STATISTICAL ANALYSIS OF MICROMETEOROID AND SPACE DEBRIS IMPACTS ON THE SPACE STATION "SALYUT-4"

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ABSTRACT

It is common to consider the flow of micrometeoroids and space debris (MM/SD) as the Poisson one when calculating risks for spaceship [1]. However even the first studies of the near Earth space with the aid of spaceships where sensors of particle registrations have been set allow us to suppose that registered streams have very complex features [2-3].
The objective of this work is the statistical analysis of MM/SD registration data onboard station "Salut-4" and specification of their distribution models in the near Earth. As initial data we used results of studies performed on stations "Salyut-4" and obtained with the help of the control system. This system was used for detecting of the MM/SD impacts and consisted of condenser sensors (CS) and an electronic device.

1 THE FEATURES OF EXPERIMENTS MAKING UP

Features of making of experiments and the measurement results analysis consisted in the following:
1. Monitoring system (MS) for micrometeoroids impacts consisted of the electronic block (EB) and condenser sensors (CS). CS were made from two metal coverings isolated from each other by the lavsan film. The facial covering is the aluminum foil covered by the layer of varnish.
2. The method of measurement with the help of CS does not allow one to distinguish particles of a natural and artificial origin. Therefore we performed the statistical analysis using the assumption, that events at issue (registrations of particles) are events of the same type.
3. MS registered the facts and the moments of particles hit into sensors only.
4. "The dead time" is the time of restoration of CS sensor was $\Delta t_м \leq 0.001$ sec.
5. Micrometeoroids and space debris (MM/SD) impacts within the interval $\Delta t \leq 1$ were remembered as registered during one moment of time t_r, $r = 1, 2, \dots$.
6. The part of the registered particles appears by groups with "close" registration times.
7. Now it is practically impossible to define precisely the sensors orientation in space.

We'll make the statistical analysis of data by means of the methods stated in [4]. We'll check the correspondence of the MM/SD registration number to the Poisson process. It is known that for the Poisson process the distribution of intervals among events (MM/SD registrations is a model one. If the zero hypotheses on the model distribution of intervals is rejected, then the hypothesis on the Poisson distribution of the number of registrations is rejected, too. For investigating this issue we'll use characteristics of the second order of intervals.

Parameters of the station "Salyut-4" a motion path were: the height in apogee is 270 km, the height in perigee is 219 km, the orbit inclination is 51.6 °, the revolution period is 89.1 min. In flight the station was not stabilized. "Salyut-4" was in operation from 26.12.1974 to 08.02.1975. Parameters S, δ - area and thickness of a facing covering of sensors respectively are presented in the Tab. 1.

Table 1

CS	*CS -1*	*CS-2*	*CS-4*
δ, microns	20	10	10
S, м^2	2.4	0.6	0.6

Sensitivity of the CS has been chosen in order to ensure the registration of penetrating flows MM/SD. The MM/SD masses were in the range 10^{-8} - 10^{-10} g.
The analysis is made for the data obtained from the sensors CS-1, CS-2 and CS-4. The sensor CS-3 during measurements carrying out worked unstably.

2 STATISTICAL ANALYSIS

We check the correspondence between the time distribution of impacts and the Poisson statistics. It is known that the Poisson process corresponds to exponential distribution. If the zero hypothesis on the exponential distribution of intervals is rejected, then the hypothesis of the Poisson distribution is rejected, too [4].
The analysis of data of registration MM/SD we will begin with the graphic methods allowing qualitatively to estimate the basic property of data and degree of its conformity to a Poisson's stream. There are estimations for constant of variation $C(\xi)$ of the quantity ξ for all period of measurements in the Tab. 2.

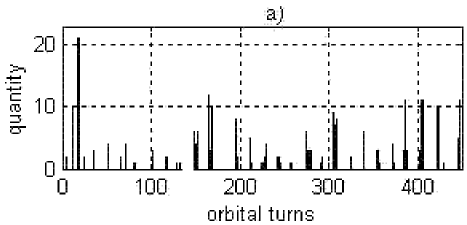
Table 2

CS	CS-1		CS -2		CS -4	
Unit of measure	sec	Orbital turns	sec	Orbital turns	sec	Orbital turns
$C(\xi)$	4.0	6.5	2.4	2.9	4.1	5.0

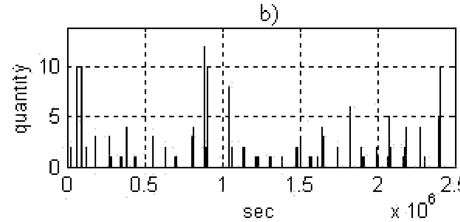
From the Tab. 2 it is seen that $C(\xi) > 1$, that mismatches Poisson's law, for which $C(\xi)=1$.

Non-uniformity of registration of microparticles is proved also by curves of individual number of registrations with respect to turns, presented in Fig. 1-3. The graphs in these figures with the indexes "a" and "b" are observed during the period t_0, the graph with index «c» is recorded during the first day, the figures with the index «d» are recorded during the first orbit. Here N_Σ are numbers of the registered particles, N_{avr} are average intensities of impacts.

