SPACE DEBRIS SIZE ESTIMATION BASED ON NEW STATISTICAL CHARACTERISTICS OF RADAR CROSS SECTION

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ABSTRACT

Radar Cross Section (RCS) of space debris is generally related to space debris geometrical structure. The estimation of space debris size from RCS is investigated in this paper. We first establish an ellipsoid model. Then, by researching the RCS series of ellipsoid, it is denoted that the ratio of the number of the bigger than the mean value of RCS series to the number of the smaller is equal to the ellipsoid's curvature. And this statistical characteristics is demonstrated in the paper. Based on it, we estimate the long and short axial size of space debris. A number of practical data is processed using this method, and the estimated values are compared with the result of MIT Lincoln Laboratory's method. The result suggests that the method has a high precision, and the estimated size can provide much information to space debris surveillance.

1. INTRODUCTION

A mass of space debris was produced by the growing increased space activities, which made the space environment deteriorative gradually, it is the time to surveil space debris environment systematically [1]. Large air surveillance radar is one of the important instruments used for space target detection and recognition. As important signature information, Radar Cross Section (RCS) can be available from any radar, and RCS depicts the incident electromagnetic scattering capability of target [2]. Some early research has shown RCS's potential capability in charactering the structure of space debris, such as size, shape and so on. Therefore, RCS can be applied in target discrimination and attributes analysis, and how to extract characteristics from RCS effectively, which could be used for target recognition, has been attracting the wide attention from domestic and oversea.

RCS of space debris is relevant to many factors, such as the shape, volume of target, the roughness of surface and so on; however, it is very difficult to ascertain the target's structure and size information direct from RCS of target. Reference [3] works out the equivalent metallic sphere's diameter by using RCS series; estimates orbit debris's size using the equivalent diameter's statistical mean. It originates a new technical approach to estimate the target size using RCS series. Reference [4] proposes a method which can be used to divide target RCS data into long axis portion and short axis portion, after that, estimates the real long and short axis size of target. Firstly, this paper makes the target be equivalent to an ellipsoid model based on L-band radar, then deduce an important property that the mean value of RCS series splits the whole data into two parts: data in one part are all bigger than the mean value while data in the other are smaller than it, and the ratio of the number of the bigger to the number of the smaller is equal to the ellipsoid's curvature. Subsequently, according this property, we estimate the space debris long and short axis size based on the minimum variation rule. Finally, a number of practical data is processing using this method, and the result suggests that this paper's algorithm is efficient in estimating target size. The size of space debris estimated by this means can afford important information to space debris surveillance.

2. SIZE ESTIMATION OF SPACE DEBRIS

2.1. Space Debris Simplified Model

When estimate the size of space debris size based on RCS data, usual model first made the target be equivalent to a metallic sphere, and then established a function which mapped RCS into metallic sphere's diameter. At last, the equivalent sphere's diameter was regarded as the target size. This method was first applied in estimating the shape and quality of orbit debris. However, the method completely ignored the effect caused by the ratio of space debris long axis size to the short. The ellipsoid model is shown in Fig. 1. For two ellipsoids, their short axis length are the same (a=1 m), but curvature of each ellipsoid are different (e=1/2, 1/3), RCS value is not equal to invariable optical projection area. RCS is plotted in Fig. 2. When the ellipsoids are irradiated by the radar beam from 0 degree, RCS of two ellipsoids are not the same value πa^2 . And the fact is that the RCS is cut down with the decrease of ellipsoid's curvature. Therefore, when RCS is mapped into the diameter of equivalent sphere, the estimation of target's size must be bias. This problem is well solved when the ellipsoid model which considered the curvature's effect is applied. In addition, the rule of space debris movement in atmosphere was researched later indicated that the effects of atmospheric

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drag on the space debris were absolutely different with metallic sphere. According to this point, the space debris can not be simplified into an equivalent sphere. But, when the space debris is equivalent into an ellipsoid, these differences among different space debris can be depicted more precisely [3]. Also, the ellipsoid model gains the two dimension information of target, expressed the space debris structure characteristics more intuitively, contained more abundant information, and no double contributed to space debris surveillance.

