMS within the limits from 0.003 to 0.005 sec. At the last measurement instant the accuracy of determination of time using the OFM and LST techniques (with an optimum fit span) differs insignificantly. Such a behavior of residual discrepancies in the OFM technique illustrates advantages of the given technique. Namely, the change of residual discrepancies over a rather large fit span (58 revolutions in this case) allows one to estimate automatically the time variability of drag (noise) and to use these estimates for forecasting the noise. Another important advantage of the OFM technique consists in the possibility of changing (increasing) the fit span with conserving the accuracy of estimates. This property is not present in LST: increasing of a fit span in relation to the optimum one results in essential worsening the accuracy of estimates at the updating instant and in short-term forecasting.

6. RESULTS OF TESTS BASED ON THE REAL INFORMATION

6.1. Satellite SL-4 R/B.

The example of tests of the OFM technique based on the real information is given below. The orbital data in the form of so-called two-line elements (TLE) [18] for the SL-4 R/B (No. 20967) satellite were used as the initial data. The time interval was chosen in such a manner, that the real period change per revolution to correspond to the initial data used in modeling (Section 5). The orbital parameters have been updated 108 times over the time interval of November – December, 2005. The average time interval between successive TLEs was 0.54 days ≈ 8.3 revolutions.

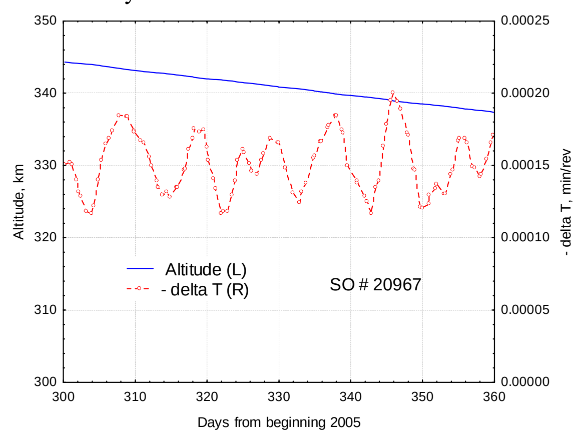

Figure 6. Orbital characteristics of the satellite

Fig. 6 presents the data on the satellite altitude and on the period change per revolution (ΔT), and Fig. 7 gives the data on the solar and geomagnetic activity. These data clearly show correlation of variations of indices and parameter ΔT.

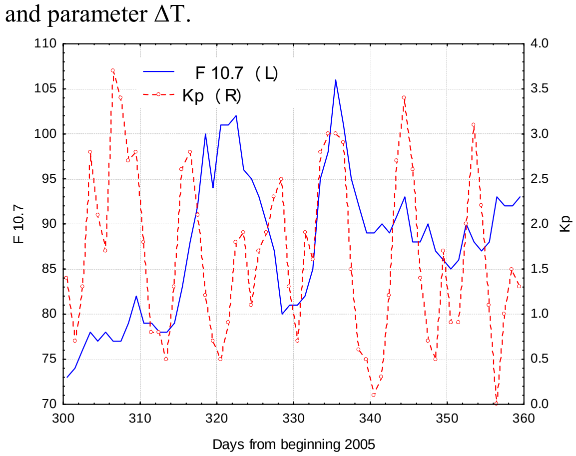

Figure 7. Solar and geomagnetic activity indices

The initial conditions for forecasting have been updated on the basis of application of two techniques: OFM and LST. In so doing, the numerical model of motion was used, in which the major harmonics of the geopotential and the dynamic model of the atmosphere were taken into account. Fig. 8 presents the relative variations of SO drag characteristics obtained as a result of TLE processing.

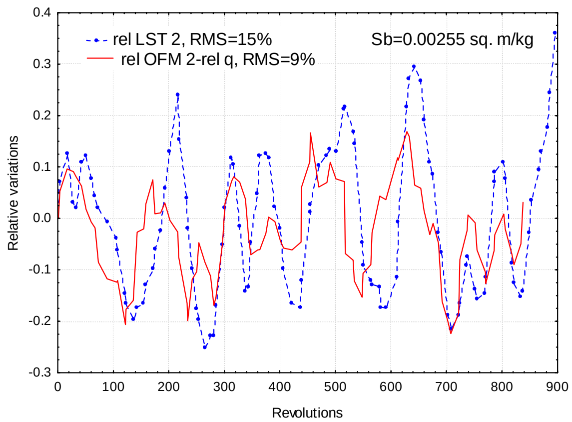

Figure 8. Variations of drag characteristics

The estimates of drag variations, presented in Fig. 8, well agree with corresponding data of Fig. 6. This testifies to the objective character of observed variations. Existing relatively small divergences are explained by the effect of random errors and various time "attribution" of obtained estimates. The order of divergences (≈10 %) correlates with generally accepted ideas about the drag estimates accuracy.