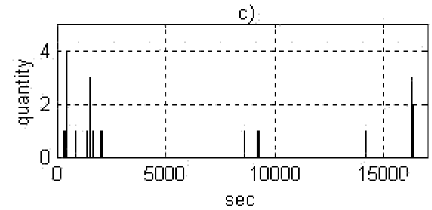
Proc. '5th European Conference on Space Debris', Darmstadt, Germany
30 March – 2 April 2009, (ESA SP-672, July 2009)



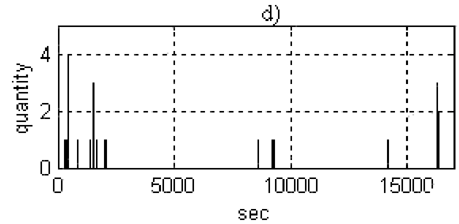
T_0=420 trn, N_{Σ}=256, N_{avr} =4.6×10^{-5} pcs /m^2×s



t_0=2.2*10^6, N_{Σ}=256, N_{avr}=4.6×10^{-5} pcs /m^2×s

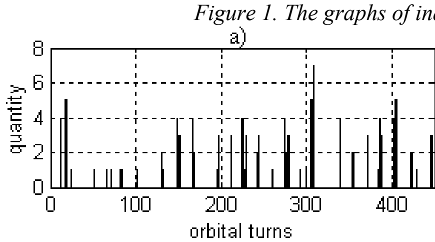


28.01.75, N_{Σ}=26, N_{avr}=6.6312×10^{-4} pcs /m^2×s

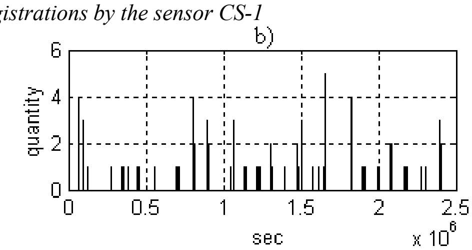


28.01.75, N_{Σ}=15, N_{avr}=3.2×10^{-3} pcs /m^2×s

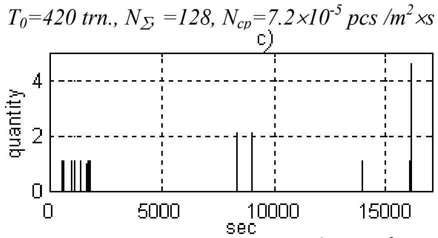
Figure 1. The graphs of individual registrations by the sensor CS-1



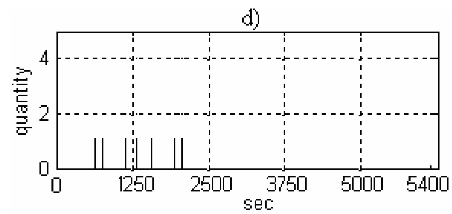
T_0=420 trn., N_{Σ} =128, N_{cp}=7.2×10^{-5} pcs /m^2×s



t_0=2.2*10^6, N_{Σ} =128, N_{cp}=7.2×10^{-5} pcs /m^2×s

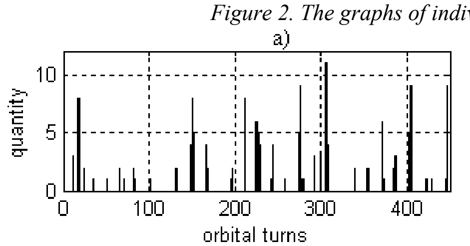


28.01.75, N_{Σ}=13, N_{avr}=2.7×10^{-4} pcs /m^2×s

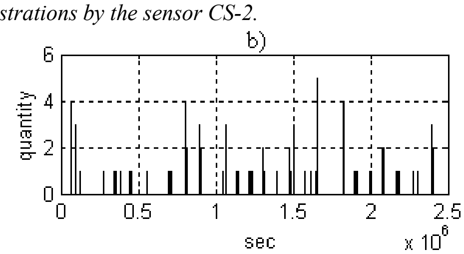


28.01.75, N_{Σ}=7, N_{avr}=2.2×10^{-3} pcs /m^2×s

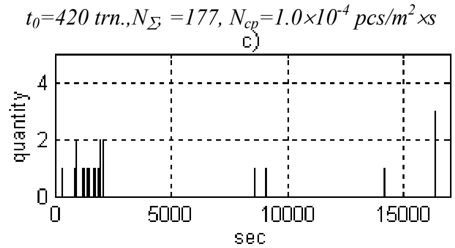
Figure 2. The graphs of individual registrations by the sensor CS-2.