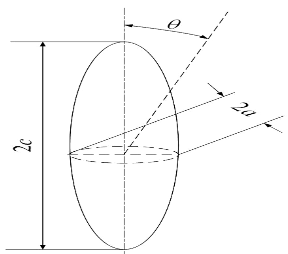

Figure 1. Ellipsoid model

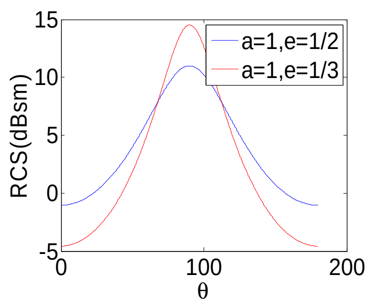

Figure 2. RCS curve of different curvature

2.2. Characteristics of Radar Cross Section Based on Ellipsoid Model

The model established for size estimation of space debris is shown in Fig. 1, and model's RCS characteristics is [5]:

$$\sigma = \frac{\pi a^4 c^2}{\left(a^2 \sin^2\theta + c^2\cos^2\theta\right)^2} = \frac{\pi a^2 e^2}{\left(e^2 \sin^2\theta + \cos^2\theta\right)^2} \quad (1)$$

Where, $e = a/c$, θ is the angle between radar incidence beam direction and ellipsoid long axial direction.

Fig. 3 gives a plot of model's RCS data whose angle range from 0 degree to 180 degree. The RCS is integrated in Eq. 2:

$$\begin{aligned} \int_0^{\pi/2} \sigma\, d\theta &= \int_0^{\pi/2} \frac{\pi a^2 e^2}{\left(e^2 \sin^2\theta + \cos^2\theta\right)^2}\, d\theta \\ &= \pi a^2 e^2 \left[\left(1+\frac{1}{e^2}\right)\frac{1}{2e}\times\frac{\pi}{2}\right] \end{aligned} \quad (2)$$

Then, the mean value can be obtained in Eq. 3:

$$\bar{\sigma} = \int_0^{\pi/2} \sigma\, d\theta \Big/ \frac{\pi}{2} = \pi a^2 e^2 \left[\left(1+\frac{1}{e^2}\right)\frac{1}{2e}\right] \quad (3)$$

The incidence azimuth angle $\bar{\theta}$ is mapped into the mean value of ellipsoid's RCS:

$$\begin{aligned} &\bar{\sigma} = \frac{\pi a^2 e^2}{\left(e^2 \sin^2\bar{\theta} + \cos^2\bar{\theta}\right)^2} \\ &\Rightarrow \bar{\theta} = \arcsin\left(\frac{1-((1+1/e^2)/2e)^{-1/2}}{1-e^2}\right) \end{aligned} \quad (4)$$

Therefore, when $0 \le \theta \le \pi/2$, it is obtained that the angle ranges from 0 to $\bar{\theta}$ where the value of RCS is below its mean value; the angle ranges from $\bar{\theta}$ to $\pi/2$ where the value of RCS is above its mean value. For practical RCS data, the angle range is in proportion to the number of RCS data, so that the radio of the number l of bigger than the mean value of RCS data to the number s of the smaller is equal to the angle range where the value of RCS is bigger than its mean value to the angle range where the RCS value is smaller. It is shown in Eq. 5, and Fig.3 gives a plot of these parameters:

$$\frac{l}{s} = \frac{\pi/2 - \bar{\theta}}{\bar{\theta}} = \frac{\pi/2 - \arcsin\left(\frac{1-((1+1/e^2)/2e)^{-1/2}}{1-e^2}\right)}{\arcsin\left(\frac{1-((1+1/e^2)/2e)^{-1/2}}{1-e^2}\right)} \quad (5)$$

We define that the estimated curvature in Eq. 6:

$$\hat{e} = \frac{\pi/2 - \arcsin\left(\frac{1-((1+1/e^2)/2e)^{-1/2}}{1-e^2}\right)}{\arcsin\left(\frac{1-((1+1/e^2)/2e)^{-1/2}}{1-e^2}\right)} \quad (6)$$

Then MATLAB is applied in simulating the relation between e and $\hat{e}$. Because that the value of curvature e ranges from 0 to 1, 90 samplings are equally obtained from the range of e. The result of simulation is shown in Fig. 4, and the conclusion $\hat{e} \approx e$ can be easily obtained from it. The precision fills the requirement of engineering computation. Based on it, in this paper, we take the ratio of the number l of the bigger than RCS mean value to the number s of the smaller as the estimated value of ellipsoid curvature e.

