ORBIT EVOLUTION AND UNCONTROLLED RE-ENTRY OF THE "MOLNIYA" TYPE SATELLITES

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ABSTRACT

The Russian telecommunication satellites of the "Molniya" series are inserted into special high elliptical orbits with a period of ~ 12^h and typical minimal altitude of about 500 km. Such kinds of satellites have been launched since the mid of 1960s. At present more than 160 spacecrafts of this type were inserted into orbits. After the end of mission (or a spacecraft emergency) the satellite turns into an uncontrolled mode and the parameters of its orbit are changing due to the perturbing forces action. With the lapse of time the perigee altitude can become so low, that a spacecraft cannot orbit the Earth and it re-entries. The conditions of the final phase of the flight and the re-entry of the "Molniya" type satellite as a rule have the other character, than the re-entry of the space objects which were flying on circular (or near circular) orbits.

In the paper the influence of the perturbation forces (first of all the luni-solar attraction) on changing the parameters of "Molniya" type orbit is researched. In the example of a concrete "Molniya" spacecraft the orbit evolution of such a kind of satellites is examined. The variation of the main parameters, determining the lifetime of space vehicle, and a condition at which the satellite re-enters are studied.

INTRODUCTION

The perigee altitude of a "Molniya" type orbit can vary depending on mission purposes and a spacecraft (SC) modification; however for standard orbits of SC of "Molniya-1" series the value of this parameter is close to ~500 km. Usually satellites of a concrete series form the constellation. The planes of satellite orbits are disposed such as to ensure requirements of full coverage of all Russia (CIS) territory, and also of foreign countries. In principle only three "Molniya" type satellites are needed to give 24 hour coverage of Russia. It is necessary to point out, that afterwards other satellites belonging to the different countries were inserted into the "Molniya" type orbits.

The period and the inclination (which is near to the critical one i ~ 63.4 degrees) of the "Molniya" type orbits promote a small and slow modification of an apsides line of these orbits. During the flight time these orbits suffer the significant modifications at the expense of various perturbing factors. To counteract perturbations and to keep back the given properties of the orbits, the SC "Molniya" are equipped with a correcting propulsion system, and from time to time the orbital parameters are corrected. However after the end of a mission (or a SC emergency) the satellite turns into uncontrolled mode and nothing can stop the change of its orbital parameters due to the perturbing forces. At the expense of orbital evolution the altitude of perigee also varies, becoming in certain time so low, that the spacecraft cannot orbit the Earth and it re-entries. The first SC of "Molniya" series re-entered in March 1967, having lifetime of ~1.5 years. Further similar re-entries of such SC became regular enough and now only *50* satellites of the "Molniya" series continue their orbital flights with the various remaining lifetime.

Having the mass exceeding 1.5 tons, "Molniya" spacecrafts at uncontrollable re-entry can be considered as the real hazard objects that demands the accurate permanent orbital control of them especially at the final phase of a flight.

EVOLUTION OF THE "MOLNIA" TYPE ORBITS

Under investigation of evolution of the given class orbit we will consider as the base an orbit with the following initial parameters:

draconic period T = 718 *min,*
minimum altitude H_{min} = 550 *km*,
inclination i = 62,8 *deg*,
argument of perigee ω = 280 *deg*

To the indicated parameters there is a semi-major axis $a \approx 26600$ *km* and an eccentricity $e \approx 0.74$ osculating at the time of an ascending node.

The orbit with mentioned above parameters was most often used as the nominal one at launching the SCs of "Molniya" series. At the same time, for wider representation of orbit evolution and SC' lifetime in these orbits, in some cases, in addition to the base orbit we will examine also the orbits slightly different from it on parameters i and ω.

The analysis of orbital evolution concerned a space vehicle passive flight, i.e. it was supposed, that, in case of the active SC, all corrections of its orbit have already been fulfilled, and residual impulses of its operating

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attitude control systems are enough small, or, that SC is not capable to fulfill any more of its goal functions, and it became a space debris.

Flying in high elliptical orbit the SC "Molniya" periodically approaches the Earth entering the atmosphere and expose itself to its drag influence, and disposes from the Earth in the significant distances, filling strong enough luni-solar gravitational influence. Because of a design, including the shape and size of solar batteries, the SC "Molniya" has a significant reflecting area promoting solar pressure radiation.

The purpose of the paper is the orbit evolution study but not a generation of SC ephemerides with the high accuracy (up to some *mm* level) that is why in the analysis of the "Molniya" type orbits the influence of the following factors was estimated:

- non-central part of the Earth gravity field,
- luni-solar gravitational attraction,
- solar radiation pressure,
- aerodynamical drag of the Earth's atmosphere.

The estimation of the orbital parameters evolution were fulfilled on the base of a numerical integration of the differential equations of the SC' motion for the given time periods applying one or another model of perturbing forces, or by means of using the certain analytical relations. For transformation from rectangular coordinates and velocities of SC, being particularly a result of numerical integration of the SC' motion equation, to the appropriate orbital elements and for inverse transformation were used known formulas.

The system of the differential equations, realizing numerical model of the SC motion and taking into account the complete composition of the indicated above forces, can be presented in the rectangular inertial geocentric coordinate system (IGCS) related to the mean equinox and equator of standard epoch J2000 in the following vector form:

$$\ddot{\bar r} = -\mu \frac{\bar r}{r^3} + \mathrm{M}\,\overline{grad}\,U(\bar r') + \bar F_{atm}(\bar r, \dot{\bar r}) + \sum_{\alpha = L,S} \bar F_{\alpha}(\bar r, \bar r_{\alpha}) + \bar F_{spr}(\bar r, \bar r_s) \quad . \tag{1}$$

Here $\bar r, \dot{\bar r}, \ddot{\bar r}$ - position vector, velocity vector and acceleration vector of a SC in a mentioned inertial system of coordinates; μ - a gravitation constant of the Earth; $U(\bar r')$ - a non-central part of the Earth gravity field represented in decomposition to a series by spherical functions (harmonics), $\bar r' = M^T \bar r$ - a SC position vector in geocentric Earth-fixed rotating coordinate system (*M* - a matrix of transformation from the rotating to the inertial system of coordinates); $\bar F_{atm}$ - an acceleration called by atmospheric drag; $\bar F_{\alpha} = \bar F_L, \bar F_S$ - an acceleration due to a gravitational attraction of the "third" body (the Moon or the Sun); - an acceleration caused by the solar radiation pressure; $\bar r_L, \bar r_S$ - position vectors of the Moon and the Sun in IGCS.

