ACCURACY OF ESTIMATING SOLAR RADIATION PRESSURE FOR GEO DEBRIS WITH TUMBLING EFFECT*

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ABSTRACT

The accuracy of estimating solar radiation pressure for GEO debris is examined and demonstrated, via numerical simulations, by fitting a batch (months) of simulated position vectors. These simulated position vectors are generated from a "truth orbit" with added white noise using high-precision numerical integration tools. After the long-arc fit of the simulated observations (position vectors), one can accurately and reliably determine how close the estimated value of solar radiation pressure is to the truth. Results of this study show that the inherent accuracy in estimating the solar radiation pressure coefficient can be as good as 1% if a long-arc fit span up to 180 days is used and the satellite is not tumbling. The corresponding position prediction accuracy can be as good as, in maximum error, 1 km along in-track, 0.3 km along radial and 0.1 km along cross-track up to 30 days. Similar accuracies can be expected when the object is tumbling as long as the rate of attitude change is different from the orbit rate. Results of this study reveal an important phenomenon that the solar radiation pressure significantly affects the orbit motion when the spin rate is equal to the orbit rate.

NOMENCLATURE

GEO = geosynchronous Earth orbit
TLE = two-line elements
C_R = index of surface reflection of the spacecraft ($0 < C_R < 2$)
A/m = area-to-mass ratio of the spacecraft (projected area normal to Sun's ray)
k = dimensionless coefficient of the sine term
G = spin rate of the debris object
ECI = Earth centered inertial coordinates
UD = upper diagonal factorization, an efficient algorithm for computing a covariance

1. INTRODUCTION

Results of a recent study [1] show that the solar radiation pressure effect of a geosynchronous Earth orbit (GEO) debris object can be estimated by fitting a long-arc (months) of the two-line elements (TLE) data determined from the U.S. Air Force Space Command [2]. As a result, the long-term orbit prediction accuracy can be significantly improved with potential position accuracy considerably better than the TLE data noise (10 km). To prove the above potential improvement in orbit prediction accuracy, a well determined GEO or super synchronous orbit using independent measurements is needed. However, the accurate ephemerides determined from ranging data for most of the operational GEO spacecraft are corrupted by periodical stationkeeping maneuvers and the daily momentum wheel dumping. The ephemerides for GEO debris with accuracy better than the TLE data are not available due to primarily the poor understanding of the tumbling motion of an inactive space object.

The purpose of this study is to examine and demonstrate, via numerical simulations, the inherent accuracy of estimating the solar radiation pressure coefficient (C_R*A/m) by fitting a batch (months) of simulated position vectors. The solar radiation pressure coefficient is defined, similar to the ballistic coefficient, as the product of the index of the surface reflection, C_R, and the area-to-mass ratio, A/m, of the space object. These simulated position vectors are generated from a "truth orbit" with added white noise using high-precision numerical integration tools specially designed for this study. After the long-arc (months) fit of the simulated observations (position vectors), one can accurately and reliably determine how close the estimated value of C_R*A/m is to the truth. In the same orbit determination solution, one can also evaluate the accuracy of orbit prediction by comparing the predicted orbit with the truth orbit. In this analysis, the attitude or tumbling motion of the debris object is also simulated in order to understand the effect on estimating the solar radiation pressure.

2. DATA GENERATION AND COMPUTATION

To verify the prediction accuracy of this system using a numerically generated truth model with simulated data noise similar to that of the TLE data, we implemented the following procedure:

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1) Use the high-precision numerical integration to create a "truth orbit" with perturbing forces: a 12x12 EGM/WGS84 Earth gravity, luni-solar attractions, and solar radiation pressure effects (flat plate).
2) Generate similar TLE data (position vectors) from the truth orbit by adding random noise to the simulated data.
3) Perform long-arc fit to the above data with a batch least squares method using an efficient UD (Upper Diagonal) filter [3].
4) Evaluate the accuracy of the estimated constant solar radiation pressure coefficient by comparing with the truth.
5) Demonstrate the improved ephemeris accuracy of long-term prediction by comparing the predicted orbit with the "truth orbit" with no noise.

