ON RE-ENTRY PREDICTION OF NEAR EARTH OBJECTS WITH GENETIC ALGORITHM USING KS ELEMENTS

Sharma, R. K..[(1)], Anilkumar, A. K.[(1)], Xavier James Raj, M. [(1)], and Sabarinath, A. [(1)]

[(1)] Applied Mathematics Division, Vikram Sarabhai Space Centre, Thiruvanathapuram – 695022, India.
e-mail: rk_sharma@vssc.gov.in

ABSTRACT

The accurate orbit prediction of the near-Earth objects is an important requirement for the re-entry and the life time estimation. The method of Kustaanheimo and Stiefel (KS) total energy element equations is one of the powerful methods for orbit prediction. Recently, due to the reentries of large number of risk objects, which posses threat to the human life and property, a great concern is developed in the space scientific community. Consequently, the prediction of risk objects re-entry time and location has got much importance for the proper planning of mitigation strategies and hazard assessment. This paper discusses an integrated procedure for orbit life time prediction combining the KS elements and genetic algorithm (GA). The orbit prediction is carried out by numerically integrating the KS element equations. In this methodology, the ballistic coefficient is estimated from a set of observed orbital parameters in terms of the Two Line Elements (TLE) by minimizing the variance of the predicted re-entry time from different TLE using GA. A software, KSGEN, systematically developed in-house using KS elements and genetic algorithm is utilized for predicting the re-entry time of the risk objects. This software has been effectively used for the prediction of the re-entry time in the past seven re-entry exercise campaigns conducted by the Inter Agency Space Debris Coordination Committee (IADC). The predicted re-entry time matched quite well with the actual re-entry time for all the seven IADC re-entry campaigns. A detailed analysis is carried out with two case studies.

1. INTRODUCTION

The accurate estimation of the orbital lifetime (OLT) of decaying near-Earth objects is of considerable importance for prediction of risk object re-entry time and proper planning of mitigation strategies and hazard assessment. It has become necessary to use extremely complex force models, which are realistic to match with the present operational requirements and observational techniques. The problem becomes all the more complicated in the near-Earth environment due to the fact that the object is influenced by the non-spherical effects of the Earth's gravitational field as well as the dissipative effects of the Earth's atmosphere. The database available for the re-entry time or orbital lifetime prediction of the debris objects is based on the set of Two Line Elements (TLEs) provided by United States Space Surveillance Network (USSSN). These TLEs provide information regarding the orbital parameters together with rate of mean motion decay and an equivalent ballistic coefficient B*. The objects physical parameters like mass, area of cross section, shape and dimensions are not available accurately and the modeling of the atmosphere in which objects decay takes place is also uncertain. Besides, the tumbling effect of the body and gas molecular interaction will make the prediction of re-entry time a very complicated exercise. The method of the KS total-energy element equations provided by Stiefel and Scheifele [1] is a powerful method for numerical solution with respect to any type of perturbing forces, as the equations are less sensitive to round-off and truncation errors in the numerical algorithm. These equations are everywhere regular in contrast with the classical Newtonian equations, which are singular at the collision of the two bodies. The equations are smoothed for eccentric orbits because eccentric anomaly is the independent variable.

Towards improving the OLT prediction accuracies, it is well recognized that one needs to incorporate a procedure that estimates the drag related ballistic coefficient (m/C_D*A_{eff}, C_D being the drag coefficient, A_{eff} - effective area and m – mass of the object) in analyzing a decaying trajectory from a set of its initial observations. In such an approach it can be intuitively appreciated that short-periodic variations in the ballistic coefficient (BC) get averaged out while the data noise also gets smoothed out. It may also be noted that the uncertainties in predicting the atmospheric density gets absorbed to the BC estimation. However, the purpose of the BC estimation using earlier observations in this context is to improve the OLT prediction accuracy, not necessarily to improve the BC estimate.

It is possible to think of different strategies in this connection. For example, as indicated by Chao and Plat [2] the success of an approach based on differential correction of the BC over a simplified semi analytical model for orbit decay via minimization of residual errors on two parameters namely, the semi major axis (a) and the eccentricity (e). Also, Anil. V. Rao [3] has

Proc. '5th European Conference on Space Debris', Darmstadt, Germany
30 March – 2 April 2009, (ESA SP-672, July 2009)

proposed a minimum variance estimation in the form of a two pass filter (an extended Kalman filter for the forward estimate and a Rauch-Tung-Striebel smoother for the backward estimate) using an initial set of observations on azimuth and elevation of the decaying object, from a ground station. Inclusion of lift related BC's has also been considered**.**

This paper deals with a new approach by considering two different ballistic coefficients depending up on the radial distance of the object. This procedure estimates simultaneously the effective ballistic coefficients, first one when the radial distance is above 6500 km and the second one when the radial distance is below 6500 km. This procedure can improve the OLE predictions as the decay of the object takes a sharp nonlinear feature at about a radial distance of 6500 km. This change in the decay pattern can be absorbed by considering two values for the ballistic coefficients.