The usage of the equations (1) for the Earth satellite motion model gives a possibility simply enough to take into account the various perturbing forces and to obtain the appropriate calculation data, allowing to estimate influence of these perturbing factors on the evolution of the satellite orbit parameters both separately and jointly. Thus, the obtaining of basic calculation data – the position and velocity vectors of the SC at the given epoch in each case was carried out by means of the numerical integration of appropriate variants of the equations (1) with the help of the effective numerical method [1].

As the orbital elements used at the analysis of an orbit evolution, were considered classical Keplerian elements a, e, i, ω, Ω, and also perigee distance $r_\pi = a\,(1-e)$ and the minimum altitude of SC over the Earth surface - H_{min}. The last two parameters are especially important from the point of view of an estimation of the remaining SC's lifetime.

The influence of non-central part of the Earth gravity field on "Molniya" type orbit evolution

The influence of non-central part of the Earth gravity field is the main perturbing factor practically for all orbits of artificial Earth satellites. Due to this factor the orbital parameters of an artificial satellite have short-period, long-period and secular variations.

The short-period variations representing changing in orbital parameters within one revolution take place for all elements. On Fig. 1-3 the short-period variations of a semi-major axis, an eccentricity and a perigee distance of the "Molniya" type orbit due to the influence of non-central part of the Earth gravity field on two neighboring revolution are shown. (In this case the harmonics up to the order and degree (16×16) were taking into account). The represented results correspond to the basic orbit and to the orbit different from it on the parameter ω (which in the second variant had the value $\omega = 300°$). From these figures, in particular, follows that in spite of the maximum value of the parameter *a* variations on one revolution reaches ~150 *km*, due to a synchronous changing of parameter *e* the variation of perigee distance r_π, determined by dependence $\delta r_\pi = \delta a\,(1-e) - a\;\delta e$, in both cases do not exceed 3 - 5 *km*.

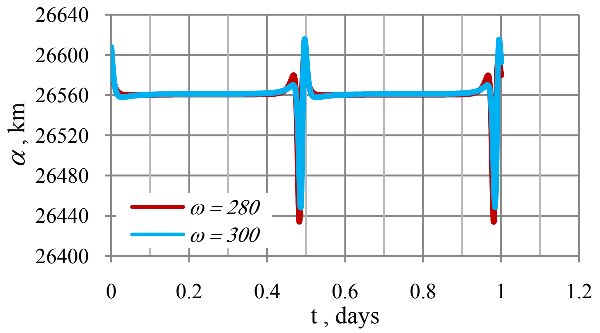


Figure 1. "Molniya" type orbit's semi-major axis variation on two revolutions.

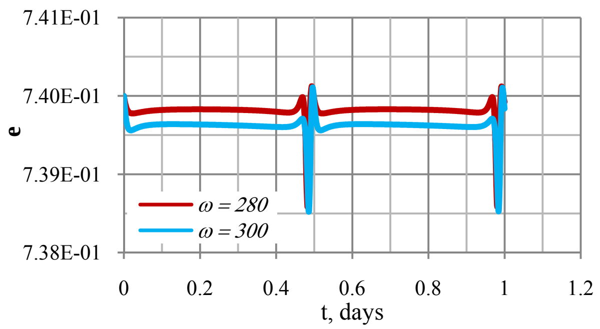


Figure 2. "Molniya" type orbit's eccentricity variation on two revolutions.

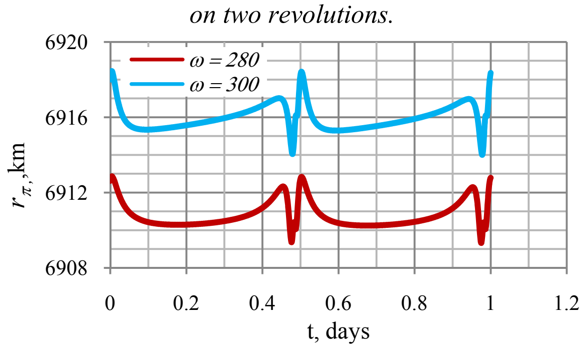


Figure 3. "Molniya" type orbit's perigee distance variation on two revolutions.

Long-period perturbations (having a considerably greater oscillation period in comparison with a SC' orbit one) appear in all elements, except for a semi-major axis. However the value of variations of orbital parameters is rather small. The character of the long-period variations caused by the influence of the Earth gravity field of order and degree (16×16) on the example of parameter r_π is shown on Fig. 4. Here the results of calculations for the basic orbit and for the orbits different from it or only on inclination (i = 65°), or only on perigee longitude (ω=300°) are presented. As it is seen from the figure, for the basic orbit the value of r_π decreased on ~50 *km* within the period of 15 years. In the case of orbit with ω = 300° the degradation of r_π for the same time has made ~35 *km*, and for an orbit with i = 65° the fifteen-year history of the perigee changing has led to a r_π rising on ~10 *km*.

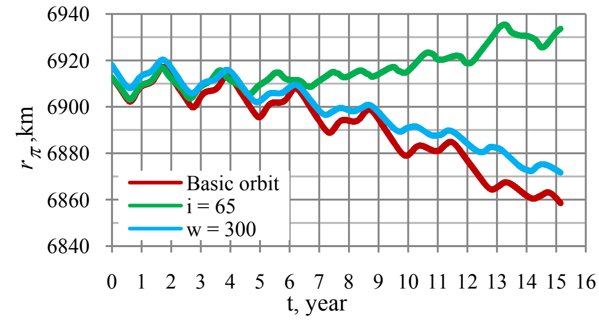


Figure 4. Variations of perigee distance of "Molniya" type orbit's over fifteen-year period due to the Earth gravity field.

Secular drift of orbital parameters become much more important from the point of view of long-term evolution. Among the parameters interesting for us these drifts are presented at the longitude of ascending node Ω and in argument of perigee ω. Secular drift of the indicated parameters are stipulated by the influence of even zonal harmonics of geopotential, first of all – by the influence of the second zonal harmonic. Secular drift of these parameters $\delta\Omega_{sec}$, $\delta\omega_{sec}$ due to influence of the second zonal harmonic can be presented in the form:

$$\delta_{\sec}\Omega = \dot{\Omega}_2\left(t - t_0\right), \qquad \delta_{\sec}\omega = \dot{\omega}_2\left(t - t_0\right).$$

Where within the first order accuracy the coefficients $\dot{\Omega}_2$, $\dot{\omega}_2$ are determined as follows [2]:

$$\dot{\Omega}_2 = -\frac{3}{2} J_2 n \left(\frac{a_e}{a}\right)^2 \frac{\cos i}{\left(1-e^2\right)^2},$$

$$\dot{\omega}_2 = \frac{3}{4} J_2 n \left(\frac{a_e}{a}\right)^2 \frac{5\cos^2 i - 1}{\left(1-e^2\right)^2}.$$

Here n - mean motion of SC, a_e - equatorial radius of the Earth.

