ON BUNCHING OF ORBITAL DEBRIS MICROPARTICLES IN CIRCULAR AND ELLIPTIC ORBITS

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ABSTRACT

In the presented work we estimate the possible bunching of space debris microparticles due to the drag changes in near-Earth circular and elliptical orbits caused by quasi-periodic changes in atmospheric density. The study shows that collective effects in orbital evolution of microparticles are essentially displayed for high elliptical orbits where variations of the atmosphere density along the orbit and lifetime of the microparticles are relatively large. However there is no limiting distribution. The clusters are formed and destroyed several times during orbital lifetime of the microparticles.

1. INTRODUCTION

It is known that a spacecraft functioning in near-Earth space (NES) undergoes the influence of many space factors including impacts of microparticles with sizes ranging from submicrons to hundreds of microns from artificial and natural sources. The existing space debris models and calculations of spacecraft protection are based on Poisson distributions of microparticle impacts on spacecraft's outer surfaces. However, a lot of observations [1-3] show a non-uniform character of the space and temporary distributions of the space debris in NES. The real nature of such "clouds" is still unknown to present day. It can be due to mutual interactions of microparticles or due to some focusing factors of physical fields in NES, or the microparticles are recent degradation products of orbital object which have had no time for randomization over the space. It is known [4, 5] that the collective properties of particles in high density systems are related to inelastic collisions of particles, or to the Coulomb interactions of charged particles. But the density of particles in NES is very low, and therefore as a matter of fact the particles in NES move independently from each other.

Under the circumstances we need for every particle to consider gravitational force, electromagnetic forces, pressure of solar radiation, and drag force caused by residual atmosphere. If the particle size is greater than 0.1 μm the electromagnetic forces are negligible [6, 7]. Also it is known that in low Earth orbit the atmospheric drag prevails over solar radiation pressure and it is a main factor that limits the orbital lifetime of a particle [8, 9]. The atmospheric drag is proportional to the atmosphere density and directed oppositely to the particle velocity. It is known that the atmosphere density depends on the altitude, the illumination of particle orbit and the solar activity. Therefore the atmospheric drag can be described by a force field with the structure of characteristic size. As mentioned above we suppose that the particles move independently. But if the distance between particles lesser than the characteristic size of the force field acting on the particles we can expect occurrence of collective features in the dynamics of the ensemble of non-interacted particles. In the presented work we estimate the possible bunching (clustering) of space debris microparticles due to the drag changes in near-Earth circular and elliptical orbits caused by quasi-periodic changes in atmospheric density. The simple deterministic dependencies for atmospheric density variations along the particle orbit are used.

We consider a central gravitational force and an atmospheric drag in accordance with the Newton law. The equation of a particle motion in NES has the form

$$m\frac{d\mathbf{V}}{dt} = -\frac{\mu m}{r^3}\mathbf{r} - C_x S_M \frac{\rho_a V^2}{2}\cdot\frac{\mathbf{V}}{V}, \qquad (1)$$

where m is the particle mass, $\mathbf{V}$ is the velocity vector, $\mu = G\cdot M_E$ is the gravitational parameter of the Earth (G is the gravitational constant, M_E is the mass of the Earth), $\mathbf{r}$ is the radius-vector of the particle in a geocentric frame, ρ_a is the atmosphere density, C_x is the drag coefficient (it can be considered equal to 2), S_M is the particle cross-section area. The piece-wise exponential model of the atmosphere density is used:

$$\rho_a(h) = \rho_a(h_i)\exp\left(\frac{h-h_i}{H_i}\right), \qquad (2)$$

where $\rho_a(h_i)$ is the atmosphere density at the lower boundary of the i-th layer (with altitudes from h_i to h_{i+1}), and H_i is an appropriate altitude scale for the i-th layer. The atmospheric density in the joint points is given by the standard atmosphere model [10].