The above procedure is then repeated with a simulated attitude motion, or tumbling, of the debris object. In this study, a simplified sinusoidal variation of the effective area normal to the Sun's rays is assumed. The area-to-mass ratio is assumed to be:

$$A/m = (A/m)_0 * [1 + k * \sin(G * T)] \quad (1)$$

where $(A/m)_0$ is the constant initial value of area-to-mass ratio. The factor k is the coefficient of the sine term (k <1), and G is the rate of change or spin rate of the debris object. T is time in the same units as G. It is believed that when a communication satellite becomes inactive after it is disposed, the spacecraft tends to gradually spin up due to the small angular momentum increase caused by solar radiation torque [4]. However, no study has been done in understanding the long-term attitude motion of an inactive spacecraft with large solar arrays. In this paper, several values of k and G are assumed to understand the effects due to the variation of the solar radiation coefficient. The index of surface reflection, C_R, is assumed to be a constant value of 1.3 for all cases of the simulations.

3. DATA PREPARATION AND TOOLS

A special tool RK78R is coded in FORTRAN on PC for generating the simulated observations (position vectors in ECI reference frame) with user specified random noise. RK78R is modified from an in-house orbit propagator for orbit integration using the 7/8th order Runge-Kutta integrator (RK78) developed by NASA (Fehlberg). The second tool FITGEO7, which was used in a parallel study [1] for the differential corrections of TLE generated observations, is used here for fitting the simulated data. FITGEO7 estimates seven parameters: six states at epoch and the constant solar radiation coefficient (C_R*A/m), using a simple batch- least-squares UD filter [3]. The numerical precision of both tools (RK78R and FITGEO7) have been verified against an independent high-precision orbit propagator TRACE [5].

The "truth orbit" generated by RK78R can either be an orbit with a constant SRP coefficient or an orbit with simulated tumbling motion following Eq. 1. In the same time, assumed data noises are added to the clean orbit with a random number generator. The data noises are added to the first 30 to 150 days of the orbit integration. The remaining span of 30 to 50 days is the "truth orbit" without noise and will be used to compute the deviations in the prediction span. The computation is performed by FITGEO7 after the differential corrections process is converged.

4. RESULTS AND DISCUSSION

4.1. No Tumbling Effects

In the first group of simulations, no tumbling effects are considered. Three cases are assumed to reflect the differences in initial orbit conditions. Case A1 assumes an orbit with small eccentricity and inclination (0.0002 and 0.05 deg) and a Sun-pointing condition; Case A2 has increased values in eccentricity and inclination (0.002 and 5.0 deg) and an approximate Sun-pointing condition; Case A3 has higher values in both eccentricity and inclination (0.003 and 15 deg) with no Sun-pointing condition. These three cases try to simulate the GEO disposal orbits at three stages: Case A1 at a few days after disposal maneuvers, Case A2 at 5 to 6 years after disposal, and Case A3 at about 25 years after disposal. All three cases have a mean orbit altitude of 300 km above GEO. The constant area-to-mass ratios in the truth orbit are 0.035 m^2/kg for Case A1, 0.030 m^2/kg for Case A2 and 0.025 m^2/kg for Case A3. A white data noise of 10 km (1 sigma) similar to that of the TLE data is added to the clean data generated from the truth orbit. The assumed data noise along the ECI coordinates has the distribution of 6.85 km along x and y components and 1.5 km along the z component.

Figure 1 gives an example of a 60-day fit with the assumed data noise. The position errors in the prediction span of 40 days are computed from the truth orbit and are an order of magnitude smaller than the data noise. Figure 2 shows the accuracy in estimating the solar radiation pressure coefficient, C_R*A/m, as a function of fit span. Figures 3 through 5 show the maximum errors from truth in the first 30 days of prediction versus the fit span. The results of the first group of simulations reveal:

1) The error of estimating solar radiation pressure coefficient decreases to 3% or smaller when the fit span is longer than 60 days; the

error drops to about 1% or better when the fit span is 180 days;

2) The 30-day maximum prediction errors in in-track, radial and cross-track position are, respectively, 3 km, 1.2 km and 0.3 km when the fit span is between 60 days and 150 days; and 1 km, 0.3 km and 0.1 km when the fit span is 180 days.