The values of secular drift (over a year) for parameters Ω and ω of the "Molniya" type orbits at different values of inclination i are given in Tab. 1.

Table 1. Rates of secular drift for angular elements of the "Molniya" type orbits

Inclination, i	62°	62.8°	63.4°	65°
$\dot{\Omega}_2$, *deg/year*	- 56.36	- 54.87	- 53.75	- 50.73
$\dot{\omega}_2$, *deg/year*	6.12	2.68	0.15	- 6.42

As it follows from the table, the rates of secular drift in longitude of ascending node for the considered orbits due to the Earth oblateness has made over 50 degrees per a year. The rates of secular drift in argument of perigee ω are much lower. At inclinations more than critical value (i = 63.4°) the parameter ω tends to constant reduction, and at i <63.4° the ω variation tendency is the inverse.

Luni-solar perturbations of the "Molniya" type orbits

The gravitational attraction of an artificial satellite by the Moon and the Sun perturbs parameters of its orbit. Luni-solar perturbations can be divided into three groups: short-term, long-term and secular.

Short-term variations of orbital elements induced by these perturbations are small enough and are not interesting for our investigations. Secular and long-term perturbations due to the Moon and the Sun essentially depend on a semi-major axis and an eccentricity of orbit: they become greater if the value a and e increase. That is why these perturbations substantially appear in high elliptical orbits, in particular in the "Molniya" type orbits.

Secular perturbations of the orbital elements

Secular variations of the parameters of the considered orbits due to the luni-solar perturbations have a complicated enough character, and depend not only on a semi-major axis and an eccentricity, but also on other orbital elements: i, Ω and ω.

For an estimation of secular variations of orbital parameters first of all we will examine such variations for one revolution of SC. The estimation of perturbations due to the "third body" we will carry out in a geocentric rectangular coordinate system $\tilde{X}\tilde{Y}\tilde{Z}$, its plane $\tilde{X}\tilde{Y}$ coincides with the orbital plane of perturbing body (the Moon or the Sun). In this coordinate system for one revolution the variations of orbital elements $\{\tilde{q}_k\}$ can be written (as a first approximation) by the following formulas [3]:

$$\begin{aligned}
&\delta\,\tilde{a} \cong 0,\\
&\delta\,\tilde{e} = \frac{1}{2}Ae\sqrt{1-e^2}\cdot \sin^2\tilde{i}\cdot \sin 2\tilde{\omega},\\
&\delta\,\tilde{i} = -\frac{1}{4}A\frac{e^2}{\sqrt{1-e^2}}\cdot \sin 2\tilde{i}\cdot \sin 2\tilde{\omega}, \qquad (2)\\
&\delta\,\tilde{\omega} = A\frac{1}{\sqrt{1-e^2}}\cdot\left[\left(e^2-\sin^2\tilde{i}\right)\cdot \sin^2\tilde{\omega}+\frac{2}{5}\left(1-e^2\right)\right],\\
&\delta\,\tilde{\Omega} = -A\frac{1}{\sqrt{1-e^2}}\cdot \cos\tilde{i}\cdot\left[e^2\cdot \sin^2\tilde{\omega}+\frac{1}{5}\left(1-e^2\right)\right].
\end{aligned}$$

Here $\tilde{i}$ - inclination of the SC orbit to the orbital plane of perturbing body; $\tilde{\omega}$ - angular distance of perigee of the SC orbit, relative to its ascending node in $\tilde{X}\tilde{Y}$ plane; $\tilde{\Omega}$ - the angular distance (counted out in $\tilde{X}\tilde{Y}$ plane) between a direction in a cross point of $\tilde{X}\tilde{Y}$ plane and the Earth equator and a direction in an ascending node of the SC orbit on this plane (see Fig. 5).

In the formulas (2) the value A, designated further as A_α to indicate its dependence on the concrete perturbing body, looks like the following:

$$A_\alpha = \frac{15}{2}\pi\cdot\frac{\mu_\alpha}{\mu_0}\left(\frac{a}{a_\alpha}\right)^3\cdot\sqrt{1-e_\alpha^2}\,.$$

In this formula index α, accepting value L or S, specifies the perturbation (lunar or solar) in the relevant parameters, μ_0 − gravitation constants of the Earth. In the assumption, that a semi-major axis of the "Molniya" type orbit $a \approx 26600\,km$, A_L and A_S will have the following average values:

$$A_L = 0{,}95\cdot 10^{-4},\qquad A_S = 0{,}44\cdot 10^{-4}\,. \qquad (3)$$

Thereby the value of perturbations of elements $\{\delta\,\tilde{q}_k\}$ due to lunar influence will exceed more than 2 times the similar perturbations induced by the solar attraction.

Angular elements in the equation (2) depend on angular elements of SC orbit in the basic equatorial system of coordinates (IGCS), and also on inclination angle I of a perturbing body orbit to the Earth equator. When the Sun is a perturbing body the angle I will coincide with the obliquity of the ecliptic to Earth equator ε. Dependence between angular orbital elements referred to IGCS and elements $\tilde{i}$, $\tilde{\omega}$, $\tilde{\Omega}$ referred to the coordinate system $\tilde{X}\tilde{Y}\tilde{Z}$ is determined by the following relations:

$$\begin{aligned}
&\cos\tilde{i} = \cos I\cdot\cos i + \sin I\cdot \sin i\cdot\cos\Omega',\\
&\sin\tilde{\Omega}\cdot\sin\tilde{i} = \sin\Omega'\cdot\sin i,\\
&\cos\tilde{\Omega} = \cos\Omega'\cdot\cos d - \sin\Omega'\cdot\cos i\cdot\sin d \qquad (4)\\
&\tilde{\omega} = \omega - d,\\
&\sin d\cdot\sin\tilde{i} = \sin I\cdot\sin\Omega',
\end{aligned}$$

Similarly, there are formulas for transformation from elements $\tilde{i}$, $\tilde{\omega}$, $\tilde{\Omega}$ to elements i, ω, Ω'.

