# ON GENERALIZATION OF THE PRINCIPLE OF EQUIVALENCE OF THE SEARCH PLAN ELEMENTS FOR DIFFERENT TIMES


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## ABSTRACT

The main notion and the stem of the theory of optimum planning the search for a space object (SO) by the incomplete orbital information on it is the principle of equivalence of the search plan elements for different times. So far the theory as well as the equivalence principle were developed constructively (up to the real search methods and working programs) for the important case of taking into account the initial state vector error only along the track. For this case practical application of the equivalence principle appeared to be very convenient which makes the search planning procedure practically simple enough. But except this case there exist many search situations where one cannot neglect the state vector errors in different directions. For such situations the equivalence principle as the main tool of optimum planning the search for a SO should be generalized to the majorizing equivalence principle. After a short reminder of the former formulation of the principle its new generalization is introduced. The theoretical account is accompanied by some elucidating examples. The generalized equivalence principle can be used for practical constructing optimum plans of search for SOs by rough a priori orbital information in the most common case of the state vector errors.


## 1. INTRODUCTION

The main tool of the theory of optimum planning the search for a space object (SO) by its incomplete a priori orbital information is the principle of equivalence of the search plan elements for different times. This theory was developed some years ago for the important case of having respect to the state vector error only along the track [1]. The principle is strictly defined in [1] and [2]. Let us shortly remind it.

The rough (incomplete) a priori orbital information on every SO is available if in the 6-dimentional phase space $X_6$ there is given a domain $D_6(t_0)$ of possible values of the sought for SO motion parameters vector (the state vector) on the time $t_0$, id est the initial state vector (its mathematical expectation)

$$R_6(t_0) = \left\| x, y, z, v_x, v_y, v_z \right\|^{\mathrm{T}} \in D_6(t_0) \subset X_6 \quad (1)$$

and the related covariance matrix

$$K_{R6}(t_0) = \left\| \begin{matrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} & \sigma_{XVx} & \sigma_{XVy} & \sigma_{XVz} \\ 0 & \sigma_Y^2 & \sigma_{YZ} & \sigma_{YVx} & \sigma_{YVy} & \sigma_{YVz} \\ 0 & 0 & \sigma_Z^2 & \sigma_{ZVx} & \sigma_{ZVy} & \sigma_{ZVz} \\ 0 & 0 & 0 & \sigma_{Vx}^2 & \sigma_{VxVy} & \sigma_{VxVz} \\ 0 & 0 & 0 & 0 & \sigma_{Vy}^2 & \sigma_{VyVz} \\ 0 & 0 & 0 & 0 & 0 & \sigma_{Vz}^2 \end{matrix} \right\|, \quad (2)$$

that is the related probability distribution density function $f_{t0}(R_6)$ defined on the domain $D_6(t_0)$.

The celestial mechanics laws define on the domain $D_6(t_0)$ a homeomorphic mapping $F$ which transfers each point $R_6(t_0)$ of $D_6$ at the time $t_0$ to another point $R_6(t_1)$ of $X_6$ at the time $t_1$ :

$$R_6(t_1) = F\,[t_0, R_6(t_0), t_1]. \quad (3)$$

The property of homeomorphism of $F$ means that the domain $D_6(t_0)$ is one-to-one and to-and-fro continuously transferred by $F$ into the domain $D_6(t_1)$ :

$$D_6(t_1) = \bigcup_{R_6 \in D_6(t_0)} F[t_0, R_6(t_0), t_1] = F\,[t_0, D_6(t_0), t_1]. \quad (4)$$

## 2. THE EQUIVALENCE PRINCIPLE FORMULATION

Checking the point $R_6(t_0)$ of $D_6(t_0)$ at the time $t_0$ (to learn if the sought for SO is present or absent at the point) is equivalent to checking the point $R_6(t_1) = F\,[t_0, R_6(t_0), t_1]$ of $D_6(t_1) = F\,[t_0, D_6(t_0), t_1]$ at the time $t_1$ in the sense that it is not necessary to accomplish both acts of checking – it is sufficient to check only one of the two equivalent points.

