# ANALYTICAL FORMULAS FOR EVALUATION OF METEOROID DISTRIBUTIONS IN THE NEAR-EARTH SPACE

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## ABSTRACT

The distributions of stream meteoroids in the near-Earth space is investigated analytically using the method of general functions (MGF). The analytical formulas for the focusing and shadowing effects by the Earth are obtained. These results reflect clearly the main features of meteoroid distributions, and can be used for interpretations of on-board measurements and ground observations by meteor radars and optical telescopes. The results can also be used for estimations of spacecraft safety. Then there are presented new results on transitory dust dynamics obtained by MGF. The temporary distributions and a gravitational drag for a moving gravitating center are evaluated in linear approach (without self-gravitation).

## 1. INTRODUCTION

The analytical formulas by E.J. Opik, D. Kessler, and other authors are the techniques to calculate the risk of collisions with objects disposed in elliptical orbits. In this way the problem of calculations of space debris risk obtained a simple and exact mathematical foundation.

The problem of statistical description of hyperbolic motion of interplanetary or interstellar dust clouds which move from infinity through the vicinity of a gravitation center have been attracting attention of many explorers starting from the beginning of the previous century at least. The analytical works [15, 12, 6] can be regarded as examples. The first work had a little bug, the second one had a bug too, but the last work proposed accurate results.

The method of general functions (MGF) as alternative approach for orbital statistical mechanics was firstly developed for elliptical orbits in the following works: in [7] and afterwards [8,9] this method was used for determination of densities of distributions of space debris; in [2-5] the MGF was proposed for use in similar problems of statistics of orbital motions. The most detailed description of MGF foundations was given in [5]. The averaging over an orbital period was an essential part of this technique, so implementations of this method were initially limited to statistical analysis of space objects in elliptical orbits.

Then in [10,11] the extension of the MGF on hyperbolic problems was proposed. Hence it was shown that the MGF is a powerful tool for a more extensive class of statistical orbital mechanics problems including infinite motion.

The new results facilitates the risk analysis of spacecraft because of interplanetary dust poses enough serious hazards to spacecraft along with orbital debris, and now the serious attention is paid to their protection against meteoroids.

## 2. DIMENSIONLESS PARAMETERS

Like to the usual hydrodynamics, the problem of motion in vicinity of a gravitational center with gravitational constant μ can be formulated in terms of the following dimensionless parameters: velocity $\tilde{v}$, radious-vector $\tilde{r}$, impact parameter $\tilde{\chi}$, elapsed time $\tilde{t}$, space density $\tilde{\rho}$, and force $\tilde{F}$:

$$\tilde{v} = \frac{v}{v_0}, \qquad \tilde{r} = \frac{r \cdot v_0^2}{\mu}, \qquad \tilde{\chi} = \frac{y_0 \cdot v_0^2}{\mu},$$

$$\tilde{t} = \frac{t \cdot v_0}{\mu}, \qquad \tilde{\rho} = \frac{\rho}{\rho_0}, \qquad \tilde{F} = F \cdot \frac{v_0^2}{\rho_0 \mu^2}.$$

Besides, the following relations for a flux and density increment can be written:

$$f_0 = \rho_0 \cdot v_0, \qquad \delta = \tilde{\rho} - 1.$$

Furthermore, the sign of nondimensionalization is omitted.

## 3. THE MOTION OF A MONODIRECTIONAL METEOROID STREAM IN THE NEAR-EARTH REGION

The motion of a meteoroid stream in the near-Earth space can be represented generally as follows.

There is a parallel flow of dust with velocity $v_0$ at infinity (the nondimensional speed equals 1). The flow incidents on a gravitation center. It is a 2D-problem with cylindrical symmetry.