The element Ω' in the equation (4) represents an arc of the Earth equator counted out from a cross point of orbital plane of a perturbing body with equator to an ascending node of SC orbit on equator. Obviously, that in a case, when the plane $\tilde{X}\tilde{Y}$ is an ecliptic plane (a perturbing body - the Sun) $\Omega' = \Omega$. In the case when the Moon is a perturbing body, will take place: $\Omega' = \Omega - \Omega_L$, where Ω_L - a longitude of ascending node of the Moon orbit on the Earth equator.

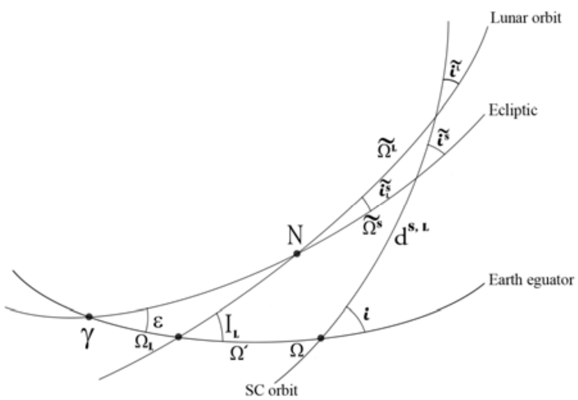


Figure 5. Geometry of orbital motion of the SC "Molniya" *and the "third" body.*

On the basis of the known formulas for transformation of angular elements dependences for determination of secular variations for one revolution for elements *i, ω, Ω* in the basic coordinates system IGCS can be obtained. Namely, for δ*i, δω, δΩ* it takes place:

$$\delta i = \cos d \cdot \delta \tilde{i} - \sin \Omega' \cdot \sin I \cdot \delta \tilde{\Omega},$$

$$\delta \omega = \delta \tilde{\omega} + \cos \tilde{i} \cdot \delta \tilde{\Omega} - \cos i \cdot \delta \Omega, \tag{5}$$

$$\delta \Omega = \frac{1}{\sin i} \cdot \left(\sin d \cdot \delta \tilde{i} + \cos d \cdot \sin \tilde{i} \cdot \delta \tilde{\Omega} \right)$$

At obtaining these formulas it was supposed, that the inclination angle *I* of a perturbing body orbit to the Earth equator remains constant within one revolution of SC and even during more significant periods of time. Such assumption is quite comprehensible within the accepted accuracy. For calculation of perturbations from the Sun *I*=*ε*, however, by examining the perturbations from the Moon it is necessary to consider, that the value of inclination *I* will vary within the limits from *I* = 18.2° to *I* = 28.6° with a period ~ 18.6 years due to precession of the Moon orbit (that is revealed in the moving of the ascending node *N* of this orbit on the ecliptic).

It is necessary to note, that a semi-major axis and eccentricity variations are invariant concerning various rectangular coordinate systems. Therefore in the case of IGCS it will take place: $\delta a = \delta \tilde{a} \approx 0$, $\delta e = \delta \tilde{e}$.

As $\delta a = \delta \tilde{a} \approx 0$, for an estimation of variation of perigee distance $\delta \tilde{r}_\pi$ the following dependence will be valid:

$$\delta \tilde{r}_\pi = \delta r_\pi = -a \delta \tilde{e} = -a \delta e. \tag{6}$$

According to (2) the variations of elements $\{\tilde{q}_k\}$ after one revolution generally it is possible to present in the form:

$$\delta \tilde{q}_k = A \cdot f_{\tilde{q}_k} \quad \left(\tilde{q}_k = \tilde{e}, \tilde{i}, \tilde{\omega}, \tilde{\Omega} \right), \tag{7}$$

Where functions $f_{\tilde{q}_k} = f_{\tilde{q}_k}\left(e, \tilde{i}, \tilde{\omega}\right)$ explicitly depend on Ω if take into account (4).

Using (7) and (5) it is possible to establish, that in the basic geoequatorial coordinate system the variation of the elements e, i, ω, Ω for one revolution can be presented in the form similar to (7):

$$\delta q_k = A \cdot f_{q_k}; \quad \left(q_k = e, i, \omega, \Omega \right)$$

and $f_{q_k} = f_{q_k}\left(e, i, \omega, \Omega, I\right)$.

We will denote $\delta' q_k$ the adjusted value of δq_k , so: $\delta' q_k = \dfrac{\delta q_k}{A} \cdot 10^{-4}$.

The variation of functions $\delta' q_k$ depending on the parameter Ω' at the different inclination angles I $\left(I_1 = 18.2^\circ, I_2 = \varepsilon = 23.4^\circ, I_3 = 28.6^\circ\right)$ for the case of basic "Molniya" type orbit is given on Fig. 6-9.

From the shown graphics follows, that depending on the current value Ω' (and also a declination angle of perturbing body' orbital plane to the Earth equator) orbital elements will increase, or decrease from a revolution to a revolution.

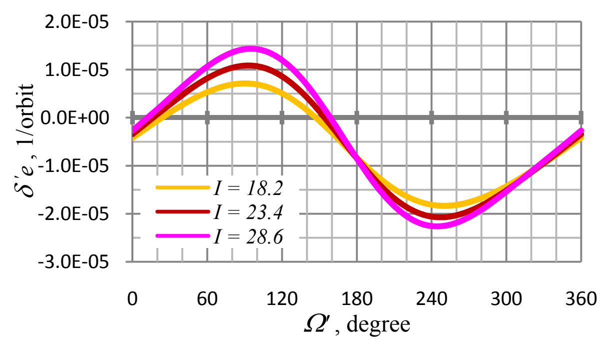


Figure 6. Dependence of $\delta' e$ *on* Ω'

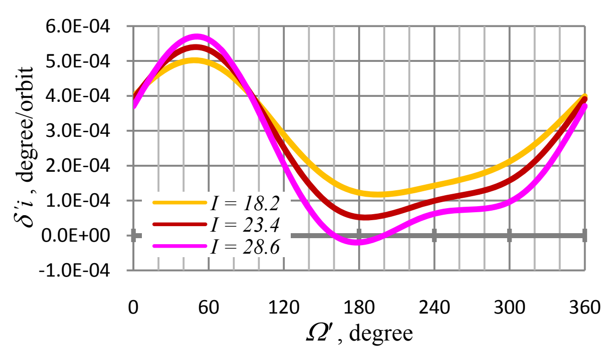


Figure 7. Dependence of $\delta' i$ *on* Ω'