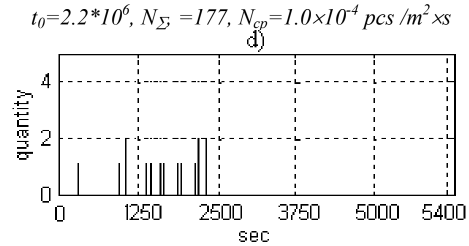
t_0=420 trn.,N_{Σ} =177, N_{cp}=1.0×10^{-4} pcs/m^2×s



t_0=2.2*10^6, N_{Σ} =177, N_{cp}=1.0×10^{-4} pcs /m^2×s



28.01.75, N_{Σ}=21; N_{avr}=2.5×10^{-3} pcs/m^2×s



28.01.75, N_{Σ}=15, N_{avr}=1.3×10^{-2} pcs /m^2×s

Figure 3. The graphs of individual registrations by the sensor CS-4.

On the graphs there are local fluctuations of numbers of registrations during various intervals for all sensors. The obtained results are in qualitative agreement with the results obtained in experiment LDEF [5].

Change of the average intensity of registrations of microparticles in course of time is illustrated by the plots of collected number of events N_t occurred till the instant *t* inclusive as a function of t (Fig. 4-6). Time *t* is measured in orbital turns (Fig. 4a-6a) and seconds (Fig. 4b-6b). In Fig. 4-6 the curves denoted by the numeral 1 correspond to the sequence including all registered micro particles (type 1), by the numeral 2 correspond to sequence of individual particles and centers of groups of particles with “close” registration times (type 2).

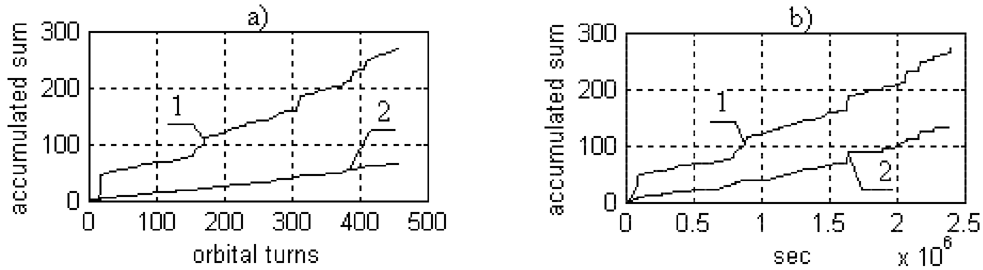


Figure 4. The graph of cumulative number of registrations by the sensor CS-1.

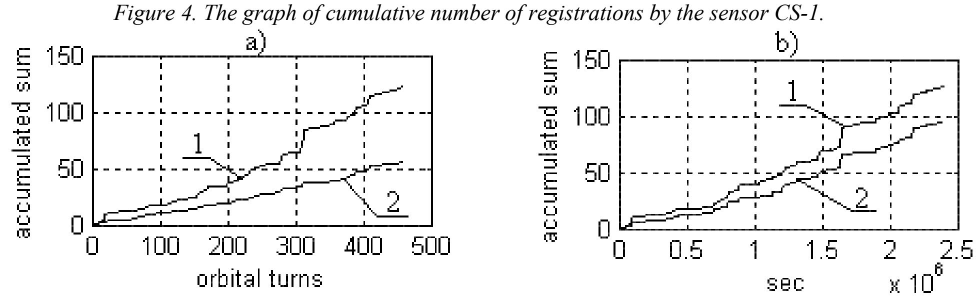


Figure 5. The graph of cumulative number of registrations by the sensor CS-2.

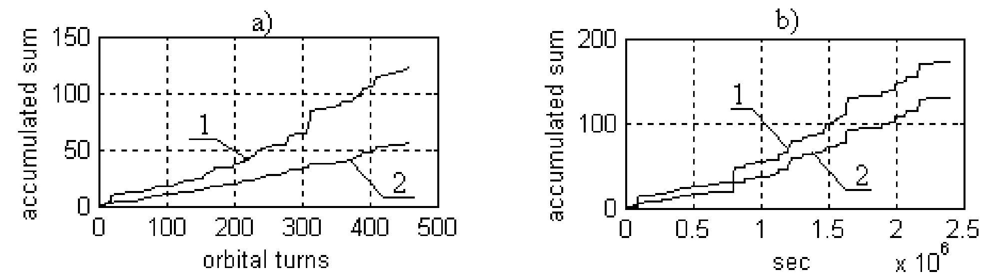


Figure 6. The graph of cumulative number of registrations by the sensor CS-4.

The property of these graphs is following: an inclination of the straight line joined any two points of the graph defines intensity of MM/SD impacts in the time unit. In theses figures there are fluctuations of impact intensities; it is available for both groups and separate impacts.

Fig. 7-9 represents the logarithmic graph of empirical function of reliability *logRn(x)* for sequences of intervals x of the 1st and 2nd types. As a whole the graphs differ from straight lines. Especially it is appreciable for the sequences considering all registered micrometeoroid. It is evident because there are a lot of short and long intervals. Nonlinear character of graphs of reliability empirical function for sensors testifies to difference of intervals distribution between groups of registered micrometeoroids from exponential distribution that gives the basis to reject a hypothesis about Poisson's character of separate and group impacts.