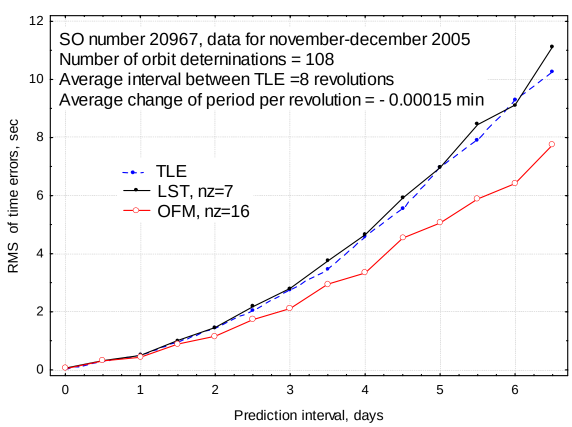

Figure 9. Estimates of forecasting errors

The forecasts were carried out for the "future" orbital data. In this case the direct forecast of TLEs (without updating) was also used, which is based on applying the known American analytical model of motion SGP 4 [19]. This model is adapted to using TLEs as initial conditions. The obtained RMS of time errors are presented in Fig. 9. The number of realizations at calculating each of points was ≈100. It is seen from figure's data that in processing the real information the comparative characteristics of accuracy of OFM and LST techniques well correlate with modeling results (Fig. 4).

Note. Rather unexpected is the fact, that the direct application of TLE as initial conditions for forecasting leads to the same errors, as application of LST and the numerical model of motion. It could be expected, that the accuracy of direct application of TLE would be worse. This result is explained, apparently, by the fact that essentially greater number of initial measurements was used in obtaining TLEs, than in our conditions (7 measurements, which correspond to the fit span of ≈3 days). In conformity with recommendations stated in the monograph [8], the optimum fit span in LST equals 3 - 5 days for considered conditions. It should be expected, that under the conditions of obtaining a greater number and more accurate measurements, the application of OFM and LST techniques would result in a better accuracy as compared to the direct application of TLE.

6.2. Analysis of the data on collision of SC Iridium 33 and Cosmos 2251.

The collision of these satellites on February, 10, 2009 at 16 hr 56 min provides a unique opportunity for getting precise estimates of forecasting errors. Calculations on the basis of using previous TLEs were performed in three ways:

1. Direct forecast of TLE on the basis of application of the SGP 4 model of motion;

2. Updating the initial data by means of LST (nz=6) and application of the numerical model of motion;

3. Updating the initial data by means of OFM (nz=20) and application of the numerical model of motion;

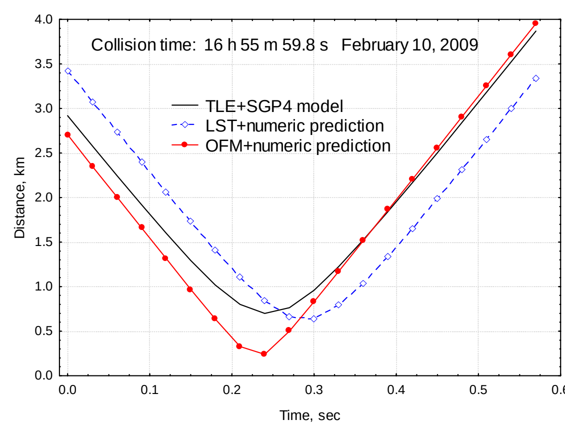

Figure 10. Change of distance between the satellites

The last, before collision, initial data on February, 9 corresponded to the time instants: 18 hours (Iridium 33) and 12 hours (Cosmos 2251). Fig. 10 presents calculated values of the distance between satellites close to the collision instant.

It is seen from the obtained results, that application of the OFM technique resulted in 2.7-fold decrease of forecasting errors as compared to application of other calculation techniques.

CONCLUSION

The application of the technique of optimum filtration of measurements is a perspective direction in perfecting the software applied for operative solution of various ballistic problems. This technique combines in itself the advantages of the classical least square technique and modern measurement filtration techniques with allowance for the errors of the model of motion.

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