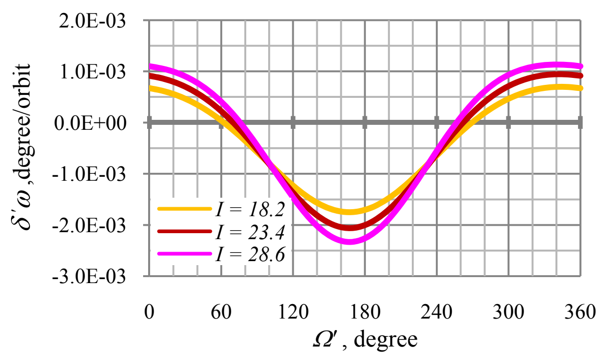


Figure 8. Dependence of $\delta'\omega$ on Ω'

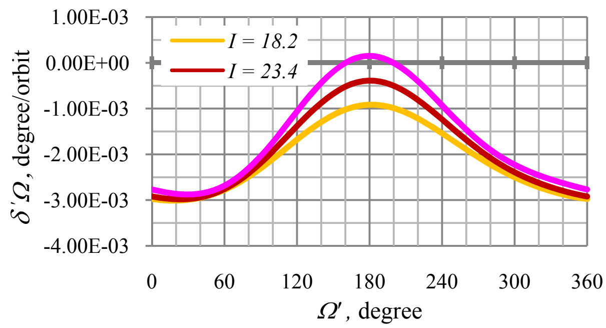


Figure 9. Dependence of $\delta'\Omega$ on Ω'

As a result of the analysis of presented graphics, taking into account that secular drifts of examined elements are proportional to A_L or A_S, determined in (3), and for δr_π it is right (6), it is possible to conclude, that due to the gravitational attraction of the Moon and the Sun the “Molniya” type orbit parameters i, ω, Ω and r_π tend to a secular variation. Thus, depending on initial value Ω' rates of this changing will be different, but, as a whole, at recalculation for a year for the case of the basic orbit they are in the limits of effective range presented in Tab. 2.

Table 2. Limits of secular drifts of parameters of the “Molniya” type orbit due to luni-solar perturbations

Perturbing body	Rates of secular drift for orbital parameters			
	δi, *deg/year*	$\delta\omega$, *deg/year*	$\delta\Omega$, *deg/year*	δr_π, *km/year*
The Moon	-0.01÷0.40	-1.62÷0.79	-2.09÷0.10	-263÷416
The Sun	0.02÷0.17	-0.66÷0.30	-0.96÷-0.13	-96÷177

<u>Long-term perturbations</u>

Long-term oscillations of elements of SC orbit due to the gravitational attraction of the Moon and the Sun - $\{\delta_l q_k\}$ have frequency $2\lambda_\alpha$ and period $\frac{P_\alpha}{2}$, where λ_α is an angular velocity and P_α - a period of revolution of a perturbing body in its motion around the Earth. Amplitudes of these oscillations depend on the SC orbital parameters and parameters of perturbing bodies. In particular, for an estimation of the amplitude E of long-term oscillations of the eccentricity inducing respective variations of perigee distance r_π, the following formula [4] can be applied:

$$E = \frac{15}{16} C_\alpha D(\tilde{\omega}, \tilde{i}) e\sqrt{1-e^2}, \quad (\alpha = L, S)$$

Where

$$C_L = \frac{\mu_L}{\mu_0 + \mu_L}\frac{P}{P_L} = \frac{1}{82{,}3}\frac{P}{P_L}, \quad C_S = \frac{P}{P_S}, \quad P = 2\pi\sqrt{\frac{a^3}{\mu_0}},$$

$$D(\tilde{\omega}, \tilde{i}) = \sqrt{4\cos^2\tilde{i}\cdot\cos 2\tilde{\omega} + \left(1+\cos^2\tilde{i}\right)^2\cdot\sin^2 2\tilde{\omega}}.$$

The character of luni-solar perturbations, including their secular and long-term component, on an example of parameter r_π for the case of basic “Molniya” orbit at initial $\Omega = 280^\circ$ is shown on fig. 10. Here the variations of r_π within one year interval due to the influence of the Moon and the Sun separately and due to joint perturbation effect of these bodies are shown.

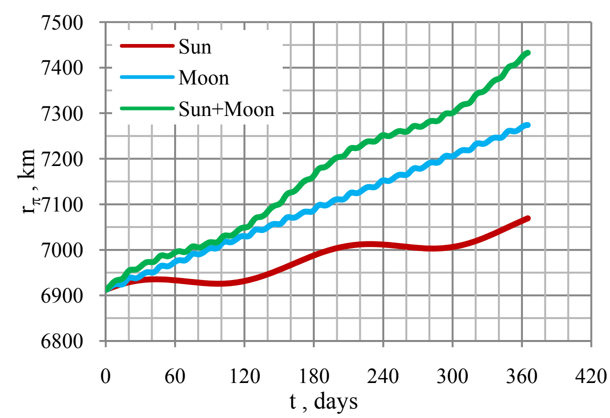


Figure 10. Variations of perigee distance of the “Molniya” type orbit due to luni-solar perturbations.

As it is seen from the Fig. 10 for the basic orbit the value of r_π may increase on ~ 500 *km* within the period of a year due to luni-solar perturbations.

The influence of solar radiation pressure and atmospheric drag on the SC’ “Molniya” orbit

The pressure of solar radiation on a surface of the SC “Molniya” leads to short-term and long-term perturbations of its orbital elements. Thus, short-term perturbations are small enough. For example, for a semi-major axis they do not exceed ~10 *m*. Long-term perturbations are present in all elements, except a semi-major axis.

For a “Molniya” type SC with the standard mass-dimensional specifications the value of orbital parameters variations throughout enough long-term time periods (several years) due to solar radiation pressure are estimated as follows:

$|\Delta a| < 10\,м$, $|\Delta e| < 4 \cdot 10^{-5}$, $|\Delta i| < 0.002^\circ$,

$|\Delta \omega| < 0.007^\circ$, $|\Delta \Omega| < 0.007^\circ$.

The amplitude of parameter r_π oscillations does not exceed 1 *km*. Thereby solar radiation pressure does not take any significant influence on evolution of the “Molniya” type orbits.

Moving along the high elliptical orbit, SC “Molniya” is situated within the atmosphere spreading area only small part of its orbit revolution time. So for the nominal orbit with a period ~12 hours and the minimum altitude H_{min} ~ 500 *km* SC “Molniya” will have the altitude H <1000 *km* for no longer than 15 *min* for one revolution and the atmospheric drag in this flight part will be rather insignificant in the value, and its integral effect on variation of SC orbital parameters will be small in comparison with the influence of the Moon and the Sun.

