SPATIAL DENSITY EVOLUTION OF SPACE DEBRIS ENVIRONMENT

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ABSTRACT

It is known that the space debris provide great risks to the safety of on-orbit space vehicles, which makes it necessary to establish a model to evaluate their impact risks. In order to provide the scenario of the future space debris environment for space vehicle designers, this paper addresses a simplified mathematical model for the future evolution of space debris environment. While the spatial density is of more concern to engineers, this paper is devoted to study the evolution of the distribution of spatial density of space debris. Given the initial spatial density, this paper develops the algorithm to obtained the spatial density of space debris at any moment of future, by considering the orbital propagation and collisions.

1. INTRODUCTION

It is known that the space debris provide great risks to the safety of on-orbit space vehicles, which makes it necessary to establish a model to evaluate their impact risks. In order to provide the scenario of the future space debris environment for space vehicle designers, this paper addresses a simplified mathematical model for the future evolution of space debris environment.

Currently there exist some models of the space debris environment[1–3], either engineering models or evolutionary models. However, most of these models do not provide the description of the evolution of the spatial density of space debris, i.e., the spatial density at any future moment. For engineering models, only the distribution of spatial density at current time is available. For evolutionary models, only the evolution of the total number of space debris is provided. While the spatial density is of more concern to engineers, this paper is devoted to study the evolution of the distribution of spatial density of space debris. Given the initial spatial density, this paper develops the algorithm to obtained the spatial density of space debris at any moment of future.

In order to carry out the evolution process and develop the mathematical expressions of the algorithm, the paper firstly assumes that the change of space debris environment only subjects to orbital propagation and collisions between space debris. New launches are currently not considered. For simplification, space debris under concerned are catalogued into a number of classes according to their size, since both orbital propagation and collisions are greatly dependent on the size of space debris. As a result, each class of space debris has its own spatial density distribution. The paper's main work is to obtain the evolution of these distributions, by solving which gives the evolution of spatial density of the whole space debris environment. For orbital propagation, a simplified model is developed, which only concerns the J2 term in the Earth gravitational potential, and provides fast calculation for large population. By applying kinetic theory of gases, one single collision between space debris is analyzed. Summing up all of the potential collisions, the change of space debris environment due to collision is obtained, which also gives the spatial density as a result. Combining the work above, the mathematical model for evolution of spatial density of each space debris class is obtained.

2. BASIC CONCEPT AND SCHEME

For simplification, space debris are assumed to have shape of sphere and has the same mass density with aluminium, which is $\rho_{al} = 2700\mathrm{kg/m}^3$. Then the mass of a space debris of size d can be given by

$$m = \frac{1}{6}\pi d^3 \rho_{al} \tag{1}$$

Also, to simplify the evolution procedure, space debris are classified into S categories, according to their average size, which is described by its efficient diameter d. All of the space debris in the same category are assumed to have the same efficient diameter d. Thus, there is a spatial density function $\rho_i(h,t)$ associated with each space debris category, where $i = 1, 2, \ldots, S$ is the category index. Denote $N_i(h_1, h_2, t)$ as the number of space debris which are in the ith category and falls into the altitude range of $[h_1, h_2]$ at time moment of t, $N_i(t)$ as the number of all the space debris in the ith category at time

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moment of t. Hence, the following can be achieve:

$$N_i(h_1, h_2, t) = \int_{h_1}^{h_2} \rho_i(x,t)\mathrm{d}x \quad (2)$$

$$N_i(t) = \int_H \rho_i(x,t)\mathrm{d}x \quad (3)$$

where $H = [h_{min}, h_{max}]$ is the altitude range being considered.