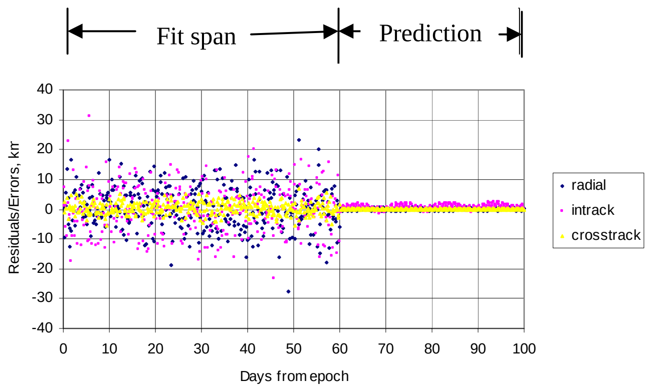

Figure 1 Residuals of a 60-day fit and the 40-day prediction errors from the truth orbit

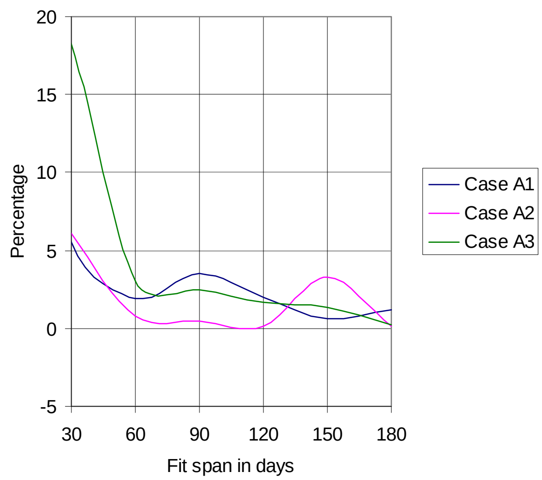

Figure 2 Percentage error of estimated solar radiation pressure coefficient vs. fit span (no tumbling)

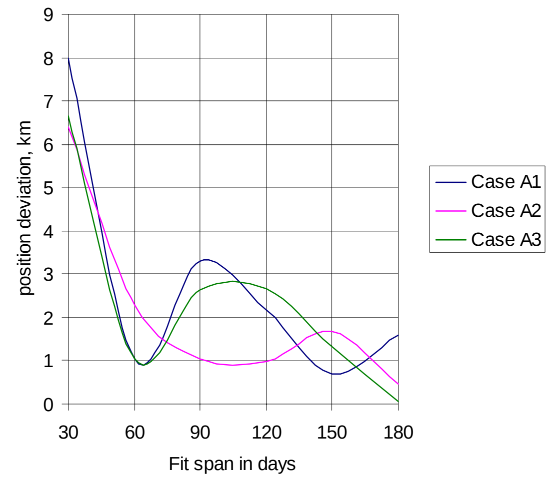

Figure 3 30-day maximum in-track prediction errors vs. fit span (no tumbling)

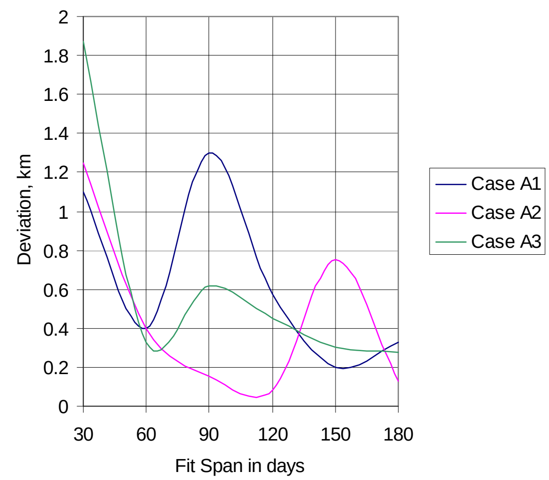

Figure 4 30-day maximum radial prediction error vs. fit span (no tumbling)