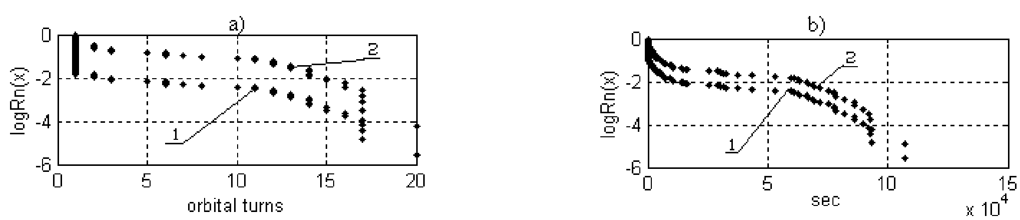


Figure 7. The logarithmic graphs of empirical function of reliability according to the sensor CS-1

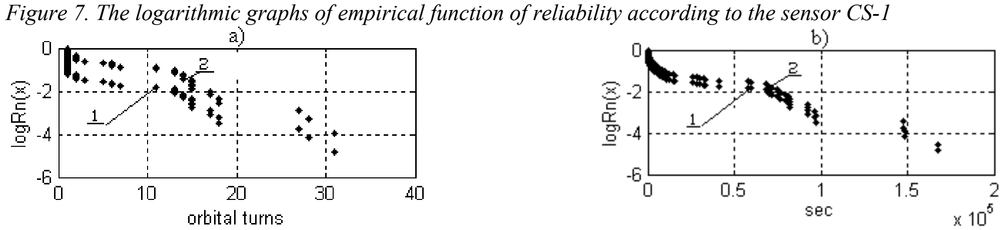


Figure 8. The logarithmic graphs of empirical function of reliability according to the sensor CS-2

Figure 9. The logarithmic graphs of empirical function of reliability according to the sensor CS-4

Let's pass to research of properties of the registration stream by formal methods. Because of limitation of experimental data the statistical analysis we will fulfill on the basis of characteristics of the first and second order of interval lengths between micrometeoroids registration.

Correlation properties of the sequence of intervals between registrations $\{X_i\}$ are set by the sequence of the correlation coefficients ρ_j. The theoretical value of the coefficients is $\rho_j = 0$. If estimates $\tilde{\rho}_j$ are in agreement with theoretical values ρ_j then the original sequence is stationary and independent.

Estimates of the sequential coefficient of correlation $\tilde{\rho}_j$ for the sequence of intervals between registrations of microparticles are presented in Fig. 10-12. Horizontal dashed lines are 5% confidence bounds of the significance for estimates.

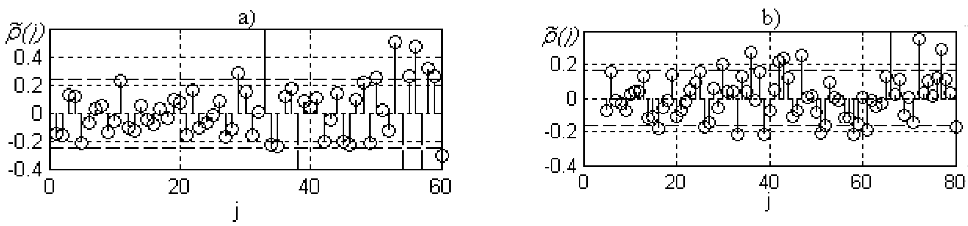


Figure 10. Estimation of correlation coefficient according to the sensor CS-1

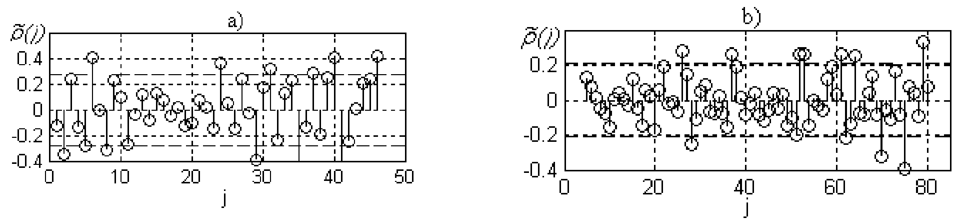


Figure 11. Estimation of correlation coefficient according to the sensor CS-2

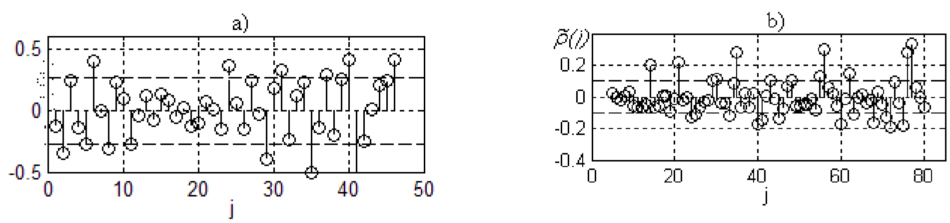


Figure 12. Estimation of correlation coefficient according to the sensor CS-4.