Hence the new approach improves the OLT prediction accuracy, from an initial set of TLE's via minimization of the variance of the OLT predictions using the orbit prediction code KSNUM, using KS element equations. These equations are numerically integrated with a suitable integration step size with the 4th - order Runge-Kutta-Gill method till the end of the orbital life (90 km altitude), by including the Earth's oblateness terms J_2 to J_6 terms, and modeling the air drag forces through an analytical oblate diurnal atmosphere with the density scale height varying with altitude. Jacchia (1977) atmospheric model [6], which takes into consideration the epoch, daily solar flux (F10.7) and geomagnetic index (Ap) for computation of density and density scale height, is utilized. Variance minimization is achieved through the application of a simple version of the Genetic Algorithm [4, 5], which has received a great deal of attention regarding potential as an optimization technique for complex functions.

The basic feature of the present approach is that the model and measurement errors are accountable in terms of adjusting the ballistic coefficient and hence the estimated BC is not the actual ballistic coefficient but an effective ballistic coefficient. The inaccuracies or deficiencies in the inputs, like F10.7 and Ap values, are absorbed in the estimated BC.

The re-entry prediction carried out for the decayed objects SL-12 R/B (Sat No. 34267) and SROSS-C2 Satellite (Sat No. 23099), which re-entered the Earth's atmosphere on 3rd March 2009 and 12th July 2001, respectively, are also provided.

The predicted re-entries were found to be all along quite close to the actual re-entry time, with quite less uncertainties bands on the predictions. Part of the studies carried out in this paper with an earlier version of simple genetic algorithm was presented in [6].

2. ORBIT PREDICTION SOFTWARE 'KSNUM'

The orbit prediction is carried out by numerically integrating the KS element equations [1] by including the forces due to Earth's flattening with Earth's zonal harmonic terms J_2 to J_6 and atmospheric drag is modeled through an analytical atmosphere that takes into consideration the oblateness of the atmosphere and in which the density behavior approximates to the observed diurnal variation.

The KS element equations are

$$\frac{dw}{dE} = -\frac{r}{8w^2}\frac{\partial V}{\partial t} - \frac{1}{2w}(\vec{u}^*, L^T\vec{P}),$$

$$\frac{d\tau}{dE} = \frac{1}{8w^3}(K^2 - 2rV) - \frac{r}{16w^3}(\vec{u}, \frac{\partial V}{\partial \vec{u}} - 2L^T\vec{P}) - \frac{2}{w^2}\frac{dw}{dE}(\vec{u}, \vec{u}^*)],$$

$$\frac{d\alpha}{dE} = \left\{\frac{1}{2w^2}\left[\frac{V}{2}\vec{u} + \frac{r}{4}\left(\frac{\partial V}{\partial \vec{u}} - 2L^T\vec{P}\right)\right] + \frac{2}{w}\frac{dw}{dE}\vec{u}^*\right\}\sin\frac{E}{2},$$

$$\frac{d\beta}{dE} = -\left\{\frac{1}{2w^2}\left[\frac{V}{2}\vec{u} + \frac{r}{4}\left(\frac{\partial V}{\partial \vec{u}} - 2L^T\vec{P}\right)\right] + \frac{2}{w}\frac{dw}{dE}\vec{u}^*\right\}\cos\frac{E}{2},$$

where

$$\vec{u} = \vec{\alpha}\cos\frac{E}{2} + \vec{\beta}\sin\frac{E}{2},$$

$$\vec{u}^* = \frac{d\vec{u}}{dE} = -\frac{\vec{\alpha}}{2}\sin\frac{E}{2} + \frac{\vec{\beta}}{2}\cos\frac{E}{2},$$

$$\tau = t + \frac{1}{w}(\vec{u}, \vec{u}^*),$$

$$r = \sqrt{x_1^2 + x_2^2 + x_3^2} = u_1^2 + u_2^2 + u_3^2 + u_4^2,$$

$$x = L \quad \vec{u},$$

$$L(\vec{u}) = \begin{bmatrix} u_1 & -u_2 & -u_3 & u_4 \\ u_2 & u_1 & -u_4 & -u_3 \\ u_3 & u_4 & u_1 & u_2 \\ u_4 & -u_3 & u_2 & -u_1 \end{bmatrix}$$