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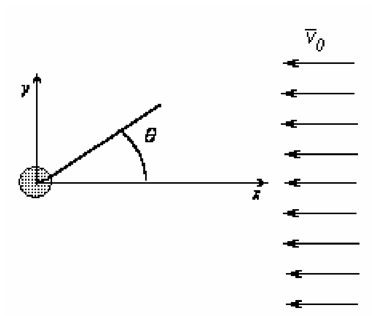


*Figure 1. The problem on a meteoroid flow in the the near-Earth space.*

The density of space distribution in a time *t* induced by a particle distributed uniformly (i.e. with equal probabilities) over azimuthal angle φ, is given by expression

$$\rho(r,\theta,t) = \frac{1}{2\pi \cdot R^2(t)\cdot \sin\Theta(t)}\delta(R(t)-r)\cdot\delta(\Theta(t)-\theta) \tag{1}$$

where $R(t)$ and $\Theta(t)$ are the functions which describe radial and angular motion of the particle in time.

The norm integral (the total number of particles in the whole space) is as follows:

$$\rho(r,\theta,t) = \int_0^\infty \int_0^{2\pi} \rho(r,\theta,t)\cdot 2\pi\cdot r^2 \sin\theta\cdot dr\cdot d\theta = 1$$

A mean space density induced by one particle during time *T* can be written as

$$\overline{\rho}(r,\theta) = \frac{1}{T}\cdot\int_0^T \rho(r,\theta,t)\cdot dt\,.$$

This equation can be integrated using one of the available δ-functions and an elementary relation for angular motion $\left|\frac{d\Theta}{dt}\right| = \frac{|v_\theta|}{r}$. Equation $\Theta(t)-\theta=0$ has two solutions for a given radious-vector *r* corresponding to two different impact parameters.

$$\chi_{1,2} = \frac{1}{2}\left(r\sin\theta \pm \sqrt{r^2\sin^2\theta + 4r(1-\cos\theta)}\right). \tag{2}$$

In other words, commonly there are two streamlines going through an every point of space $(r,\theta,\varphi)$. The streamline with positive impact parameter corresponds to "direct" particles, the streamline with negative impact parameter corresponds to "dispersed" particles. The "dispersed" particles move "around" the gravitational center. And have "longer" pathes

$$\overline{\rho}(r,\theta) = \frac{1}{T}\cdot\sum_{i=1}^{2}\frac{1}{2\pi\cdot R'^2(\theta)\cdot\sin\theta}\cdot\delta(R'(\theta)-r)\cdot\frac{1}{\left|\frac{v_\theta}{R'(\theta)}\right|}\,. \tag{3}$$

In the formula (2) the function $R'(\theta)$ is a dependency of radious-vector via angle θ. Velocity $v_\theta$ is a function of the angle θ also.

From the conservation law for the orbital angular momentum it follows that $\frac{|v_\theta|}{r} = \frac{\chi}{R^2(\theta)}$.

Then we can integrate over impact parameter to get a total density of distribution. The number of particles in an elementary interval of the impact parameter for a unit of time is as follows.

$N = 2\pi\chi\cdot d\chi\cdot T\cdot f_0$,

where $f_0$ is a particle flux in infinity.

The total density is described by the sum of two integrals.

$$\overline{\overline{\rho}}(r,\theta) = f_0\sum_{i=1}^{2}\int_0^\infty \frac{\delta(R(\theta)-r)}{\sin\theta}\cdot d\chi\,.$$

Using properties of the δ-function we have:

$$\overline{\overline{\rho}}(r,\theta) = f_0\sum_{i=1}^{2}\frac{1}{\sin\theta\cdot R'_\chi}\,, \tag{4}$$

where

$$R'_\chi = \frac{dR}{d\chi} = 2\frac{r}{\chi} - \frac{r^2}{\chi^2}\sin\theta \tag{5}$$

Thus, Eqs. (2), (4), and (5) determine an average density of distribution of interplanetary dust in the near-Earth space during a meteoroid storm.

It is easy to check that this solution coincides numerically with the earlier results [6, 10, 11], although they have different forms.