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2. MODEL PROBLEM: ANALYTICAL EXAMPLE OF THE BUNCHING OF PARTICLES

First we consider a one-dimensional model problem, which allows us to track the bunching of particles analytically. Eq. (1) is taken without the gravitational force and it is considered that the density is varied periodically $\rho_a = \rho_{a0}(1+\varepsilon\cos kx)$, where k=2π/λ, λ is a space period (a structural parameter), ρ_{a0} is constant and $0<\varepsilon<1$. In this statement Eq. (1) can be once integrated. The result has the form

$$\frac{dx}{dt} = V(x)\,,\ V = V_0\exp\{-\kappa[x - x_0 + \frac{\varepsilon}{k}(\sin kx - \sin kx_0)]\}$$
$$x(0) = x_0\,, \qquad (3)$$

where $\kappa = C_x S_M \rho_{a0}/(2m)$, x_0 and V_0 is the initial coordinate and initial velocity of a particle. The solution of Eq. (3) depends on the time t and the parameter x_0:

$$x = x(t, x_0)\,. \qquad (4)$$

Let assume that the damping over the period is small: $\kappa' = \kappa\lambda \ll 1$. This assumption allows us to evaluate the time of the *1*-st and *n*-th "cycle" of the particle. From (3) one gets

$$T_1 = \int_{x_0}^{x_0+\lambda}\frac{dx}{V(x)} = \frac{1}{V_0}e^{-\frac{\kappa\varepsilon}{k}\sin kx_0}\frac{1}{\kappa}(e^{\lambda\kappa}-1)(1+\mathrm{O}(\kappa'^2))\,,$$
$$T_n = e^{\kappa\lambda(n-1)}T_1 \qquad (5)$$

It is seen from (5) that the cycle time depends on the initial position of the particle x_0. The time of each subsequent cycle increases in $e^{\kappa\lambda}$ times. Thus, the particles initially uniformly distributed along the period, with the same initial velocity, in some time after the beginning of the motion (for example, during the cycle time of some "marked" particle) are drifting relatively each another.

Let $\rho(t,x)$ is the spatial density of particles and assume $\rho(0,x_0)=\rho_0(x_0)$ From the law of mass conservation for one-dimensional case one gets

$$\rho_0 dx_0 = \rho dx \ \text{ or } \ \rho(t) = \frac{\rho_0}{j(t)}\,, \qquad (6)$$

where $j = \dfrac{\partial x(t,x_0)}{\partial x_0}$ is the divergence. Eqs. (3) and (6) yield an approximate solution in the form

$$\rho = \rho_0\,\frac{e^{\frac{\kappa\varepsilon}{k}(\sin(kx)-\sin(kx_0))}(1+T'e^{\frac{\kappa\varepsilon}{k}\sin(kx_0)})}{1+(1+\varepsilon\cos(kx_0))T'}\,, \qquad (7)$$

where $T' = \kappa' t V_0/\lambda$ is the non-dimensional "slow time". In limit case $T' \gg 1$ we obtain omitting terms of order $\mathrm{O}(\kappa')$

$$\rho = \rho_0\,\frac{1}{1+\varepsilon\cos(kx_0)} \qquad (8)$$

It is evident that the particles do not tend to each other arbitrarily near, and there is some limit distribution of the particles density. Initially uniform distributed particles (ρ_0=const) form non-uniform structure with maximum density value of $\rho_{\max} = \rho_0/(1-\varepsilon)$ in the clustering phase and minimum value of $\rho_{\min} = \rho_0/(1+\varepsilon)$ in the rarefied phase. The distribution (8) depends on the initial position of the particles and does not depend on their area-to-mass ratio. Similarly the relative displacement of two arbitrary particles $\Delta = x_2 - x_1$ can be evaluated. As the calculations show, if $T' \gg 1$ then

$$\Delta_\infty = \Delta_0 + \frac{\varepsilon}{k}\left[\sin(kx_{2,0}) - \sin(kx_{1,0})\right], \qquad (9)$$

where $\Delta_0 = x_{2,0} - x_{1,0}$. I.e. there is a limit distance for moving away (or converging) particles. In such a way it is shown that structured atmospheric drag can lead to the bunching effect of the particles in stream. This result allows us to presume that the similar effect is for all debris particles in the near-Earth orbits.