$$(L^T\vec{P})_1 = u_1P_1 + u_2P_2 + u_3P_3,$$

$$(L^T\vec{P})_2 = -u_2P_1 + u_1P_2 + u_4P_3,$$

$$(L^T\vec{P})_3 = -u_3P_1 - u_4P_2 + u_1P_3,$$

$$(L^T\vec{P})_4 = u_4P_1 - u_3P_2 + u_2P_3.$$

E, w, t, r and K^2 are, respectively, the eccentric anomaly, angular frequency, physical time, radial distance and the gravitational constant.

Knowing the position and velocity $\vec{x}$ and $\dot{\vec{x}}$ at the instant $t = 0$, the values of r, w, u_i, u_i^* can be computed [1], and by adopting $E = 0$ as the initial value of the eccentric anomaly, we obtain,

$$\vec{\alpha} = \vec{u}, \quad \vec{\beta} = 2\vec{u}^*.$$

If $\vec{P}$ is the aerodynamic force per unit mass acting on a satellite of mass m, then

$$\vec{P} = -\rho\,\delta\,|\vec{v}|\,v/2$$

with

$$\delta = F \quad A_{eff} \quad C_D / m,$$
$$F = [1 - r_{p0}\Lambda\cos \quad i_0 \;/\; v_{p0}]^2.$$

where ρ is the atmospheric density, Λ is the rotational rate of the atmosphere about the Earth's axis, r_{p0} is the initial perigee radius, i_0 is initial inclination, v_{p0} is the velocity at the initial perigee, C_D and A_{eff} are, respectively, the drag coefficient and the effective area of the satellite.

If the density ρ, is assumed to vary sinusoidally with ϕ, where ϕ is the geocentric angular distance from the direction of the density maximum, we write

$$\rho = \rho_0 (1 + F\cos\phi) \exp\{-(r - \sigma)/H_0\},$$

with

$$\sigma = \frac{r_p(1 - \varepsilon\sin^2\Phi)}{(1 - \varepsilon\sin^2\Phi_p)},$$
$$F = \frac{f - 1}{f + 1},$$
$$f = \frac{\rho_{max}}{\rho_{min}},$$
$$\vec{\beta} = \frac{1}{H_0}.$$

where ρ_0 is the average density on the reference spheroid when $\phi = 90^\circ$, and H_o and Φ are average density scale height and geocentric latitude, respectively. The ellipticity of each of the oblate spheroidal atmospheric surfaces is assumed to be the same as that of the Earth's ellipticity, 0.00335. The atmospheric density model of Jacchia [7] is used to compute the values of ρ_0 and H_o. With some algebra, we get

$$\cos\phi = A\cos\theta + B\sin\theta,$$

where,

$$A = \sin\delta_B \sin i \sin\omega + \cos\delta_B \{\cos(\Omega - \alpha_B)\cos\omega - \cos i \sin(\Omega - \alpha_B)\sin\omega\},$$

$$B = \sin\delta_B \sin i \cos\omega - \cos\delta_B \{\cos(\Omega - \alpha_B)\sin\omega + \cos i \cos(\Omega - \alpha_B)\cos\omega\},$$

$$\alpha_B = \alpha_s + \lambda, \quad \delta_B = \delta_s,$$

(α_s, δ_s) and (α_B, δ_B) are the right ascension and declination of the Sun and the centre of the day time bulge, and θ is the true anomaly.
The density ρ can be written as [8],

$$\rho = k\{1 + FA\cos\theta + FB\sin\theta\}\exp\{z\cos E + c\cos 2(\omega + \theta)\},$$

where

$$k = \rho_0 \exp\{(r_{p0} - a)/H_0 - c\cos 2\omega_0\},$$
$$c = \varepsilon r_{p0} \sin^2 i / 2H_0.$$

The perturbing potential of the earth due to J_2 to J_6 is given by

$$V = \frac{K^2}{r}\sum_{n=2}^{6} J_n \left(\frac{R}{r}\right)^n P_n(\cos\upsilon),$$

where J_n, K and R are constants,

$$\cos\upsilon = x_3 / r$$

and $\boldsymbol{P_n}$ are Legendre polynomial of order n.