## 4. FOCUSING AND SHADOWING OF A METEOROID STREAM BY THE EARTH

The average density of distribution of interplanetary dust in the near-Earth region can be evaluated using the dependency of density via radious-vector that lies in the transverse plane, that goes through the center of the Earth (similar to the main plane in optics).

$$\bar{\bar{\rho}}(r,\frac{\pi}{2})\cdot v \cong f_0 \cdot \frac{|\chi_1|\sqrt{r^2+\chi_1{}^4}+|\chi_2|\sqrt{r^2+\chi_2{}^4}}{r^2\cdot\sqrt{r^2+4r}}, \quad (6)$$

where $\chi_{1,2} = \frac{1}{2}\left(r \pm \sqrt{r^2+4r}\right)$.

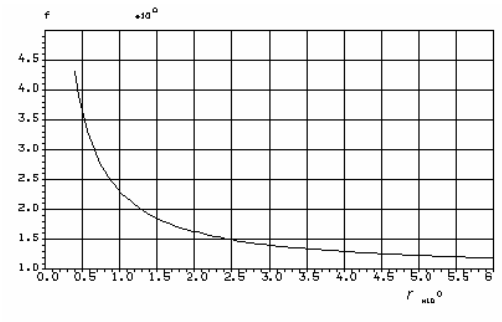


*Figure 2. A profile of density of meteoroid distribution in the transverse direction*

This profile agrees with the commonly accepted estimates of the all-directional flow ( for example, in [13, 14]) .
Radius-vector of the point where a streamline crosses the main plane is

$r_c = \frac{\chi^2}{1+\chi}$. Its value tends to the mpact parameter when impact parameter tends to infinity.

The focusing of the monodirectional flow at the axis beginning at some distance of the Earth is an important feature of the flow (Fig. 3).

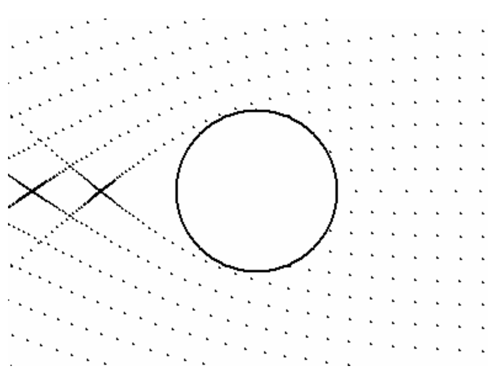

*Figure 3. The focusing of the monodirectional flow of meteoroids in vicinity of the Earth. It is an appropriate scale for initial velocity of meteoroids of 10 km/s.*

Such focusing leads to essential reduction of the shadow region. Vice versa, there is a sharp increasing of the meteoroid fluxes starting from the boundary of the "dead" zone (the region of shadow free from the dust) at some critical distance because there is a singularity in the density distribution at the axis of the parallel flow. The singularity is described by the following characteristic dependency.

$$f(r,\theta) \cong f_0 \cdot \frac{\sqrt{2}}{\sqrt{r}|\pi-\theta|}\left(1+\frac{2}{r}\right). \quad (7)$$

Such process has place for any initial speed of meteoroids but for the greater speed the frontier is enough far from the Earth. The point where a stream crosses the flow axis corresponds to $\theta = \pi$, hence, the radious-vector of this point is $r_f = \frac{\chi^2}{2}$, and sharply increases with the impact parameter.
Foe example, geostationary orbit corresponds to $\frac{r_{GSO}}{r_m} \approx 6.52$, where $r_m$ is a minimal radious (the Earth's radious plus the height of atmosphere that is about 100 km). If it is a boundary of the Earth's shadowing of the meteoroid stream then $r_{GSO} = r_f = \frac{r_m^2}{2}$, and. $r_m \approx 13$. It corresponds to velocity of 28.3 km/s, so that impact parameter is $\chi$=13. It is obvious that if the stream velocity is greater than 28.3 км/c, the frontier of the singularity is out of the geostationary orbit. Note, nevertheless, the singularity exists for the greater meteoroid velocities but somewhere far from the Earth.
Note once more, that the right calculation of risks needs a special approaches indicating the singularities.
The focusing is a real danger that is ignored now.
Even a weak meteoroid streams which are not cataloged now can be hazardous due to the singularity.
This question needs the further study; firstly, meteoroid stream structures and their evolution should be explored.