Similarly, checking the subdomain $d_6(t_0) \subset D_6(t_0)$ at the time $t_0$ is equivalent to checking the subdomain $d_6(t_1) \subset D_6(t_1)$ at the time $t_1$ in the same sense.

The search plan $M_6$ is a set of pairs

$$M_6 = \{(R_6, t)\} \; R_6 \in D_6 \quad (5)$$


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(taking into account the equivalence principle).

Each pair means checking the point $R_6$ at the time $t$ .

## 3. THE OPTIMUM CONDITION FOR THE SEARCH PLAN

The search plan $M_6$ is referred to as optimum if it is complete and non-redundant. $M_6$ is complete if realization of all its pairs $\{(R_6\ ,\ t)\}$ guarantees a coverage of the sought for SO in the phase space, id est $M_6 = \{(R_6\ ,\ t)\} \forall R_6 \in D_6$ (taking into account the equivalence principle). $M_6$ is non-redundant if among all its pairs $\{(R_6\ , t)\}$ there are no equivalent ones.

For a 6-dimentional uncertainty domain $D_6(t)$ where every its point is the full state vector $R_6(t)$, the $F$'s property of homeomorphism provides the transfer of a point $R_6(t_1)$ to some point $R_6(t_2)$ and vice versa. This fact greatly simplifies the analysis of temporal structural transformation of the SO position uncertainty domain and application of the equivalence principle for planning the search (if it were in 6-dimentional space!).
But a real sensor sounds not the 6-dimentional phase space but its 2-dimentional (for an optical sensor) or 3-dimentional (for a radar sensor) projection. Simultaneously, the mapping $F$ is projected into 2-dimentional ($F_2$) or 3-dimentional space ($F_3$), respectively. The projectional equations are given in [1]. According to it $F_k$-image ($k$ =2 or 3) of any point $R_k(t)$ is already not a point but a set of points (a domain). So the mapping $F_k$ is not a homeomorphism, id est not one-valued in both sides, not to speak about direct and reverse continuity. That means that the real optimum planning process of the search becomes very complicated.
As shown in [1, 3, 4] this process can be essentially simplified (just $F_k$ can be returned to a homeomorphism) for the case of assumption of predominant error propagation only along the track. Under this assumption one can reduce the process of constructing optimum search plans to simple operations in such a space as $ut$-plane where $u$ is argument of latitude and $t$ is time. And so, application of the equivalence principle becomes constructive and very simple [3, 4].
This case is very important and actual for situations of search for a SO in highly eccentric orbits with the help of narrow-angle optical and electro-optical sensors.
But there exist many search situations where one cannot neglect the state vector errors in different directions. For these situations the equivalence principle as the main tool of optimum planning the search should be generalized to the majorizing equivalence principle.
Here, the main distinction from the above simple case (the one of predominant state vector propagation only along the track) consists in the necessity of having respect to a very complicated influence of the different directions errors to the character of structure dynamics of the sought for SO current position uncertainty domain [5]. For certainty, let us confine ourselves by 2-dimentional searching space $D_2(t)$ in the picture plane (PP) which is natural for optical sensors.