The values of the constant utilized in the package are,

$J_2 = 1.08263 \times 10^{-3}$
$J_3 = -2.53648 \times 10^{-6}$
$J_4 = -1.62330 \times 10^{-6}$
$J_5 = -2.26194 \times 10^{-6}$
$J_6 = 5.42635 \times 10^{-6}$
$K^2 = 398600.8\ km^3 / sec^2$
$R = 6378.135$ km

Software named KSNUM was developed for orbit prediction using the above expressions.

3. GENETIC ALGORITHMS

Genetic Algorithms [4, 5] (GA) have received a great deal of attention regarding their potential as an optimization technique for complex functions. These are search algorithms based on the mechanics of natural selection and natural genetics. They combine the survival of the fittest with a structured yet randomized information exchange to form a search algorithm. GAs can be considered as a stochastic optimization techniques where search methods model natural phenomena of genetic inheritance and Darwinian strife for survival. The metaphor underlying the genetic algorithms is that of natural evolution.

GAs is different from the traditional algorithms, because GAs work with a coding of the parameter set, not the parameters themselves. GAs search from a population of points, not a single point. GAs use payoff (objective function) information, not derivatives or other auxiliary knowledge. GAs use probabilistic transition rules, not deterministic rules.
A simple genetic algorithm that yields good results in many practical problems is composed of three operators. (1) Reproduction, (2) Crossover and (3) Mutation.

Reproduction is the process by which the proper parents are selected, in accordance with their fitness, for possible mating to generate off springs. Generally this process is carried out probabilistically by taking into consideration of the fitness values of the individuals in the population.

Crossover combines the features of two parent solutions to form two offspring by swapping corresponding segments of parents.

Mutation arbitrarily alters one or more bits of the selected member of the population.

The simplest form of GA as used in the present context is given below:

1. Choose coding to represent problem parameters, Select the criteria for reproduction, Crossover and Mutation,
2. Input the initial population size, probabilities of cross over and mutation, search domain of the variables, termination criteria or maximum number of iteration as T_{max},
3. Set T=0, Generate initial population from the search domains randomly,
4. Evaluate each string of the population for fitness,
5. If Termination Criteria is satisfied, or $T > T_{max}$, then STOP,
6. Perform reproduction on the population,
7. Perform crossover on the population,
8. Perform mutation on the population,
9. Evaluate the strings of the new population,
10. T = T + 1, go to step 4.

Here we have constantly assumed probability of crossover = 0.90 and probability of mutation = 0.05.

4. BRIEF DESCRIPTION OF THE SOFTWARE

The software "KSGEN" developed in VSSC was utilized for the re-entry predictions. This software is an integrated package of "KSNUM" and Genetic algorithm. Effective ballistic coefficient BC is estimated with respect to the state vectors from different epochs that minimize the dispersions in the re-entry times from the state vectors under consideration.

5. METHOD OF ANALYSIS

The software requires suitable values of solar flux (F10.7), magnetic index (Ap) and interval for ballistic coefficient variation. F10.7 values utilized at the epoch are based on an average of the 81 days just prior to the epoch TLE and for further propagation the software uses either predicted or estimated values in day-by-day basis. The limits for the two ballistic coefficients, one is up to 6500 km and other is beyond 6500 km in the radial distance are taken as 70 to 130 and 130 to 180, which is found to be sufficient enough after test runs of 'KSGEN'. The TLEs are converted in to position and velocity components using SGP4/SDP4 theory for the numerical integration and propagation by 'KSGEN'.

The execution of the program provides the best ballistic coefficient estimated, which minimizes the predicted re-entry time variations with respect to each set of TLE data, together with the mean prediction and dispersions on the predictions at the TLEs utilized. And it also provides the prediction of the re-entry at the latest available TLE. The mean prediction from the sufficient number of TLE sets is considered as the re-entry prediction at the latest epoch. In all our prediction exercises, we assumed that the object re-enters the Earth's atmosphere when it reaches perigee height below 90 km above. This is based on the fact that there are significant variations in the atmospheric properties above 90 km with solar, magnetic activity and local time than below 90 km. Also a diffusive equilibrium predominates beyond 90 km.

6. DISPERSION LIMITS

Also it is noticed that the estimated best ballistic coefficients from different sets of TLEs were lying in a small band of ballistic coefficients. Considering these three factors, we took ±10% for BC, to obtain the dispersion limits in re-entry time.

7. CASE STUDIES

For the present analysis we considered two objects, SROSS C2 satellite and SL-12 Rocket Body, which were reenterd or decayed on 12th July 2001 and 3rd March 2009, respectively.