3. EVALUATING THE BUNCHING OF MICRO-PARTICLES IN QUASI-CIRCULAR ORBIT

Eq. (1) for the particle motion is considered in osculating elements [11]. The particles move in constant orbital plane. Varying orbital parameters are: focal parameter *p*, eccentricity $e \ll 1$ and argument of perigee ω. A parameter describing the damping during a period is $\kappa' = \kappa\cdot p_0 \ll 1$, where $\kappa = C_x S_M \rho_{a0}/(2m)$ and $p_0 = p(t=0)$. We also assume that $\kappa' \ll e \ll 1$ and $\omega(t=0)=0$. Suppose that atmospheric density along circular orbit is subjected to the following law

$$\rho_a = \rho_{a0}(1+\varepsilon\cos u)\,, \qquad (10)$$

where u is the latitude argument and ρ_{a0} is constant. The dependence (10) can qualitatively simulate the density changes due to solar illumination variations along circular orbit. At altitudes 400-600 km the relative change of atmosphere density is ≤ 50% [12] due to this factor. Therefore parameter ε in (10) is not small, but ε < 1. In this statement the solution for a particle can be easily found as a power series of small parameters κ', and e. As in the previous section the cycle time of the particle depends on its initial position u_0 :

$$T_n = \frac{1}{\mu^{1/2}} \int_{u_0}^{u_0+2\pi n} \frac{p^{3/2} du}{(1+e\cos\vartheta)^2} = \frac{p_0^{3/2}}{\mu^{1/2}} 2\pi n \left(1 - \frac{3}{2}\kappa'(\pi n - \varepsilon \sin u_0)\right), \quad (11)$$

where $\vartheta = u - \omega$. Hence the particles having initial uniform angular distribution will migrate relative to each other after $n > 1$ cycles of some "marked" particle (e.g., a particle with initial value $u_0 = 0$). Besides, this displacement will be essentially non-uniform with respect to u.

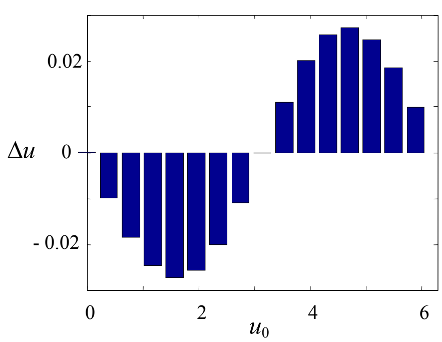

Figure1. The relative drifts of 16 particles during their orbital lifetime. Initially the particles were uniformly distributed over the quasi- circular orbit at altitude of 600 km.

Let's evaluate the relative position of the particles in the orbit depending on their initial position and number of cycles. The comparison is done at the instants according to N cycles of the particle «1», $u_{1,N} = u_{1,0} + 2\pi N$, $u_{1,0}$ is an initial value of latitude argument. During this time an arbitrary particle which we denote as «2» will have $u_{2,N} = u_{2,0} + 2\pi N + \delta_N$, where $u_{2,0}$ is an initial value, and δ_N is a small addition. After calculations in the main order we obtain

$$\Delta_N = \Delta_0 - 6\pi N \kappa' \varepsilon (\sin u_{2,0} - \sin u_{1,0}), \quad (12)$$

where $\Delta_N = u_{2,N} - u_{1,N}$. Below we consider that $u_{1,0} = 0$. Obviously the period of the mapping (12) contains two stationary points, $u_2 = 0$ and π, where $\Delta_N = \Delta_0$, $N > 0$. These points correspond to the perigee and apogee of the orbit, and the elementary analysis show that they are points of attraction and repulsion. It is also seen that the particles initially symmetrical about axis of apses will remain symmetrical about this axis for all next $N > 0$. This analysis is valid for not very large N: in (12) the growth of N is restricted by assumption of smallness of $\delta_N = \Delta_N - \Delta_0$. The essential difference of evaluation (12) from the similar evaluation (9) is an absence of the limit distance of particles' divergence (convergence). As follows from Eq. (12) at $u_{1,0} = 0$, the increment of particle's latitude argument depending on its initial orbital position may be evaluated according to the formula

$$\Delta u_N = u_{2,N} - u_{2,0} - 2\pi N = -6\pi N \kappa' \varepsilon \sin u_{2,0} \quad (13)$$