At perigee rising the influence of atmosphere on the “Molniya” orbit evolution will weaken even more. However at lowering r_π the atmospheric drag in the area around perigee will increase due to the atmospheric density increment, and at the reaching of r_π some threshold values the braking effect of the atmospheric drag will lead to the appreciable and even to the significant variations of SC orbital elements. First of all, it will concern a semi-major axis a and an eccentricity e. And even if in the course of the subsequent changing due to the influence of other factors r_π will exceed that threshold value, and the SC will prolong the orbital flight the parameters of its orbit (a and e) will be already essentially changed. In the cases when the minimum altitude H_{min}, reduced up to a critical threshold will remain lower this level during certain time the strong atmospheric drag effect in the area around the perigee will lead to rather fast reduction of semi-major axis and eccentricity of orbit. As the result, the effect of the aerodynamics resistance can become a dominant one for the termination of the SC lifetime.

Combined effect of various perturbing factors on evolution of the “Molniya” type orbits

If to regard, that the influence of solar radiation pressure on evolution of the “Molniya” type orbits is practically negligibly small, the character and the value of significant variations of orbital elements will be determined by combined effect of a non-central part of the Earth gravity field, the luni-solar perturbations and the atmospheric drag. At flight in orbits with high enough perigee distance (r_π > 400÷500 *km*) the first two of the mentioned factors will be the main forces perturbing SC motion. The special role in evolution of the “Molniya” type orbits having the important practical meaning from the point of view of these orbits lifetime gains the variation of parameter r_π.

Analyzing and comparing the effects discussed in prior sections it is possible to state, that:

- preferential influence of the Earth gravity field (in comparison with luni-solar perturbations) appears in secular variations of angular parameters Ω and ω, which rate of changing (at the given a) depends only on eccentricity and inclination; thus the rate of Ω drift considerably exceeds the appropriate rate of ω drift, being for the considered orbits small enough because of proximity of their inclination i to the critical value;
- long-term variations of eccentricity e, and perigee distance r_π due to the non-central part of the Earth gravity field, considerably less than the variations of these parameters induced by the luni-solar attraction;
- luni-solar perturbations cause secular drift of a SC orbit’s inclination, so that a direction (increase or decrease) and velocity of changing i depend on parameters ω and Ω. Because of changing these parameters, first of all due to the Earth oblateness influence, the character of a modification of parameter i will vary also;
- luni-solar perturbations are capable to generate secular drift in parameter ω as well, and the character of these perturbations also depends on concrete values of a longitude of ascending node. Simultaneously the secular drift ω also will be caused by the influence of a non-central part of the Earth gravity field. In the latter case the direction and velocity of changing ω will depend on an orbit inclination and its eccentricity which also varies eventually in dependence of parameter Ω. Therefore the value of perigee argument will vary due to the combined effect of two indicated factors, and the character of variation of this parameter will depend on the current Ω;
- the character and velocity of changing r_π will depend on current values Ω and ω. So for each concrete value ω (from the range of possible values of this parameter) the perigee distance of an orbit at the given values Ω will increase, and at others - on the contrary decrease, thus the velocity of changing r_π will depend on this concrete value ω (see Fig. 11);
- the gravitational attraction from the Moon and the

Sun can lead to such variations of perigee distance (minimum altitude) of an orbit, that only due to this reason further SC orbital motion becomes impossible. At the same time, the influence of the atmospheric drag will promote in these cases an acceleration of process of finishing a SC flight, and in some cases this perturbing factor can become even a principal reason of terminating of space vehicle life in an orbit.

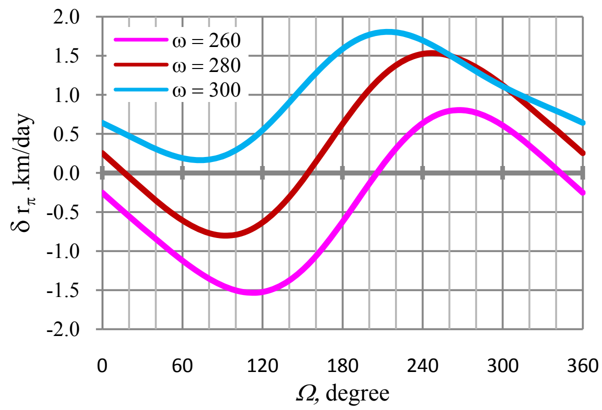


Figure 11. Velocity of changing of parameter r_π of the "Molniya" type orbit depending on ascending node longitude Ω and argument of perigee ω due to joint influence of the Moon and the Sun (for the case $I_L = \varepsilon$).

On Fig. 12 a long-term variation of parameter r_π for the basic orbit of SC "Molniya" is shown at the different initial values of longitude of ascending node Ω. Evolution was calculated at taking into account all significant perturbing factors. The presented results show the character of the influence of parameter Ω on the change of r_π. As one can see from the figure, for the great number of considered variants of computation, evolution r_π has led, eventually, to the termination of ballistic life of chosen virtual space vehicle though this finish has need sufficiently long-duration time (~ from 12 till 19 years). Only in one of considered variants (at initial value Ω=310°) hypothetical SC has orbited within more than 25 years. But also in this case approximately in 21 years in the course of orbit evolution its minimum altitude decreased to the critical values ~ 130 km. In this time the SC underwent a strong enough effect of the atmospheric drag in the area of the low altitudes. And though at the expense of subsequent evolution r_π (first of all - because of the luni-solar perturbations) a space vehicle has left dangerous band of flight, its orbit has already been essentially changed. Afterwards the perigee distance continued to vary and approximately in 22 years at next lowering H_{min} to a critical threshold the object has finished its existence in the space.

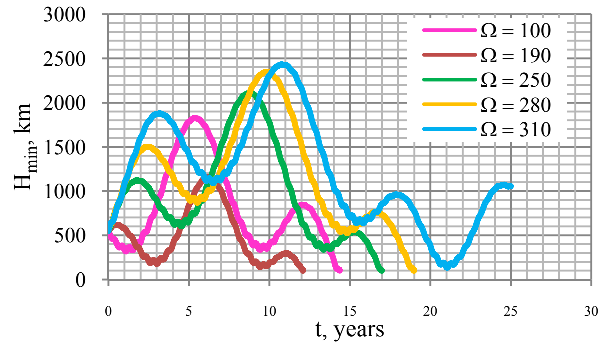


Figure 12. Evolution of minimal altitude H_{min} for the basic orbit of SC "Molniya" at different initial values Ω.