The re-entry prediction of SROSS C2 satellite was carried out with a total of 14 TLE's available for the last five days (from 8th July 2001). From the latest TLE epoch of 12th July 2001, 00:38:01, the prediction is

made as 12th July 2001, 4 hours 43 minutes against the actual re-entry time of 4 hours 37 minutes. The difference of 6 minutes is noted. The details of all the 14 predictions made from different TLE's with their percentage error, upper and lower bounds are provided in Tab 1.

The percentage errors are computed by the following formula.

$$\%error = (T_{COM}-T_{REF})*100/(T_{REF}-T_{IN})$$

where, T_{COM} is the time of actual reentry, T_{REF} is the predicted time of reentry, T_{REF} is the time corresponding to the initial TLE propagated.

Fig 1 depicts all the 14 predictions with their upper and lower bounds. It is observed that in all predictions, the actual re-entry time is with in the bounds.

The re-entry prediction of SL-12 rocket body was carried out with a total of 13 TLEs available for the last three days (from 1st March 2009). From the latest TLE epoch of 03rd March 2009 at 16:54:51, the prediction is made as 03rd March 2009 at 17hours 25 minutes against the actual re-entry time of 03rd March 2009, 17hours 30 minutes. The difference of 5 minutes is noted. The details of all the 13 predictions made from different TLE's with their percentage error, upper and lower bounds are provided in Tab 2.

Fig 2 depicts all the 13 predictions with their upper and lower bounds. It is observed that in all predictions, the actual re-entry time is with in the bounds.

8. IADC RE-ENTRY CAMPAIGNS

Till today the IADC [9] conducted re-entry prediction campaign for 10 identified objects. We have been participating in the last 7 IADC re-entry campaigns since 2002. Our re-entry predictions were in general quite satisfactory. In the last 5 re-entry campaigns our re-entry predictions with the last available TLE was one of the best predictions among all participating agencies including NASA and ESA.

9. CONCLUSION

The re-entry prediction made by the software KSGEN match very well with the actual re-entry time of the objects.

Table 1 Re-entry prediction results for the object SROSS

TLE epoch	Prediction	Lower bound	Upper bound	Difference in minutes	% error
12/07/2001 00:38:01	12/07/2001 4:43	12/07/2001 4:40	12/07/2001 4:56	5.8	2.4
11/07/2001 22:05:35	12/07/2001 4:48	12/07/2001 4:33	12/07/2001 5:18	10.1	2.7
11/07/2001 21:43:13	12/07/2001 5:20	12/07/2001 4:30	12/07/2001 5:42	24.5	5.9
11/07/2001 12:57:07	12/07/2001 4:58	12/07/2001 4:27	12/07/2001 5:49	20.2	2.1
11/07/2001 05:37:27	12/07/2001 5:9	12/07/2001 4:23	12/07/2001 6:5	31.7	2.3
11/07/2001 02:41:21	12/07/2001 5:6	12/07/2001 4:13	12/07/2001 7:3	28.8	1.9
10/07/2001 17:52:20	12/07/2001 5:29	12/07/2001 4:10	12/07/2001 7:13	51.8	2.5
10/07/2001 13:27:29	12/07/2001 5:24	12/07/2001 3:50	12/07/2001 9:23	46.1	2.0
10/07/2001 04:37:20	12/07/2001 8:2	12/07/2001 3:41	12/07/2001 9:43	204.5	7.1
09/07/2001 16:49:34	12/07/2001 8:16	12/07/2001 3:44	12/07/2001 9:53	218.9	6.1
09/07/2001 10:55:22	12/07/2001 8:48	12/07/2001 3:56	12/07/2001 12:28	226.1	5.7
09/07/2001 05:00:57	12/07/2001 8:38	12/07/2001 3:23	12/07/2001 14:25	240.5	5.6
08/07/2001 21:37:38	12/07/2001 9:20	12/07/2001 2:28	12/07/2001 14:19	265.0	5.6
08/07/2001 11:16:33	12/07/2001 8:22	12/07/2001 5:38	12/07/2001 14:25	259.2	4.8