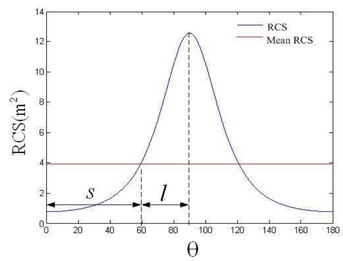

Figure 3. RCS data of ellipsoid

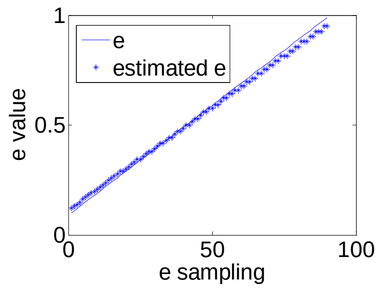

Figure 4. Estimated curvature in theory and real curvature

2.3. Size Estimation of Space Debris Long and Short Axis

According to the statistical characteristics of ellipsoid RCS, the ellipsoid curvature $\hat{e}$ is estimated first, and then the RCS curve of ellipsoid in theory is used to approach the real RCS data. At last, the space debris long and short axis size is estimated based on the minimum variance rule. The algorithm is detailed description as follow:

1. Calculate the mean value of RCS series, l is the number of RCS data bigger than its mean, s is the number of RCS data smaller than its mean. Then, the estimated ellipsoid curvature $\hat{e} = l/s$ is obtained.
2. Only the statistical characteristics of target RCS is needed for estimating the target size, so the performance of size estimation will not be affected by permutating the RCS series. The practical RCS data is sorted ascending, so that it can accord with the data distribution characteristics of ellipsoid RCS in theory. The permutated RCS series takes the first data as the RCS at the incident angle of 0 degree, the last data as the RCS at the incident angle of 90 degree. The RCS series permutated is represented as $X = \{x_1, x_2, \cdots x_N\}$, where N is the number of the data, angle variation is $\theta_i = (\pi/2N-1)\times i \;\; i = 0,1,\cdots(N-1)$. Fig. 5 gives a plot of permutated RCS series.
3. After the curvature e was estimated, for an ellipsoid model, only the parameter a is needed for calculating the RCS of ellipsoid in theory. The RCS in theory should approach the permutated practical RCS by changing the short axis length based on the minimum variance rule. The first approach to practical RCS should change the value of short axis length in steps of 1 m. The values of short axis length are A={$a_1, a_2,$ ... a_M}={1, 2, ... *mean (X)*}, estimated variance is $$\delta_m = \sum_{i=0}^{N-1} [\pi a_m{}^2 \hat{e}^2 \Big/ \left(\hat{e}^2 \sin^2\theta_i + \cos^2\theta_i\right)^2 - x_{i+1}]^2 \;,$$ where $m = 1,2,\cdots M$. Variance series is Δ = {δ_1, δ_2, ... δ_M}. Based on the minimum variance rule, we take A to obtain the minimum Δ as the optimal estimation of short axis length.
4. Decrease the value and range interval of A, i.e. $A = a_{m-1} : 0.1 : a_{m+1}$, repeat step 2 to 4. According to this approach, the more precise value of estimated short axis length $\hat{a}$ and long axis length $\hat{c} = \hat{a}/\hat{e}$ can be obtained.

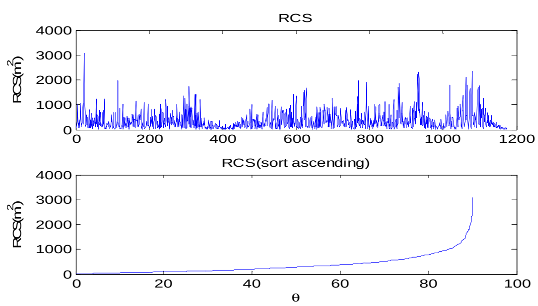

Figure 5. Practical RCS and permutated RCS

3. EXPERIMENTAL ANALYSIS

In this section, the estimated precision of this algorithm is analyzed. First, one RCS series of space debris is simulated in L-band, and then the size of space debris is estimated by the algorithm of this paper. After that, RCS series of two space targets whose size is known are analyzed, which can also testify the efficiency of this method. The estimation values are compared with the result of MIT Lincoln Laboratory's method.

The size of simulated space debris (Fig. 6) is estimated using the method in this paper. The real size of space debris is 37.777 *43.68501 cm. The RCS series is shown in Fig. 7.