## 5. THE ABSORPTION CROSS-SECTION OF THE EARTH IN A METEOROID STREAM

Evolution of the meteoroid streams crossing Earth's orbit is determined by two processes: dispersion and absorption. The perigee of the orbit of the meteoroid streamline at the boundary of absorption zone is determined by the following equation

$$tg\theta_m = -\chi \quad (8)$$

If the radius-vector of the perigee is smaller than some value $r_m$, the particle will burn in the atmosphere.

Considering the signs the critical impact parameter can be written as follows

$$r_m = \frac{\chi_m{}^2}{1+\sqrt{1+\chi_m{}^2}} \qquad (9)$$

If $\chi_m$>>1 then $r_m \cong \chi_m$

The simple expression for the minimal impact parameter is

$$\chi_m^2 = r_m^2 + 2r_m . \qquad (10)$$

Thus, the effective cross section of absorption $S = \pi \cdot \chi_m^2$ is described as

$$S = \left(1+\frac{2}{r_m}\right) \cdot S_0 , \qquad (11)$$

where $S_0 = \pi \cdot r_m^2$ is a usual cross section of the Earth. The effective cross-section is proportional to inverse square of the initial speed of the stream. The gravitational effect is essential even for meteoroid streams of moderate speeds.

## 6. ABOUT THE INFLUENCE OF THE GRAVITATIONAL FIELD OF THE EARTH ON THE FLOW OF ALL-DIRECTIONAL METEOROIDS

The stream component of meteoroid environment brings an essential local hazard, but the sporadic component is more dangerous in average. The initial velocity distribution of sporadic meteoroids is closed to isotropic distribution. A simpler but useful evaluation for the isotropic case was developed in [1]. Using the principle of symmetry and the conservation law allowed deriving simple and reliable expressions for the dependency of dust density via radious-vector. But this approach doesn't allow obtaining velocity characteristics of meteoroid distributions. In this case the above estimates made for the main plane can be useful.
But note, the Earth's motion leads to violation of the initial symmetry. Besides, the modern models of meteoroid environment consider nonisotropic features in sporadic components too. In this case there is only the way of integration of the Eqs. (2), (4), and (5) for the monodirectional flows.

As it was shown before (Fig. 3), the gravitational focusing leads to reduction of the "dead" zone in space but it also leads to enlargement of the "dead" angular zone. You see in Fig.4 that there is $\gamma \geq \gamma'$ ).

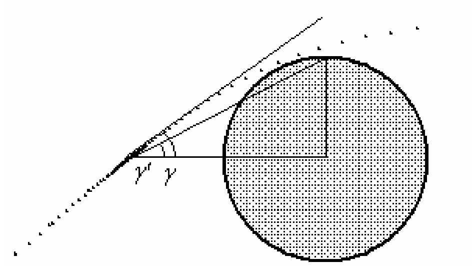


*Figure 4. A "dead" angular zone considering the effect of gravitational focusing.*

## 7. THE GRAVITATIONAL DRAG IN AN UNBOUNDED DUST MEDIA

A problem of transitory dust particles flow in an unbounded (infinite) dust region comprising a gravitation center can be reformulated in the following way.
At the initial time $t$=0 the dust particles are distributed uniformly over an infinite region. Instantly a gravitation center appears and acquires a unit velocity (Fig.5). The problem has a cylindrical symmetry.