One can see from Fig. 10-12 that estimates $\tilde{\rho}_j$ exceed these levels, and so the hypothesis concerning the independence of intervals X_i should be rejected. The flow of registrations of microparticles is not stationary.

The constancy of spectral density of intervals $f_1(\omega) = 1/\pi$ may be served as the criterion for the verification of the hypothesis that the sequence of intervals between events is a stationary independent process. So one should investigate sampling properties of Estimations $\tilde{f}_1(\omega)$ are presented in Fig. 13-15 for sequence of intervals in orbits (the figures with the index "a") and in seconds (the figures with the index «b»). The horizontal line represents theoretical spectral density of Poisson's stream that equal to $1/\pi$.

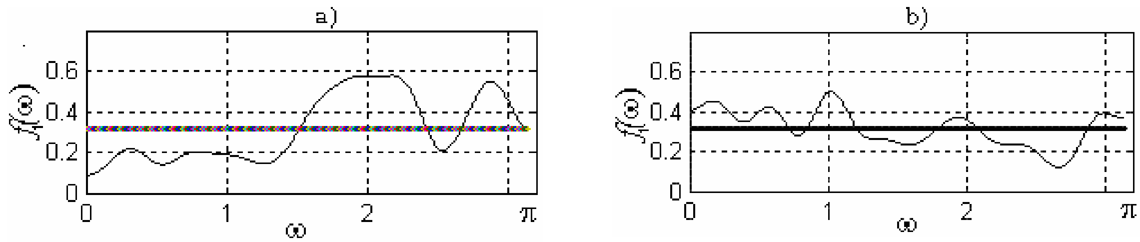


Figure 13. Estimation of spectral function according to the sensor CS-1.

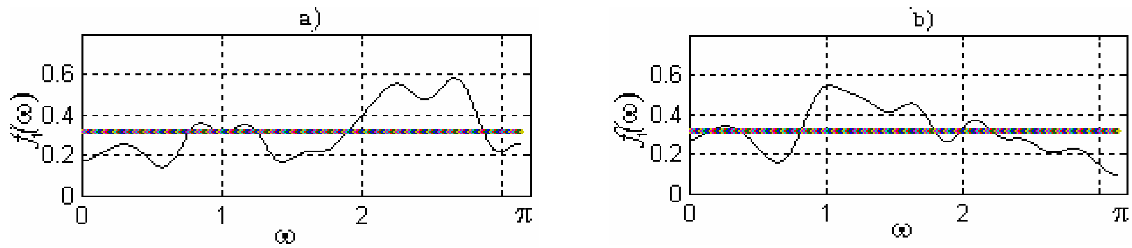


Figure 14. Estimation of spectral function according to the sensor CS-2.

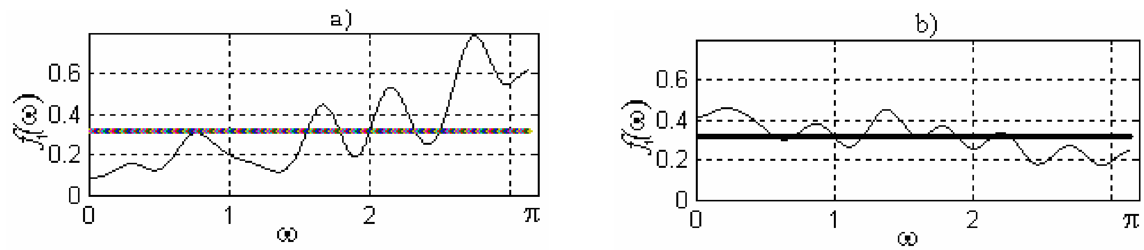


Figure 15. *Estimation of spectral function according to the sensor CS-4.*

From graphs it is possible to draw the most general conclusions only. Fluctuations of the estimation $\tilde{f}_1(\omega)$ are seen near the *1/π* value for all three sensors. The qualitative analysis of curves shows that estimations $\tilde{f}_1(\omega)$ differ from the *1/π* value. This difference is appreciable especially in Fig. 14a-14b.

The characteristic of number of events (registration of MTT), occurring on some interval τ, is the intensity function $m_f(t) = m_f(l\tau + 0.5\tau)$. Its estimations are close in value for Poisson process to average intensity of occurrences for all period of supervision.

Let's estimate the function of intensity $m_f(t)$ of approximation of micrometeoroid impacts for the sensors CS-1, CS-2, and CS-4. The result is given in Fig. 16-18. In these figures the broken lines represent the graph of the estimation $\tilde{m}_f(t)$, the horizontal dash-dotted lines are the estimation of the approximation $\overline{m}$, and curve solid lines are 5%-confidence bounds. Here $\overline{m} = N/t_0$, and N is a number of registered particles during the considered period of time t_0 in orbits (the figures with the index "a") and in seconds (the figures with the index «b»), the curves are 5 %-confidence bounds for individual values of the estimation. The intervals $\tau = 21$ (Fig. 16a-18a) and $\tau = 100000$ (Fig. 16b-18b) were used at performance of calculations, l =0, 1,2,…,n, $n = t_0/\tau$.