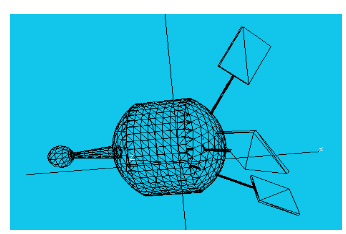

Figure 6. Space debris model

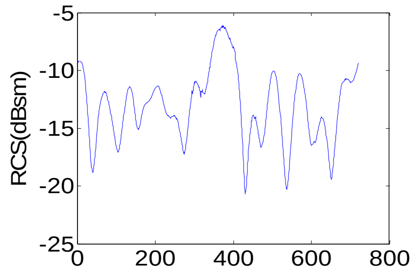

Figure 7. RCS of space debris

The result of size estimation is shown in Tab. 1.

Table 1. Size estimation of space debris

	High axis length(cm)	Short axis length(cm)
Real size	43.68	37.77
Size estimation of [3]	30.114	25.170
Size estimation of this paper	44.100	28.000

Analyze the RCS of International Space Station. At present, the three dimension size of ISS is: the length of trussed frame is 108.4 m, the cabin is 73 m broad, 44.5 m long and 27.5 m high. The solar panel of ISS has a strong wave reflection at some angle, and also has a weak wave reflection at other angle. So the size of solar panel can not be estimated using RCS data, and what is estimated is the size of cabin. In practical RCS data obtained from various angle in many circle was used for size estimation. According to this, the estimated size will be more close to the real size. The RCS series which was measured in 2008 is plotted in Fig. 8.

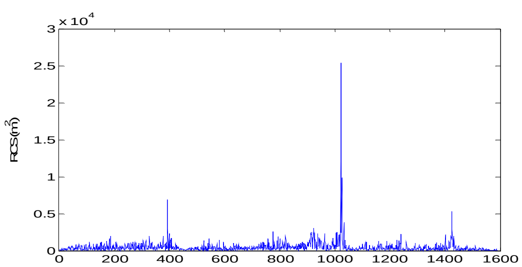

Figure 8. RCS of International Space Station

The result of size estimation is shown in Tab. 2.

Table 2. Size estimation of ISS

	Long axis length (m)	Short axis length (m)
Real size	44.5	27.5
Size estimation of [3]	30.9755	13.6929
Size estimation of this paper	45.8289	20.9000

Analyze the RCS of LCS-4 calibrated star. The RCS series of LCS-4 calibrated star is shown in Fig. 9.

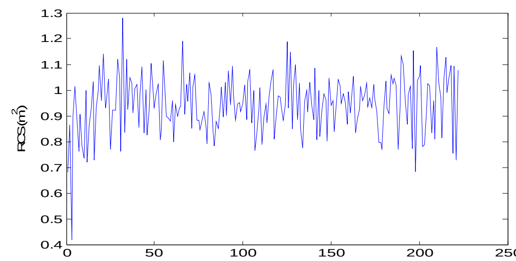

Figure 9. RCS of LCS-4

The result of size estimation is shown in Tab. 3.

Table 3. Size estimation of LCS-4

	Long axis length (m)	Short axis length (m)
Real size	1.130	1.130
Size estimation of [3]	1.1430	1.0414
Size estimation of this paper	1.1000	1.1000

When estimate the size of space debris using the method in this paper, the bigger angle that the target rotates around the radar, the more precise in size estimation. But the angle range where ground-based radar can detect is limited, thus the estimated size is the size of target at the angle range. Because that the structure of space target is simple and symmetric, at the angle rang, the estimated size of space debris is close to the real size. The estimation value is compared with the result of MIT Lincoln Laboratory's method, which testifies that the method proposed in the paper is more precise, especially for the space debris that shaped similarly with ellipsoid. Besides, more RCS data has been processed, and the result also demonstrate the method in this paper is effective.

4. CONCLUSIONS

RCS of target is available for all the characteristic measure radar, which depicts abundant information of target. How to extract features from RCS effectively has been attracting attention from the domestic and oversea. This paper established the simplified ellipsoid model based on the characteristic of space debris. By the model, an important property is obtained: the ratio of number of small RCS part to large RCS part split by the mean RCS is equal to the ellipsoid's curvature. According to the property, the size of space debris' long and short axis is estimated using the minimum variation rule. A number of practical data is processed using the approach, and the result indicates that the approach gets high accuracy in target size estimation. The approach we proposed can afford significant information of target from its RCS, which is important for space debris surveillance.

5. REFERENCES

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