By fixing i, h and t, the spatial density of the ith category space debris at the altitutde h at time moment of $t+\Delta t$ can be given by:

$$\rho_i(h, t+\Delta t) = \lim_{\Delta h \to 0} \frac{N_i(h, h+\Delta h, t+\Delta t)}{\Delta h} \quad (4)$$

As a result, the derivative of spatial density with respect to t can be given by:

$$\frac{\partial \rho_i(h,t)}{\partial t} = \lim_{\substack{\Delta t \to 0 \\ \Delta h \to 0}} \frac{\Delta_t N_i(h, h+\Delta h, t+\Delta t)}{\Delta h \Delta t} \quad (5)$$

Considering the sources that of $\Delta_t N_i$, the following list maybe achieve:

1. routine launches, its contribution is denoted as $\Delta_t N_i^l = \Delta_t N_i^{l+} - \Delta_t N_i^{l-}$.
2. orbital propagation, its contribution is denoted as $\Delta_t N_i^p = \Delta_t N_i^{p+} - \Delta_t N_i^{p-}$.
3. collisions, its contribution is denoted as $\Delta_t N_i^c = \Delta_t N_i^{c+} - \Delta_t N_i^{c-}$.
4. ...

where the term with a $+$ sign is the increment while the one with a $-$ sign is the decrement. Then the following terms:

$$\frac{\partial \rho_i^l}{\partial t} = \lim_{\substack{\Delta t \to 0 \\ \Delta h \to 0}} \frac{\Delta_t N_i^l}{\Delta t \cdot \Delta h} \quad (6)$$

$$\frac{\partial \rho_i^p}{\partial t} = \lim_{\substack{\Delta t \to 0 \\ \Delta h \to 0}} \frac{\Delta_t N_i^p}{\Delta t \cdot \Delta h} \quad (7)$$

$$\frac{\partial \rho_i^c}{\partial t} = \lim_{\substack{\Delta t \to 0 \\ \Delta h \to 0}} \frac{\Delta_t N_i^c}{\Delta t \cdot \Delta h} \quad (8)$$

represents the change rate of spatial density of the ith category space debris caused by each source.

This paper assumes that the following equation holds:

$$\frac{\partial \rho_i(h,t)}{\partial t} = \frac{\partial \rho_i^l(h,t)}{\partial t} + \frac{\partial \rho_i^p(h,t)}{\partial t} + \frac{\partial \rho_i^c(h,t)}{\partial t} + \cdots \quad (9)$$

which is called as the principle of superposition.

3. CONTRIBUTION OF ORBITAL PROPAGATION

Only those objects in the ith category are responsible for $\partial \rho_i^p / \partial t$, since orbital propagation will not change the objects' size. Denote $\Delta a(x, d_i, \Delta t)$ as the decrement of the major axis when time Δt has passed. For simplification, only the lowest-order(J_2) secular perturbations, is considered. Then the following can be achieve

$$\Delta a = \frac{\rho_a(x) d C_D \sqrt{\mu x}}{m} \left[1 - \frac{\omega_e}{n}\cos i\right]^2 \Delta t \quad (10)$$

where A is the object's cross sectional, $C_D = 2.2$ is the Coefficient of Drag, $\mu = 398600.436\text{km}^3/\text{s}^2$ is the gravitational constant, m is the object's mass, $\omega_e = 2\pi/86164 = 7.290 \times 10^{-5}$rad/s is the Earth spin rate, $n = \sqrt{\mu/x^3}$ is the mean motion, $\rho_a(x)$ is the atmospheric density. Since for issues of space debris, the altitude is hardly below 25km, $\rho_a(x)$, given by aerodynamics, can be expressed as

$$\rho_a(x) = \frac{p(x)}{0.2869(T(x) + 237.1)} \quad (11)$$

where

$$T(x) = -131.21 + 0.00299x$$

$$p(x) = 2.488 \left(\frac{T(x) + 273.1}{216.6}\right)^{-11.388}$$

are the temperature and pressure of the atmosphere respectively.