In the all figures it can be seen that individual estimations of function $\tilde{m}_f(t)$ are beyond the 5%-confidence bounds. Quantity of estimations $\tilde{m}_f(t)$, transcended the 5% bound, varies from a small number for the sensor CS-2 up to significant for the sensor CS-4.

From Fig. 16a-18a it is seen that number of individual values of estimations $\tilde{m}_f(t)$, transcended the 5% bound, it is insignificant, deviations δ of estimations $\tilde{m}_f(t)$ from the estimation of average intensity $\overline{m}$ are insignificant also and it is about 3-5 % of the value $\overline{m}$ (except two points). The opposite result is observed In Fig. 16b-18b. Deviations δ are commensurable with the value $\overline{m}$, the greater deviations amount considerably. It become clear from following: in the intervals measured in orbits the number of registered particles is increased and casual fluctuations $\tilde{m}_f(t)$ are smoothed.

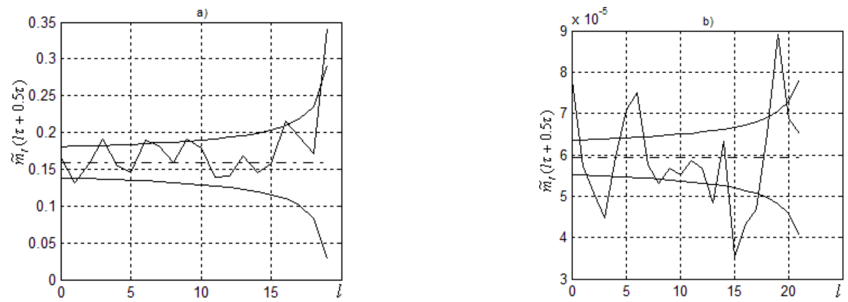


Figure 16. Estimation of function of intensity $m_f(t) = m_f(l\tau + 0.5\tau)$ for the sensor CS-1 data

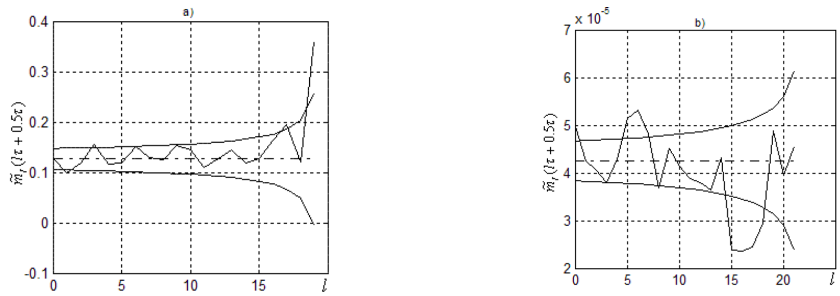


Figure 17. Estimation of function of intensity $m_f(t) = m_f(l\tau + 0.5\tau)$ for the sensor CS-2 data

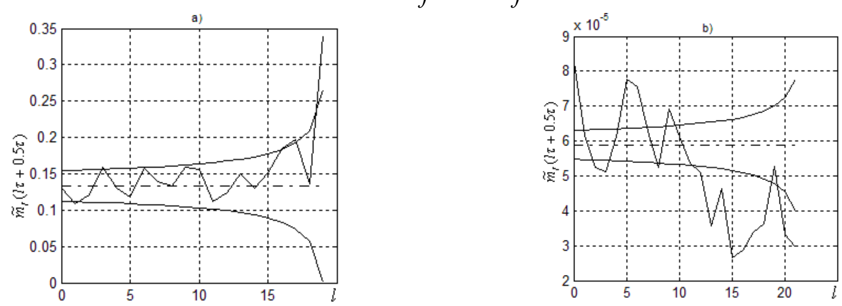


Figure 18. Estimation of function of intensity $m_f(t) = m_f(l\tau + 0.5\tau)$ for the sensor CS-4 data

Excess of the estimated values of intensity function $\tilde{m}_f(t)$ of 5%-confidence bounds gives a rough hint on difference of the registered streams from the Poisson's stream.

We use nonparametric criterion for check of zero hypothesis H_0 about exponential distributions of the Smirnov's $D_n\sqrt{n}$ and Cramer-Mises $n\omega^2$ statistics.

Calculations we fulfill for two cases:

1) one orbit is accepted as the unit of intervals duration,

2) one second is accepted as the unit of intervals duration. The result is given in the Tab. 3. In this table the value n_0 is a number of intervals, C_α are asymptotic bounds of the statistic significance, α is the significance value of corresponding statistics.

Table 3

CS	n_0	$D_n\sqrt{n}$	C_α α=5%	C_α α=1%	$n\omega^2$	C_α α=5%	C_α α=1%
Intervals in orbital turns							
CS-1	66	1.88			0.79		
CS-2	52	1.58	1.36	1.63	0.48	0.46	0.74
CS-4	55	1.64			0.51		
Intervals in seconds							
CS-1	133	4.60			8.27		
CS-2	95	3.61	1.36	1.63	5.07	0.46	0.74
CS-4	132	5.25			10.65		

From the Tab. 3 it is seen that the hypothesis about Poisson's character of MM/SD registration process for the sensors CS-1 and CS-4 is rejected with the significance level 0.01 and for the sensor CS-2 with the significance level between 0.01 and 0.05.