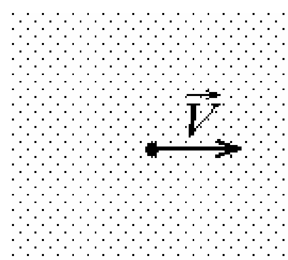


*Figure 5. The problem on instant motion of a gravitation center in a dust media.*

In the problem the motion of dust particles can be determined by following two equations:

$$t = f(\theta,\theta_0,r_0) \qquad (12)$$
$$r = g(\theta,\theta_0,r_0) ,$$

where $\theta_0$, $r_0$ are the polar coordinates of a dust particle at the initial time $t$=0, and $\theta$, $r$ are the coordinates at an arbitrary time $t$.
The line of solution of this problem is more complicated than previous. But the main ideas are the same.
As a result, at any time the density of distribution of the dust is a sum of several components

$$\rho(r,\theta,t) = \sum_i \rho_i(r,\theta,t) , \qquad (13)$$

where each of the components is described by the formula of common view:

$$\rho_i(r,\theta,t) = \frac{r_0^2 \sin\theta_0}{r^2 \sin\theta} \frac{\partial f / \partial\theta}{\left(\frac{\partial g}{\partial\theta_0}\frac{\partial f}{\partial r_0} - \frac{\partial g}{\partial r_0}\frac{\partial f}{\partial\theta_0}\right)}, \qquad (14)$$

and $r_0 = r_0^{(i)}$ и $\theta_0 = \theta_0^{(i)}$ are solutions of the system (12) (index $i$ is omitted in (14) ). There are several solutions of the equation (12) in the region of finite trajectories but only two solutions in the region of infinite trajectories where the density is composed of the following components: $\rho_p(r,\theta,t)$ is the density of direct flow and $\rho_d(r,\theta,t)$ is the density of dispersed flow. The density of the direct flow prevails everywhere, except the wake region.

The expression (14) is rather complicated really, and it needs an additional verification. Such verification can be performed in different ways. Firstly the limiting expression at $t \to \infty$ is obtained..

Nnumerical calculations show identity of limiting results from (14), and the results by [6, 10, 11] , and the results by [6, 10, 11,.

Also the temporary behavior of the direct component is investigated . Fig. 6 shows the temporary increments of the dust density when θ= 90° and $r$ = 10 and $r$ = 30.

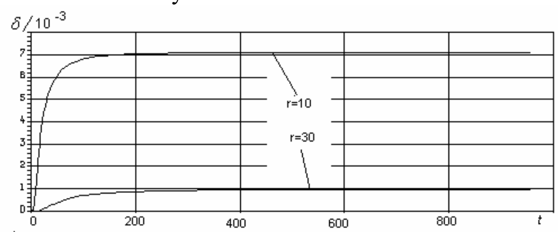


*Figure 6. Graphics of the temporary increments of primary component of dust density in direction θ=90°.*

Integration of the gravitational drag force over the full space gives the following graphic (Fig. 7)

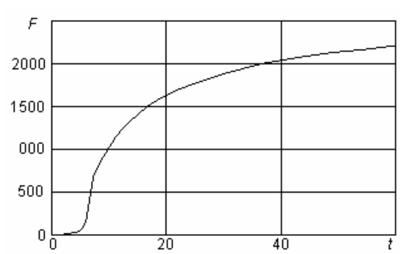


*Figure 7. Drag force via time (the disperse component is not considered).*

Note, the drag force acting on the gravitating center that moves in an infinite media grows with time unlimitedly, and the density increment at great distances contribute the main input into the drag force. .

## CONCLUSIONS

Using the method of general functions the analytical solution of the problem of a steady meteoroid flow in the near-Earth space is obtained. There are investigated the main features of this flow including effects of focusing and shadowing by the Earth.

The analytical formulas for the effects are .obtained. These results reflect the main features of meteoroid distributions more clearly than numerical calculations, and can be used for interpretation of on-board measurements and ground observations by meteor radars and optical telescopes. The results can also be also used for estimations of spacecraft safety.

The unsteady problem of dust motion in the vicinity of a gravitating center (or instant motion of the gravitating center) is resolved analytically in the linear approach (without self gravitation). At the limit of $t\to\infty$ the unsteady solution converges to the formulas for the steady flow.

The formulas for the transitory dust distribution and gravitational drag are developed in linear approach without consideration the self-gravitation processes.

The method of general functions can be used for investigation of a wide class of steady and unsteady dust flows.


## ACKNOWLEDGEMENTS

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