Obviously the maximum displacement takes place for the particles with $u_{2,0} = \pi/2$ and $3\pi/2$. The number of cycles N is essentially limited by lifetime of particles, which is rather small for circular orbits for particles with sizes ranging from micron to millimeter [9, 13]. As the estimations show the product $N_{\max}\kappa'$ remains small. Therefore Eqs. (12) and (13) may be applied for evaluating displacement of the particles in circular orbit during their lifetime. For an aluminium particle with size of 100 μm at altitude of 600 km $N_{\max} \approx 300$, $\kappa' \approx 5.1 \cdot 10^{-6}$ (the density of atmosphere was taken at the altitude of 600 km). As a result, from (13) for ε = 0.5 we obtain $\left|\Delta u_{N_{\max}}\right|$ = 0.015 This evaluation is an underestimation because the particle's orbit altitude will decrease during deceleration while the atmospheric density and the drag coefficient κ' will grow.

The results of qualitative analysis are verified by numerical modeling. Non-uniform variations of the atmosphere density caused by non-uniform solar illumination along circular orbit is simulated by (10), where the dependence of ρ_{a0} on the altitude is taken into account by means of (2) and ε is taken equal to 0.5. The initial conditions correspond to a circular orbit at altitude of 600 km. The evolution of 16 aluminium particles with size of 100 microns is studied. Initially the particles are uniformly distributed along the orbit. The Cauchy problem for Eq. (1) is calculated using the Runge-Kutta method with automatic step selection. A relative position of particle in the orbit is examined at the instants when the particle with initial coordinate u_0 = 0 completes N cycles. The calculations show that the particles tend to form cluster but have no time to generate a noticeable non-uniformity in their distribution during the orbital lifetime. It is seen in Fig. 1 where the results of calculation show how the angle coordinate variations at the moment close to the orbit lifetime depend on initial angle coordinate of the particles (the angles are measured in radians). Two stationary points $u_0 = 0$ and π are seen in this figure. One of them is a point of attraction and the other is a point of repulsion. The maximum displacement is observed for the particles initially based in the points $u_0 = \pi/2$ and $3\pi/2$. Evidently the results of calculation completely confirm the above qualitative analysis.

4. EVALUATING THE BUNCHING OF MICRO-PARTICLES IN ELLIPTICAL ORBIT

The atmosphere density varies quasi-periodically along elliptical orbit due to altitude dependence (2). In this case the bunching of microparticles may be expected in analogy with that considered in the previous section. It is also important that the effect of atmospheric drag is a factor restricting the particle's orbit lifetime, and we must consider evolution of the particles only for the time not longer than their lifetime. Relative variations of the atmosphere density are greater if particles are in high-elliptical orbit, and we can expect that the bunching effect is greater in this case. The Cauchy problem is studied numerically for Eq. (1), where the atmosphere density is determined by Eq. (2). In the initial moment all the particles are supposed to be uniformly (for the angle coordinate) distributed along a Keplerian elliptical orbit. Thereby the initial velocity of each particle is equal to Keplerian orbital velocity in the point of initial localization of the particle in this orbit. Evidently in the absence of atmospheric drag the particles will return into their initial position in the time multiple to the orbital period. The situation changes if there is the atmospheric drag. To display influence of the drag on relative displacement of particles we compare the positions of the particles at the instants corresponding to the time when the particle initially located in the perigee completes N cycles. The angular coordinates of the perigee $u_0 = 0$ and the apogee $u_0 = \pi$ are stationary points. In Fig. 2 and Fig. 3 the results of calculations for 100 μm aluminium particles injected in an elliptical orbit with perigee altitude of 350 km and apogee altitude of 29700 km are depicted.

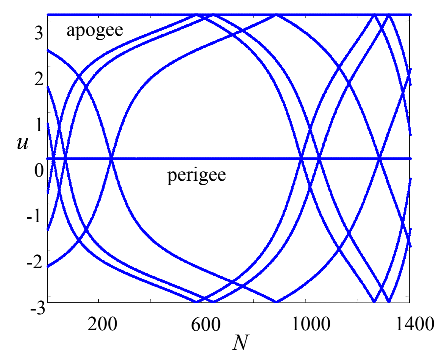

Figure 2. The positions of 8 aluminium particles in a plane of elliptical orbit as dependence of angular coordinates on the number of complete cycles N. The particles have size 100 μm.