Thus, from the analysis of MM/SD registration results made at station "Salyut-4" it is drawn the conclusion that the particles registration stream is not typical for Poisson's process.

3 MORE ACCURATE SPECIFICATION OF MODEL DISTRIBUTIONS PARTICLES

We can consider two following schemes of data representation.

1. We consider separate congestions of micro-meteoroids as the groups, the stream of as the aggregate of groups. As the general group parameters of the stream we can consider the coordinates of group centers and average Λ of point group centers at the measurement period $(0,t_0)$, as the intra group parameters we can consider the average of points λ in a group with extension Δ_0. By the terms of experiment the stream points are indiscernible.
2. We neglect the time intervals between micrometeoroids registrations in the group and consider their registrations simultaneous. That streams are named the streams of multiple point, the coordinates of multiple points are called as the centre of multiplication factor.

In accordance with these schemes we can use two models for description the streams of micrometeoroids impacts based on the method of course-of-value function [6].

1. The model of indistinguishable groups Poisson stream, when general group and intragroup streams are Poisson with parameters Λ over a period of measurements $(0,t_0)$ and λ in group with extension Δ_0.

Distribution of the number of points follows from

$$P_0(t)=\exp[\Lambda t\,(\exp(-\lambda\Delta_0)-1)], \qquad (1)$$

$$P_k(t)=\Lambda t\lambda\Delta_0/k\exp(-\lambda\Delta_0)\sum_{i=0}^{k-1} 1/(i!(k-i-1)!)P_{k-i-1}(t)(\lambda\,\Delta_0)^i,, \qquad (1)$$

where $k \geq 1$.

The average ($\tilde{n}$) and dispersion (σ^2_n) of the number of points of the groups stream

$$\tilde{n} =\Lambda t\lambda\Delta_0,\ \ \sigma^2_n=\Lambda t\lambda\Delta_0(1+\lambda\Delta_0), \qquad (2)$$

It is followed from (2) that grouping of micrometeoroids impacts brings to increase of dispersion. Impact number dispersion increase the greater, the greater the average of impacts in each group.

2. The model of multiple points Poisson stream, which $\lambda(\tau) = \lambda$, $\nu_m(\tau) = \nu_m$ and multiplication factor is subordinated to Poisson's law with probability ν with probability $\nu = 1/m!\ q^m\ (\tau)[\exp(q(\tau))-1]^{-1}$. The value q in the formula is evaluated by fitting of hypothetical distribution to empirical one.

Distribution of the number of points follows from

$$P_0(t)=\exp[\Lambda\times t\times(\exp(-\lambda\times\Delta_0)-1)], \qquad (3)$$

$$P_k(t) = \lambda t/(\exp(q)-1)\times\sum_{i=0}^{k} i!/k!\ A_{i+1,k}q^{k-i}\times P_i(t), \quad (3)$$

where $k \geq 1$,

$A_{i,k}$ is defined as follows:

$A_{i,k} = 0$ при $i=0$ и $i>k$,

$A_{i,k} = 1$ при $i=1$ и $i=k$,

$A_{i,k} = A_{i,k-1} + A_{i-1,k-1}$, при $1<i<k$.

In spite of the fact that the hypothesis about Poisson's type of the micrometeoroids registration stream was not confirmed, we'll try to adjust parameters of these models to experimental data.

For an estimation of the hypothetical models to the data of registration of MM/SD it is used the following test for concordance – the Pirson's criterion χ^2 and Kolmogorov-Smirnov's criterion $D_n\sqrt{n}$.

At using the criterion χ^2 it should be sufficiently great not only general number n of the registered micrometeoroids (hundreds), but also quantity m_i of intervals with identical number of the registered micrometeoroids (not less than 10). Here $i=1, 2, \dots$ is the number of registered MM/SD at intervals with duration τ. Kolmogorov-Smirnov's $D_n\sqrt{n}$ criterion is applied, when are known both hypothetical distribution $F(x)$ and its parameters.

On received values χ^2 and $D_n\sqrt{n}$ by means of Tables [7] we'll define values of probabilities P for each of criteria. Values χ^2 and $D_n\sqrt{n}$ have been obtained for intervals τ of averaging of data with duration of 5 turns. Results of calculation of criteria χ^2 and $D_n\sqrt{n}$ and corresponding values of probability P for various intervals of averaging according to micrometeoroids registration data are resulted in the Tab. 4.

