ABSTRACT

The accurate estimation of the space object remaining lifetime is interesting now because of many practical reasons, one of which is the selection of a disposal orbit with the settled lifetime.

The results of such a kind of estimation based on using the precise numerical integration and up-to-date dynamical models of atmosphere was obtained for different classes of near Earth orbits.

At carrying out researches the influence of initial orbital elements, including a perigee distance, an eccentricity and an inclination as well as an initial epoch of considered orbit; value of ballistic coefficient; atmospheric model and a behavior of environment data were examined.

At the calculations of the atmospheric density for long term prediction the environment data model within the fifty years, constructed by IZMIRAN was used.

INTRODUCTION

The problem of a lifetime of space objects (SO) in orbits in the near Earth space (NES) is actual at solution of many research and applied tasks concerning study and development of the NES. The examples of such tasks may be the following:

- choosing the orbits with the given properties at a stage of the space project development,
- the implementation of the operational procedure during a flight control of the launched spacecrafts,
- an estimation of a situation and the propagation of events in the case of off-design inserting of a space vehicle.

The given problem is also rather important and at solution of the problems concerning removal of man-made objects (space debris) from NES, including, determination of classes of orbits with the settled lifetime to which could be disposed the active space vehicle before the end of mission.

For a SO, being at the given initial epoch in some orbit of an artificial satellite, it is possible to carry out lifetime estimation by means of the different tools based on analytical, numerically-analytical or numerical methods of propagation of its orbiting. Thus for obtaining of reliable results and simplification of procedure of adjustment of the computing tools and fulfillment of calculations (which can be enough great amounts) it is desirable to select beforehand a necessary and a sufficient combination of considered forces and the most adequate models of their representation.

The carried out examinations allow drawing conclusions on expediency of the taking into account of those or other perturbing factors in the SO’ motion model and a selection of the environment models (atmospheric density) at solution of the given problem. Examinations were conducted on the basis of great number of the calculation variants, fulfilled with the help of the high-precision method of numerical integration of equations of SO’ motion [1].

The obtained results demonstrate a character and degree of influence on the SO’ lifetime of various factors, including initial conditions of the SO’ orbital motion and aerodynamic properties of the space object. These results can be useful for deriving the operative estimations of the considered flight characteristic of the given SO.

TASK STATEMENT, INITIAL DATA AND MODELS FOR EXAMINATIONS

In the present paper classes of researched orbits have been restricted by values of the maximum altitude $H_{\text{max}} < 1400$ km. However this limitation is not of principle for a large quantity of SOs, being in a near-Earth space, and also for the future designed space vehicles for which in the light of the new policy on space debris mitigation there is an actual a problem on their displacement at the end of missions into a special disposal orbits with the settled restricted lifetime.

It was supposed, that used models of the Earth gravity field are exact enough for spent examinations. Namely, at calculations the Russian model of a gravitational field was used.

For atmospheric density calculation the different static and dynamic models were used. In the cases of the
dynamic models, as the input data reflecting varying heliogeophysical environment, the modeling indexes of the solar and geomagnetic activity, spread during time period in 50 years, were used. These data considering some 11-year solar cycles were obtained at Institute of a terrestrial magnetism, an ionosphere and distribution of radio waves of the Russian Academy of Sciences (IZMIRAN) on the basis of the long-time historical measured data. At fulfillment of calculations in different variants were used Russian (GOST 25645.1115-84) and American (NRLMSISE-00) dynamic models of atmosphere, and also static model of it by GOST 4401-81 designation.

At the taking into account of gravitational perturbations from the Moon and the Sun these bodies were considered as the appropriate mass points, and their parameters of motion were represented by the ephemerides DE403.

Preliminary estimations have shown, that for the majority of spacecrafts, upper stages of launch vehicles and other significant space objects being in analyzed orbits, solar pressure is not capable a little noticeably to give effect in the model of motion of these objects.

SO’ orbital motion was described in the rectangular inertial geocentric coordinate system (IGCS) referred to the mean equator and the equinox of a standard epoch J2000. Generally (taking into account mentioned above) the equations of SO’ motion in the given coordinate system had the following vector form:

$$\dot{\mathbf{r}} = -\mu \frac{\mathbf{r}}{r^3} + M \mathbf{g} + \mathbf{F}_{\text{atm}}(\mathbf{r}, \dot{\mathbf{r}}) + \sum_{\alpha=L,S} \mathbf{F}_{\alpha}(\mathbf{r}, \dot{\mathbf{r}})$$  \hspace{1cm} (1)

Here $\mathbf{r}$ - SO’ vector of coordinates in the indicated inertial system of co-ordinates (IGCS);
$\mu$ - gravitation constant of the Earth;
$\mathbf{g}$ - non-central part of the Earth gravity field;
$\mathbf{r}' = M^T \mathbf{r}$ - SO’ position vector in geocentric Earth-fixed rotating co-ordinate system ($M$ - a matrix of transformation from this co-ordinate system to the main IGCS);
$\mathbf{F}_{\text{atm}}$ - an acceleration caused by an atmospheric drag;
$\mathbf{F}_{\alpha}$ - an acceleration due to a gravitational attraction of the “third” body (the Moon or the Sun),
$\mathbf{r}_\alpha$ - a position vectors of the “third” body in IGCS.