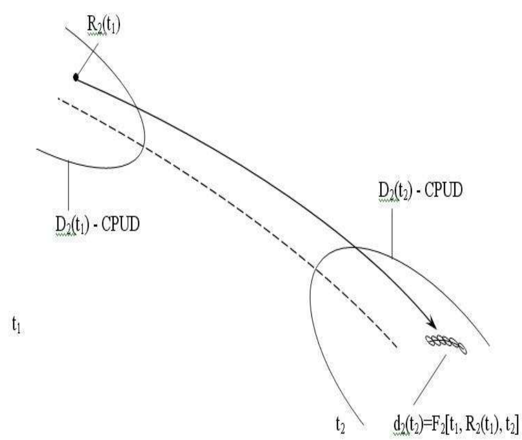


*Figure 1. Temporal transformation of a point of $D_2(t_1)$*

Unlike in the previous case, now any point $R_2(t_1) \in D_2(t_1)$ is transferred by the mapping $F_2$ not into a point $R_2(t_2)$ but into some domain $d_2(t_2) \subset D_2(t_2)$ (Fig.1). Then ***the generalized equivalence principle runs as follows:*** Checking the point $R_2(t_1) \in D_2(t_1)$ at the time $t_1$ is equivalent to checking the domain $d_2(t_2) = F[t_1, R_6(t_1), t_2] \subset D_2(t_2)$ but not vice versa, because some other points from $D_2(t_1)$ were also transferred into $d_2(t_2)$. In [5] there was shown how to calculate the domain $d_2(t_2)$.
This generalized principle together with the related calculating algorithm helps constructing (by points) $F_2$-image $d_2(t_2)$ of an optical sensor field of view $d_2(t_1)$ in PP (Fig.2). For this purpose a special mathematical model was developed and realized. One can see that the boundary of $d_2(t_2)$ became notably smeared due to the above effect of structural transformation of the domain $D_2(t_1)$ .

In terms of the generalized equivalence principle the initial field of view $d_2(t_1)$ (fixed at the time $t_1$) is equivalent to, or to be precise, majorizes the internal "volume" of $d_2(t_2)$ – excluding its boundary. Denote this subdomain of $d_2(t_2)$ as $d_2^-(t_2)$ . As follows from the updated equivalence principle, if you checked $d_2(t_1)$ at the time $t_1$ there is no need to check $d_2^-(t_2)$ at the time $t_2$ .

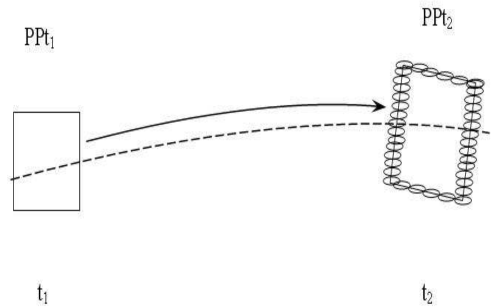


*Figure 2. Temporal transformation of a field of view*

Just in this form it will be applied to constructing optimum search plans [6], and for development of the related search methods it is fundamental.

**REFERENCES**

1. Вениаминов, С.С.( 1984 ). Оптимизация поиска объекта, движущегося циклически по замкнутой траектории. *Известия АН СССР, Техн. кибернет*., «Наука», №1, Москва, 188 – 198.

2. Veniaminov, S.S. (1993). The Methods and Experience of Detecting Small and Weakly-Contrasting Space Objects. In *Proc. 1st European Conference on Space Debris*, Darmstadt, Germany.

3. Вениаминов, С.С. (1987). Оптимизация поиска ИСЗ узкопольным телескопом по критерию времени обнаружения, *Кинематика и физика небесных тел*, «Наукова думка», №1, Киев, 14 _ 20.

4. Veniaminov, S.,S.,. Lapukhin, V.V,&. Pyrin, V.V. (2005). Some Results of Testing the New Program for Searching Space Objects in Deep Space. In *Proc. 4th European Conference on Space Debris*, ESA SP-587, ESA Publication Division, Noordwijk, The Netherlands, 107 - 110.

5. Veniaminov, S.S. & Lapukhin, V.V.( 2007). On some difficulties of planning the search for a space object by narrow-angled sensors. In *Proc 7th US/Russian Workshop on Space Surveillance*, Charlottesville Press, Virginia, USA, 63 – 67.

6. Aksenov, O., Veniaminov, S., Lapukhin, V.,& Rykin,A.(2009). Optimum planning of the search for a space object taking into account the temporal structural transformation of the space object current position uncertainty domain", In *Proc. 5th European Conference on Space Debris*, Darmstadt, Germany,.