Table 4

Sensor	Model of stream	Criterion of the consent		Criterion of the consent	
		χ^2	P	$D_n\sqrt{n}$	P
CS-1	1	6.34	0.92	0.80	0.54
	2	18.54	0.15	1.20	0.11
CS-2	1	2.82	0.98	0.57	0.89
	2	12.42	0.26	1.00	0.27
CS-4	1	10.48	0.63	0.92	0.37
	2	42.34	<0.001	1.01	0.17

On the Fig. 19-21 they are presented the histogram of registration data and curves of the indistinguishable groups Poisson stream, when general group and intragroup streams are Poisson, and the multiple points Poisson stream, which multiplication factor is subordinated to Poisson's law, with averaging interval corresponding to duration of 5 orbital turns.

From the table 1 and the Fig. 19-21 it is followed that the model of multiple points Poisson stream, which multiplication factor is subordinated to Poisson's law, is closest approach to micrometeoroids registration data obtain by Pirson's and Kolmogorov-Smirnov's criteria χ^2.

The value of probability P, obtained by criterion χ^2, is overestimated, since there is a sample limited per volume units of micrometeoroids) and the number of intervals m_i, in which it is registered $i \geq 4$, is less than 10. The value of probability P obtained by Kolmogorov-Smirnov's criterion is overestimated too, since parameters $F(x)$ are got out per the statistical data.

The stated remarks do not allow us to accept unequivocally this model of registrations stream.

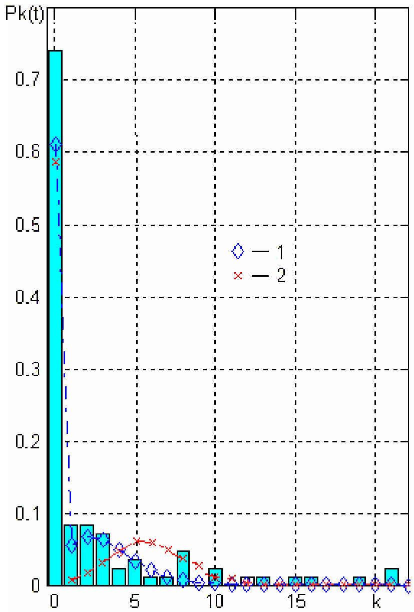


Figure 19

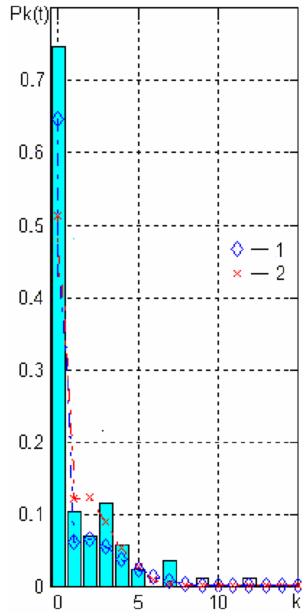


Figure 20

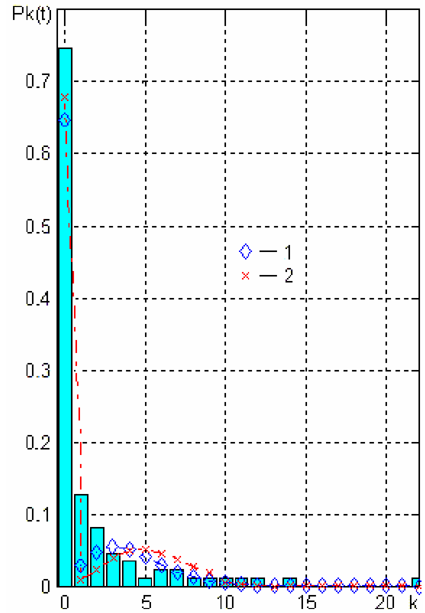


Figure 21

Histograms of registration data and curves of the indistinguishable groups Poisson stream, when general group and intragroup streams are Poisson – ***(2)****, and the multiple points Poisson stream, which multiplication factor is subordinated to Poisson's law–* ***(1)****, with averaging interval of 5 orbital turns. Figures 19-21 correspond to CS-1, CS-2 and CS-4 respectively.*

CONCLUSIONS.

Using onboard measurements data obtained at station "Salyut-4" we performed the statistical analysis of the time sequence of impacts of micrometeorites and space debris (MM/SD). There were mounted condenser sensors which allowed to register impacts of particles of 10^{-10} – 10^{-8} g.

This graphic analysis of the results of detection of meteoroids and man-caused particles allows us to come to conclusion that the flow of detecting particles is a non-Poisson process. Note that these results are similar to results of the experiments performed in the mission LDEF.

Taking into account the results of the statistical analysis we conclude that the hypothesis about Poisson behavior of the stream of registrations of particles does not true. We used for this conclusion the criteria which are based on the estimates of spectral density, correlation function, function of intensity, and nonparametric criterion and statistics by Kolmogorov – Smirnov and Cramer – Mises.

In accordance to results of the statistical analysis it is followed that the stream of microparticles registration at station "Salut-4" is close to a Poisson stream of multiple points at which the multiplicity also obeys the Poisson law.

However it is necessary to concern to this model, as well as any other models chosen on basis of low volume of experimental data, with caution because of low volume of experimental data. It is necessary for the choice between various models relatively long sequence of supervision.

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