The gravitational potential of the Earth $U$ was represented in decomposition to a series by spherical functions (harmonics):

$$U = \frac{\mu}{r} \left[ 1 - \sum_{n=2}^{\infty} \sum_{m=1}^{n} \left( \frac{a_n}{r} \right)^n P_n(\sin \varphi) + \sum_{n=1}^{\infty} \sum_{m=1}^{n} \left( C_{nm} \cos m \lambda + S_{nm} \sin m \lambda \right) P_m(\sin \varphi) \right]$$  \hspace{1cm} (2)

where $r$, $\varphi$, $\lambda$ - geodetic co-ordinates of an outer point (a SO’ position);
$a_e$ - an equatorial radius of the Earth;
$J_n$, $C_{nm}$, $S_{nm}$ - the dimensionless coefficients of the decomposition of the gravitational potential of the Earth (GPE), characterizing the form of the Earth and a mass distribution inside it,
$P_n$, $P_m(\sin \varphi)$ - the polynomials and associated functions of Legendre, defined by the formulas:

$$P_{nm}(\vartheta) = \frac{1 - \vartheta^2}{2^n n!} \frac{d^n}{d\vartheta^n} \left( \vartheta^2 - 1 \right)^n, \ \vartheta = \sin \varphi.$$  \hspace{1cm}

In case of calculation of polynomials of Legendre $m = 0$.

The acceleration created by an aerodynamic drag of the atmosphere, was represented by the formula:

$$\mathbf{F}_{\text{atm}} = -\frac{1}{2} S_b \rho(h) \mathbf{V}_{rel} \mathbf{V}_{rel},$$  \hspace{1cm}

where $S_b = \frac{C_s S}{m}$ - ballistic coefficient of SO ($C_s$ - a head resistance dimensionless factor, $S$ - mid cross section of object, $m$ - its mass);
$\rho$ - atmospheric density; $\mathbf{V}_{rel}$ - velocity of the object relative to the atmosphere.

Integration of the differential equations (1) was carried out by means of the method and technique, described in [1].

**RESULTS OF INVESTIGATIONS**

In a Fig. 1 the values of index $F_{10.7}$, characterizing solar flux on frequency of 2800 MHz, i.e. radiations with a wavelength of 10.7 cm, and a planetary index of geomagnetic activity - $Ap$ are introduced in a graphics form. In this figure data since 2001 till 2006 are measured values, and after those epochs - they were modeled in IZMIRAN.

Let's point out, that according to IZMIRAN’ model, the beginning of growth of solar activity in the nearest cycle is at the beginning of 2008, and its peak will be reached in 2010.

For an estimation of dependence of a predicted lifetime on used model of atmosphere the appropriate calculations for cases of circular orbits for altitude range $H = 400 \div 600$ km have been fulfilled. Three variants of the orbit inclinations: $i = 0^\circ$, $40^\circ$, $90^\circ$ were considered. The results of the calculations of a SO’ lifetimes for these cases are shown on Fig. 2.
The presented results testify that the usage of static model of atmosphere in orbits with altitudes more than 500 km can give values of a SO lifetime, approximately on 30% differing from variants in which dynamic models were used. Thus the results obtained with usage of different dynamic models, also differ from each other. So, the usage of model NRLMSISE-00 yields the underestimated result on a lifetime in comparison with model of GOST 25645.115-84 at the same initial conditions. Difference in a lifetime for these models can reach more than 20%. From the shown graphics a dependence of lifetime on the inclination of an initial orbit is evident also. For the analyzed cases of the circular orbits it is true that at increasing an inclination from 0° to 90° it was incremented also SO’ lifetime.

Let's note that all presented results have been obtained at an assumption that the beginning of orbiting corresponds to the beginning of increase of solar activity.

It has been stated also, that the longitude of an ascending node of an orbit \( \Omega \) slightly influences a SO’ lifetime (at least when this time is prolonged enough). Dependence of predicted SO’ lifetime on the beginning of its motion relative to a solar activity cycle for cases of circular orbits with altitudes \( H = 500, 550 \) and 600 km is shown on Fig. 3. For these calculations an orbit inclination had value \( i = 50° \), the argument of perigee was equal to 0°, and a ballistic coefficient \( S_b = 0.012 \text{ m}^2 \text{ kg}^{-1} \).