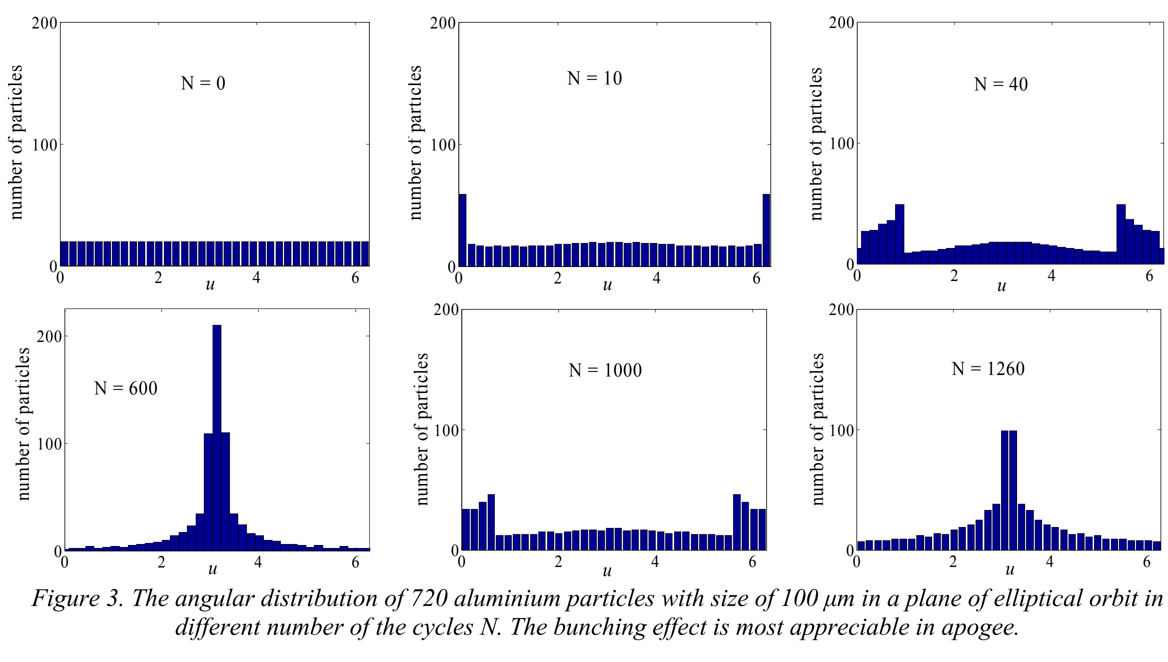

Figure 3. The angular distribution of 720 aluminium particles with size of 100 μm in a plane of elliptical orbit in different number of the cycles N. The bunching effect is most appreciable in apogee.

During their lifetime the particles complete N_{max} ~ 1433 cycles that correspond to ~ 245 days. Fig. 2 shows the positions of 8 particles in a plane of elliptical orbit during their lifetime as dependence of their angular coordinates on the number of the cycles *N*. It is visible that relative displacement of the particles is great. Intersections of trajectories in Fig. 2 correspond to bunching of the particles. Fig. 3 shows the angular distributions of 720 particles in different number of the cycles *N* (*N* = 0 corresponds to initial distribution). One can see that the clusters are formed and destroyed several times during lifetime of the particles. The bunching effect is most appreciable in the apogee of elliptical orbit in *N* ~ 600 and 1260 cycles.

5. CONCLUSION

In the presented work we estimate the possible bunching (clustering) of space debris microparticles due to the drag changes in near-Earth circular and elliptical orbits caused by quasi-periodic changes in atmospheric density. The simple deterministic dependencies for atmospheric density variations along the particle orbit are used. It is shown that the particles in quasi-circular orbit tend to formation of a cluster but they have no time to form essential non-uniformity during their orbital lifetime. For this case we have obtained both analytical and numerical estimations. The calculations show that the collective behavior of the microparticles is able to become apparent in high elliptic orbit where the relative change of atmospheric density along the orbit and the characteristic lifetime of the microparticles are greater. However there is no limiting distribution of the particles. The clusters are formed and destroyed several times during orbital lifetime of the particles. The largest cluster is formed in the apogee of the elliptical orbit.

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