If only the Lower Earth Orbit(LEO, with altitude ranges from 200km to 2000km) is considered, then the term $\omega_e \cos i/n$ can be ignored, since it's too small when comparing to 1. Replace A with the effective size d, then Δa can be simplified to

$$\Delta a(x, d, \Delta t) = \frac{\rho_a d C_D \sqrt{\mu x}}{m} \Delta t \quad (12)$$

$$\frac{\partial \Delta a(x, d, \Delta t)}{\partial x} = \frac{\mu \rho_a d C_d}{2m\sqrt{\mu x}} \Delta t \quad (13)$$

As Fig. 1 shows, when the time changes from t to $t+\Delta t$, there exist three cases for the ith category space debris:

1. In time t, space debris with altitude in range of $[h+\Delta h, \tilde{h}(h+\Delta h, d_i, \Delta t)]$ will fall into altitude range of $[h, h+\Delta h]$ in time $t+\Delta t$.
2. In time t, space debris with altitude in range of $[h, \tilde{h}(h, d_i, \Delta t)]$ will fall out of altitude range of $[h, h+\Delta h]$ in time $t+\Delta t$.
3. Otherwise, space debris will remain in the altitude range of $[h, h+\Delta h]$, or not.

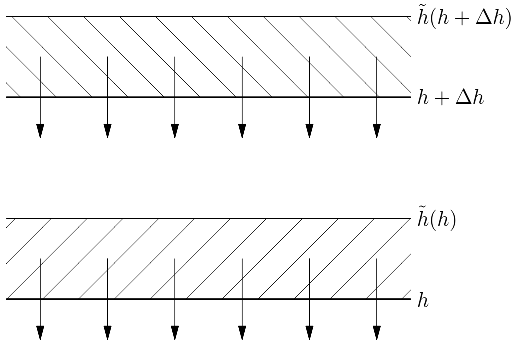

Figure 1. Propagation of altitude

where $\tilde{h}(x, d_i, \Delta t)$ satisfy

$$\tilde{h} - \Delta a(\tilde{h}, d_i, \Delta t) = x \tag{14}$$

Hence the increment of number of the i category in the altitude range of $[h, h + \Delta h]$ is given by

$$\begin{aligned}\Delta_t N_i^p &= \Delta_t N_i^{p+} - \Delta_t N_i^{p-} \\ &= \int_{h+\Delta h}^{\tilde{h}(h+\Delta h, d_i, \Delta t)} - \int_{h}^{\tilde{h}(h, d_i, \Delta t)} \rho_i(x,t)\mathrm{d}x \\ &\approx \frac{\mu \rho_a d C_d}{2m\sqrt{\mu x}} \Delta t \Delta h \rho_i(h,t)\end{aligned} \tag{15}$$

As a result, the contribution to the spatial density of the ith category by orbital propagation is given by

$$\frac{\partial \rho_i^p(h,t)}{\partial t} = \frac{\Delta_t N_i^p}{\Delta h \Delta t} = \frac{\mu \rho_a d C_d}{2m\sqrt{\mu h}} \rho_i(h,t) \tag{16}$$

4. CONTRIBUTION OF COLLISIONS

The factor of collisions considered includes the sizes of both objects and the altitude where the collision occurs. In order to obtain $\partial \rho_i^c / \partial t$, two functions are introduced, i.e., the size distribution density function of fragments $\psi(d, d_j, d_k)$ and the altitude distribution density function of fragments $\phi(x, u)$.

4.1. Distribution of fragment size $\psi(d, d_j, d_k)$

According to NASA's breakup model of EVOLVE[4], when catastrophic collision is assumed, the following distribution is given for the number of fragments of a given size x and larger when collisions occur

$$N_{j,k}(x) = 0.1(m_j + m_k)^{0.75} x^{-1.71} \tag{17}$$

where

$$m_j = \frac{1}{6}\pi d_j^3 \rho_{al}, \qquad m_k = \frac{1}{6}\pi d_k^3 \rho_{al} \tag{18}$$

are the masses of the colliding objects. Taking the distribution density function

$$\psi(x, d_j, d_k) = \frac{dN_{j,k}(x)}{\mathrm{d}x} \tag{19}$$

into account, the number of ith fragments produced when two objects with sizes of d_j and d_k occurs can be given by

$$\begin{aligned}N_i^c(d_j, d_k) = &\int_{D_i} \psi(x, d_j, d_k)\mathrm{d}x \\ =&N_{j,k}(d_i^{min}) - N_{j,k}(d_i^{max}) \\ =&23.06(d_j^3 + d_k^3)^{0.75} \\ &\times \left((d_i^{min})^{-1.71} - (d_i^{max})^{-1.71}\right)\end{aligned} \tag{20}$$

where $D_i \equiv [d_i^{min}, d_i^{max}]$ is the size range of the ith space debris category.