In the examples of the same circular orbits \((H \approx 500, 550, 600 \text{ km}; i = 50°; \omega = 0°)\) the dependence of SO’
lifetime on a value of ballistic coefficient at the different beginning of researched orbiting is shown on Fig. 4. The cases, when $S_b = 0.008$, $S_b = 0.012$, $S_b = 0.02 \text{ m}^2\times \text{kg}^{-1}$ are examined. As follows from Fig. 3 and Fig. 4 the minimal lifetime have the objects started their motion in years close to maximum of solar activity (2010), and maximal lifetime have the objects started motion under decreasing of solar activity (2012-2016).

Examinations have been carried out to estimate SO’ lifetime influenced according to taking into account different number of harmonics in the decomposition the Earth gravity field. For this purpose some models of a geopotential (2) used in the equations (1) have been handled. In these models sequentially were considered: 2nd, 2nd + 3rd, 2nd + 3rd + 4th zonal harmonics and also all zonal, tesseral and sectorial harmonics up to degree and order $(8 \times 8)$. Calculations have been fulfilled for cases of orbits with an expected lifetime ~ 4-6 years and different altitudes. Appropriate values of the maximum and minimum altitudes, and also eccentricities, in these cases were such as in Tab. 1:

<table>
<thead>
<tr>
<th>H_{max}, km</th>
<th>550</th>
<th>1000</th>
<th>1400</th>
</tr>
</thead>
<tbody>
<tr>
<td>H_{min}, km</td>
<td>500</td>
<td>350</td>
<td>300</td>
</tr>
<tr>
<td>e</td>
<td>0.0036</td>
<td>0.046</td>
<td>0.076</td>
</tr>
</tbody>
</table>

The carried out investigations have shown, that in spite of the fact that the third zonal harmonic coefficient - $J_3$, almost on three orders (modulo) is less than value $J_2$, the taking into account this harmonic leads to an appreciable modification of a lifetime of the near-polar satellite, especially at values $\omega$ close to $\pm 90^\circ$. At the same time, taking into account higher harmonics affected SO’ lifetime predictions not so significant.

Figure 3. Changing a predicted SO’ lifetime depend on a solar activity cycle. ($S_b = 0.012 \text{ m}^2\times \text{kg}^{-1}$)

Figure 4. Changing a predicted SO’ lifetime depend on a solar activity cycle and a ballistic coefficient.

Table 1. Initial parameters of orbits
Dependence of a lifetime on included harmonics of a geopotential and values of an argument of a perigee $\omega$ for the indicated cases of orbits of an artificial satellite is shown in graphics form on Fig. 5.

The presented results, in particular, testify, that for a case of an orbit with $H_{\text{max}} = 1400$ km, $H_{\text{min}} = 300$ km the non-inclusion of the third zonal harmonic can lead to errors in estimated lifetime reaching more than 10% while the non-inclusion of higher harmonics can give an error $\sim 2\%$ from a lifetime obtained in calculations at the usage of the complete model of a geopotential.

Necessity of the taking into account gravitational attraction of the Moon and the Sun has been estimated at calculation of SO’ lifetime. It has been find out, that influence of this factor on precision of calculation increases with magnification of an eccentricity of orbit. However it is possible to accept, that for examined classes of orbits (with the indicated limitations on $H_{\text{max}}$) at fulfillment of estimating calculations on a lifetime the perturbations from the Moon and the Sun in most cases may be neglected.

At the end of the present stage of investigations the estimation of an expected lifetime for orbits with $H_{\text{max}} = 1400$ km, 1000 km, 800 km, 650 km and 550 km depending on values of minimum altitudes $H_{\text{min}}$ have been obtained. The estimates were carried out in the frame of the complete model of SO motion, introduced by the equations (1), at the usage of dynamic model of atmosphere GOST 25645.1115-84 and the indicated model of solar flux and geomagnetic activity indexes, constructed by IZMIRAN. In all cases the value of ballistic coefficient $S_p$ was equal to $0.012$ m$^2$·kg$^{-1}$.

The results of the obtained estimates for SO, started their motion during increase (2008r) and decrease (2014r) of the solar activity are shown on Fig. 6 and Fig. 7.
CONCLUSION

The carried out researches have shown, that for a class of orbits with the values of maximal altitudes $H_{\text{max}} < 1400 \text{ km}$ at prediction of SO’ lifetime it is more preferably to use the dynamic models of atmosphere. Besides the account of influence of the third zonal harmonic in the decomposition of the Earth gravity field, especially for the orbits with the values $\omega$ close to $\pm 90^\circ$, also can raise accuracy of the lifetime prediction more than on 10%.

Influence of zonal harmonics of higher order as well as tesseral and sectorial ones is insignificant for a problem of an estimation of SO’ lifetime with a reasonable accuracy.

REFERENCES