ESTIMATION OF POSSIBILITY OF THE ORBIT DETERMINATION AND THE REMAINING LIFETIME PREDICTION OF SPACE VEHICLE "MOLNIYA" TYPE

For getting the answer to a problem on the possibility of determination of orbital parameters and re-entry window prediction for the space vehicles "Molniya" type, which have completed their missions, 2 real SC have been selected: SC «Molniya 3-43» (the COSPAR international designator 1992-085A, the US SSN catalog number 22255) and SC «Molniya 3-46» (the COSPAR international designator 1994-051A, the US SSN catalog number 23211). The estimation of possibility of solving the indicated problems was conducted for the final phase of the SC flight, i.e. for the time periods covering the last year of their orbital life. The SC' orbit determination and prediction was fulfilled with the help of the methods, procedure and tools developed and used in MCC for solution of similar tasks, as it is described in [5].

For the solution of the orbit determination (OD) task the accessible measuring data in a format of two-line elements (TLE) was used. The selected measurement intervals contained the data that allow to determine (improve) the parameters of SC motion with the given precision. Usually such interval covered 2-4 days span. In the case of the SC « Molniya 3-46» shortly before end of its flight for solving the considered problem in addition to the TLE' sets the optical measurements of the right ascension and the declinations of space vehicle fulfilled under control of the Russian Academy of Sciences were also used.

On the basis of the generated packages of the measurement data, related to the considered variants of solution, the task of OD with the subsequent re-entry prediction was solved for each of the mentioned SC.

The analysis of the SC « Molniya» OD solutions under using TLE has shown, that these measurement data are not always well fitted among themselves on the consecutive arcs of the SC trajectories. It has been settled, that in the complicated cases of the TLE fitting the adding of the angular optical measurements allows to improve the OD result, in particular, for determination of the orbit inclination. On Fig. 13, 14 the dynamics of the re-entry time prediction for SC «Molniya 3-43» and SC «Molniya 3-46», corresponding to the obtained satisfactory OD solutions is represented in the graphic form. In these figures the left and the right borders of an estimated window of uncertainty of the re-entry time are also shown. The analysis of these graphics allows to state that the error of SC re-entry time prediction did not exceed 23 % from the remaining lifetime. It is necessary to mention that the most complicated arcs of trajectory for OD were the arcs corresponding to the last month of SC's life cycle, in particular, because of irregular distribution at that time of the available measurement data.

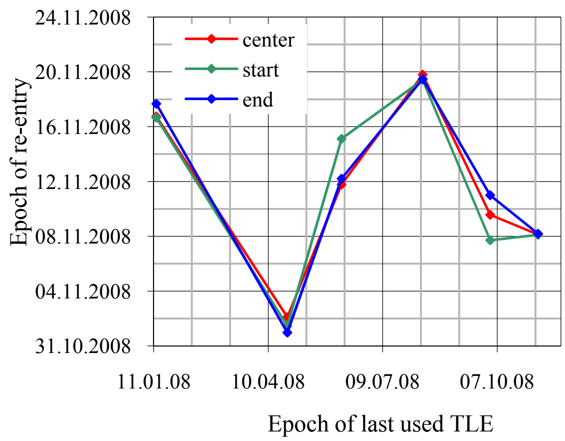


Figure 13. Dynamics of the SC «Molniya 3-43» re-entry time predictions during the last year of orbital life.

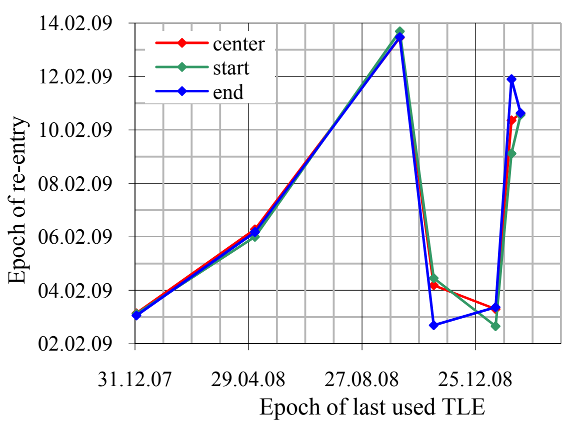


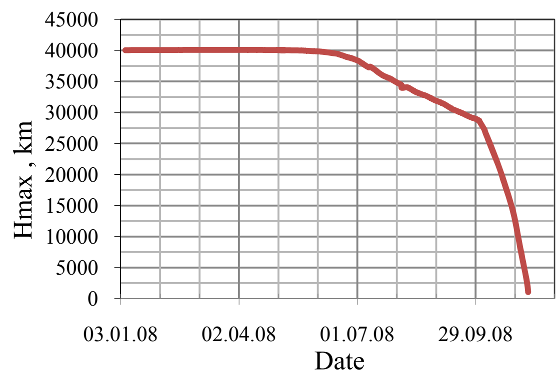
Figure 14. Dynamics of the SC «Molniya 3-46» re-entry time predictions during the last year of orbital life.

The last accessible TLE for the SC «Molniya 3-43» were referred to t_f = 08.11.2008, 00:05:20 UTC. The last OD solution obtained with using this TLE, has given the following estimation of the center of impact window (COIW) for «Molniya 3-43»:

Epoch: November, 8th 2008, 01:18:25 UTC;
Impact coordinates: 56,8° S, 218,0° E.

For the SC «Molniya 3-46» last accessible TLE corresponded to epoch t_f = 10.02.2009, 11:11:06 UTC. According to the OD solution obtained in MCC with the usage of the last TLE, space vehicle has finished its fly on February, 10th, 2009 at 14:41:14 UTC at a location: 54,6° southern latitude and 334,4° east longitude. At the same time according to the information from other public sources the SC «Molniya 3-46» decayed on February, 10th, 2009 at 14:33:00 UTC ± 1 hour at the location: 34,4° S and 310,2° E.

Dynamics of degradation of maximum (H_{max}) and minimum (H_{min}) altitudes of the SC «Molniya 3-43» orbit during the last year of its flight is shown on Fig. 15.



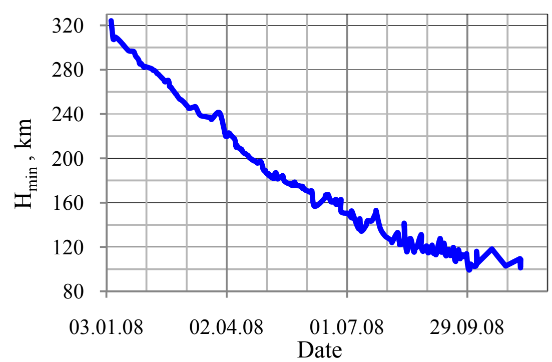


Figure 15. Dynamics of the SC «Molniya 3-43» maximum and minimum altitudes decay during the last year orbiting.