4.2. Distribution of fragment altitude $\phi(x, u)$

In this paper, the distribution of fragment altitude is assumed to be described by normal distribution, which is expressed as

$$\phi(x, u) = \frac{1}{\lambda\sqrt{2\pi}} e^{\frac{-(x-u)^2}{2\lambda^2}} \tag{21}$$

where the standard deviation.

4.3. Spatial density increment due to collision

With ψ and ϕ introduced, it's possible to establish the spatial density increment $\partial \rho_i^c / \partial t$ due to collision.

Consider the collisions occured in the altitude range of $[u, u + \Delta u]$ between the jth and the kth category objects. The number of objects within the altitude range $[u, u + \Delta u]$ in each category is given by

$$N(d_j, u, u + \Delta u) = \rho_j(u) \cdot \Delta u \tag{22}$$
$$N(d_k, u, u + \Delta u) = \rho_k(u) \cdot \Delta u \tag{23}$$

respectively. Similar to molecular dynamics, it's generally assumed that the number of collisions is proportional to the product of both objects, the time interval and is inverse proportional to the considered space volume. As a result, the number of collisions $c(d_j, d_k, u, u + \Delta u)$ in altitude range of $[u, u + \Delta u]$ between the jth and the kth category is given by

$$\begin{aligned}&c(d_j, d_k, u, u + \Delta u) \\ =&\gamma \times \frac{N(d_j, u, u + \Delta u) \cdot N(d_k, u, u + \Delta u) \cdot \Delta t}{\Delta u} \\ =&\gamma \rho_j(u) \rho_k(u) \Delta u \Delta t\end{aligned} \tag{24}$$

where the collision rate factor $\gamma = 3 \times 10^{-3}$ is obtained by a simple partical-in-box computation[5].

Among the generated fragments, the number of those falls into the ith category and with altitude in range of $[h, h + \Delta h]$ is given by

$$\int_H \gamma \rho_j(u)\rho_k(u) \times N_i^c(d_j, d_k) \times \phi(h,u)\Delta h \Delta t \mathrm{d}u$$

By dividing Δh and Δt, and summing up all of the collisions, $\partial \rho_i^c / \partial t$ is given by

$$\begin{aligned}\frac{\partial \rho_i^c(h,t)}{\partial t} =& \sum_{j \le k} N_i^c(d_j, d_k)\gamma \\ & \times \int_H \rho_j(u)\rho_k(u)\phi(h,u)\mathrm{d}u \end{aligned} \quad (25)$$

5. SUMMARY

By combining Eq. (16) and Eq. (25), the differential equation for the change rate of spatial density as

$$\begin{aligned}\frac{\partial \rho_i(h,t)}{\partial t} =& \frac{\partial \rho_i^p(h,t)}{\partial t} + \frac{\partial \rho_i^c(h,t)}{\partial t} \\ =& \frac{\mu \rho_a d C_d}{2m\sqrt{\mu h}} \rho_i(h,t) + \sum_{j \le k} N_i^c(d_j, d_k)\gamma \\ & \times \int_H \rho_j(u)\rho_k(u)\phi(h,u)\mathrm{d}u \end{aligned} \quad (26)$$

when only orbital propagation and on orbit collisions are considered. Hence, a new mathematical model to describe the future space debris environment is obtained. Given the initial scenario of current space debris environment, the spatial density of space debris at any future moment can be obtained by solving Eq. (26), which is suitable for fast calculation. As a result, the model is usefule for judging the trend of the future space debris environment. This work does not represent the final word on models based on ideas presented in the paper. But it's an important step towards understanding the mechanism of the evolution of the space debris environment.

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