Table 2 Re-entry prediction results for the object SL-12

TLE epoch	Prediction	Lower bound	Upper bound	Difference in minutes	% error
03/03/2009 16:54:51	03/03/2009 17:25	03/03/2009 17:21	03/03/2009 17:31	4.3	-12.4
03/03/2009 15:27:54	03/03/2009 17:18	03/03/2009 17:13	03/03/2009 17:34	11.5	-9.5
03/03/2009 14:00:53	03/03/2009 17:10	03/03/2009 16:56	03/03/2009 18:11	18.7	-9.0
03/03/2009 11:06:41	03/03/2009 17:22	03/03/2009 16.49	03/03/2009 18:48	7.2	-1.9
03/03/2009 03:49:31	03/03/2009 17:30	03/03/2009 14:26	03/03/2009 19:14	25.9	-3.2
02/03/2009 20:31:15	03/03/2009 16.55	03/03/2009 14:31	03/03/2009 20:29	34.6	-2.7
02/03/2009 17:35:50	03/03/2009 16:32	03/03/2009 13:39	03/03/2009 19:45	57.6	-4.0
02/03/2009 11:44:46	03/03/2009 16:40	03/03/2009 13:36	03/03/2009 19:56	49.0	-2.7
02/03/2009 02:57:39	03/03/2009 16:19	03/03/2009 13:27	03/03/2009 20:12	70.6	-3.1
01/03/2009 21:05:55	03/03/2009 16:40	03/03/2009 13:48	03/03/2009 20:13	85.0	-3.2
01/03/2009 16:42:01	03/03/2009 15:30	03/03/2009 12:30	03/03/2009 19:39	113.8	-3.9
01/03/2009 10:49:55	03/03/2009 15:14	03/03/2009 12:50	03/03/2009 19:17	138.2	-4.2
01/03/2009 04:57:40	03/03/2009 15:21	03/03/2009 12:00	03/03/2009 19:13	128.2	-3.5

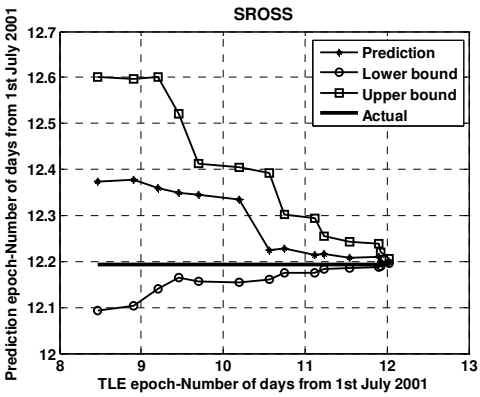

Figure 1 Re-entry predictions for the object SROSS C2

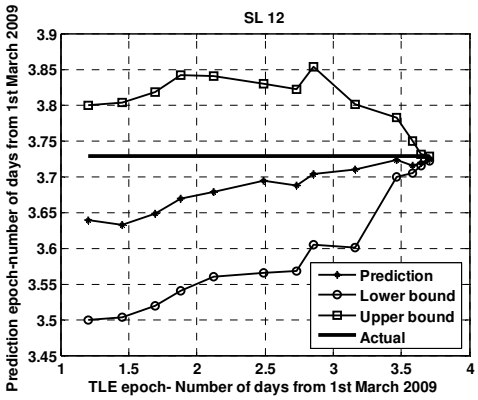

Figure 2. Re-entry predictions for the object SL-12

REFERENCES

1. Stiefel, E.L. & G.Scheifele. (1971). *Linear and Regular Celestial Celestial Mechanics*, Springer, Berlin.
2. C C Chao and M H P Latt. (1995). An accurate and efficient tool for orbital life time predictions@, AAS 91-134.
3. Anil V Rao, AMinimum variance estimation of re-entry debris trajectories, Jl. of Spacecrafts and Rockets. (2000). Vol. 37, No. 3, pp. 366 -373.
4. Deb, K.(1995). *Optimization for engineering design: Algorithms and examples,* Printice Hall of India.
5. L Darrel Whitley (ed.). (1993). A Foundations of Genetic Algorithms. 2", Morgan Kauffmann Publishers.
6. Sharma, R.K., Anil Kumar, A.K., Xavier James Raj, M. (2006). An integrated approach for risk objects re-entry predictions in terms of KS elements and genetic algorithm, Celestial Mechanics Recent Trends, Narosa Publishing House, 169-186.
7. Jacchia, L. G. (1977). Thermospheric temperature, density and composition: new model, SAO SR 371, 1977.
8. King Hele, D.G. (1987). Satellite orbits in an atmosphere: Theory and applications, Blackie, Glasgow and London, 1987.
9. Klinkrad, H. (2004). “IADC Re-entry Test Campaign 2003”, IADC/SG/2004-01, Issue 0.1, Steering Group IADC, 08 April 2004.