On Fig. 16 dynamics of the SC «Molniya 3-43» orbit altitudes changing during the last two days is presented.

As it follows from the figure, the maximum altitude of the satellite has decreased for this time with ~ 4500 *km* to ~ 810 *km*, and parameter H_{min} was practically invariable remaining within the limits 88÷83 *km*.

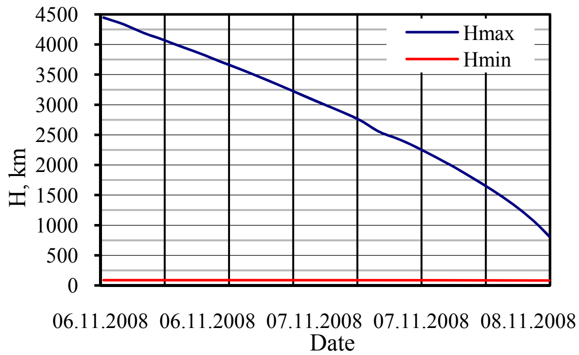
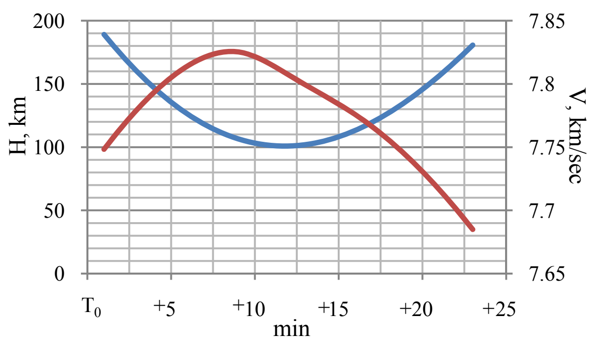
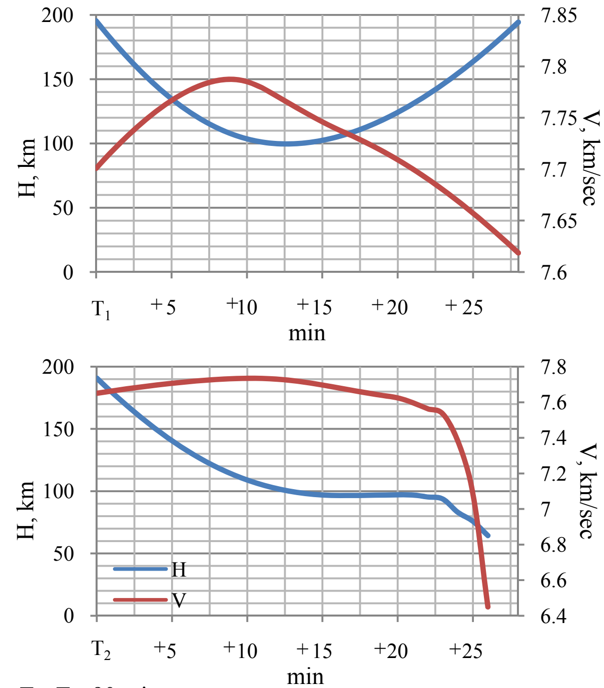


Figure 16. Dynamics of the SC «Molniya 3-43» maximum and minimum altitudes decay during the last two days orbiting.

The analysis of the trajectories of the SCs «Molniya 3-43, 3-46» reaching the Earth's atmosphere interface has shown, that the velocity of these objects at altitude of ~ 110 km was ~ 7,7 km/s, and the value of atmosphere entry angles made up ~-0,6°. Note for matching that appropriate parameters at uncontrolled re-entry of space objects from the near-circular orbits fluctuate on the velocity from 7,5 km/s to 7,1 km/s, and on the entry angles – from -0.07° to -0.2°.

Dynamics of changing the motion parameters (altitude and velocity) in area around the minimal distances from the Earth for the latest revolutions of «Molniya 3-43», directly before it decay is shown on Fig. 17.





T_1=T_0+90 min
T_2=T_1+87 min

Figure 17. Dynamics of changing the minimum altitude and velocity of the «Molniya 3-43» on the latest revolutions.

CONCLUSION

The introduced methodology and procedure as well as the presented calculation results allow to estimate the influence of the various significant perturbing factors and their joint effect on the long-term evolution of the orbits and on lifetime of the "Molniya" type space vehicle. It is find out, that the major factor influencing the lifetime of a space vehicle in an orbit is the gravitational attraction from the Moon and the Sun. The lifetime of SC "Molniya" depends on the initial values of the longitude of ascending node Ω and also on the value of argument of perigee ω of an orbit.

In the example of concrete real spacecrafts of the "Molniya-3" series the possibility of orbit determination and prediction of the re-entry time and impact window for "Molniya" type satellites is estimated at the usage of the measurement information presented in a different form. The character of variation of the key parameters determining the lifetime and conditions of SC de-orbiting for the last year and directly before the end of the flight of such a kind of space vehicles is given.

The carried out examinations allow to draw conclusion, that as a result of orbit evolution due to the influence of the considered perturbing factors all the “Molniya” type SCs sooner or later will terminate their orbital flight.

The methods, techniques and obtained results represented in the paper can be useful for the fulfillment of the analysis and the practical work on the surveillance of the flight and re-entry of the space objects inserted in the “Molniya” type orbits.

REFERENCES

1. Kolyuka Yu. F., Afanasieva T. I., Gridchina T.A. (2005). Precise long-term prediction of the space debris objects motion. In *Proc. 4th of the European Conference on Space Debris*, ESA SP-587, Darmstadt, Germany, pp315-319.

2. Reference guidance on celestial mechanics and astrodynamics (G.N. Duboshin’s redaction), Book, “Nauka”, Moscow, 1976 (in Russian).

3. Lidov M.L. (1961). Evolution of orbits of artificial satellites of planets due to gravitational perturbations of outer bodies. “Artificial Earth Satellite”, release 8 (in Russian).

4. Eliyasberg P. E. (1965). Introduction in the theory of flight of artificial satellites of the Earth, Nauka, Moscow, (in Russian)

5. Ivanov N.M., Kolyuka Yu.F., Afanasieva T.I., Gridchina T.A. (2007). Coronas-F orbit monitoring and re-entry prediction. In *Proc. 20-th International Symposium on Space Flight Dynamics*, Annapolis, USA.