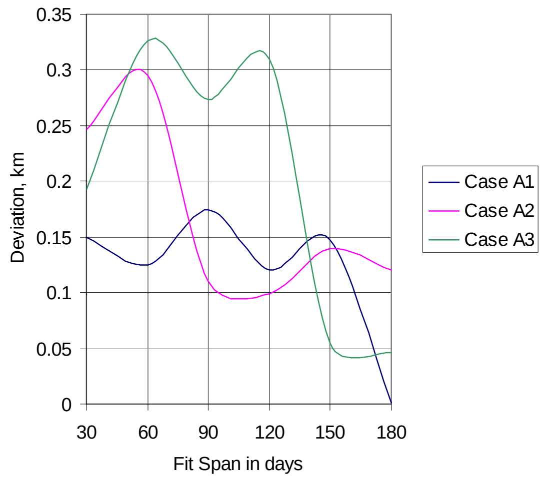

Figure 5 30-day maximum cross-track prediction error vs. fit span (no tumbling)

4.2. With Tumbling Effects

In the second group of simulations, the above three cases are repeated with simulated tumbling motion. The assumed value of k in equation (1) is 0.35, and six values of G are used for the simulations. Each value of the tumbling rate, G, is repeated for Case A1 (now Case B1) and Case A3 (now Case B3) with two fit spans, 60 and 150 days. Results from these 24 cases are tabulated in Table 1 and they reveal:

1) Very large errors were found both in the estimated solar radiation pressure coefficient as well as in predicted intrack positions when G is equal to the orbit rate;
2) The corresponding errors in 1) increased (from the first group) only slightly, by about 50%, when G is either twice or half of the orbit rate;
3) For Case B3, when the G value is set at one revolution per 120 days, the 30-day maximum in-track error has increased to 7 km and the radial error has increased to about 4 km. The error in estimated value of solar radiation pressure is also increased slightly;
4) The cross-track error in 3) is not affected by the rate of tumbling motion and stays at 0.4 km.

It is encouraging to note that the maximum deviations along the radial, intrack and cross-track remain small as long as the spinning rate of the spacecraft is faster than the orbit rate. In other words, the long-arc (150-day) fit of TLE data has the potential of providing accurate position predictions (better than 0.29 km in radial, 1.17 km in intrack and 0.11 in cross-track) up to 30 days. Based on theory, it is more likely that the solar radiation pressure torque will cause the spin rate of an inactive spacecraft or space object to remain faster than orbit rate. It is hoped that the findings of this report will prompt the study of measuring or determining the spin rate of an inactive GEO spacecraft via radar or optical observations.

Table 1 Maximum deviations in km along radial/intrack/cross-track vs. tumbling rates

Tumbling Rate (Period, day)	60-day fit (Case B1)	60-day fit (Case B3)	150-day fit (Case B1)	150-day fit (Case B3)
½	0.99/3.4/0.12	0.59/1.57/0.4	0.12/0.81/0.11	0.29/1.17/0.11
1	**5.9/427/0.32**	**1.8/194/0.6**	**9.7/143/0.65**	**16.2/143/10.5**
2	0.77/3.80/0.19	0.6/1.4/0.4	0.22/1.31/0.12	0.26/0.82/0.22
10	1.24/3.5/0.04	1.04/2.23/0.23	0.33/1.6/0.12	0.51/1.64/0.13
30	0.95/3.1/0.04	1.04/2.68/0.23	0.79/3.3/0.06	0.49/1.43/0.10
120	5.7/15.2/0.08	3.8/7.2/0.4	3.1/7.2/0.17	2.2/6.2/0.44

4.3. Tumbling at Orbit Rate

To better understand the significant effect of solar radiation pressure when spin rate is equal to orbit rate, the intrack deviations at three different values of k are generated and plotted in Figure 6. The intrack deviations are the differences between the truth orbit generated from RK78R with tumbling effect and the predicted orbit generated from FITGEO7 with the same initial state and solar radiation pressure coefficient before differential corrections. The deviation reaches 450 km after 30 days with the k value equal to 0.5 and there is no noise added to the data. These non-linear decreases in intrack motion caused by tumbling seem to suggest that the semi-major axis of the orbit is gradually pushed up due to a deep resonance. After differential corrections, the residual errors of the 30-day fit follow parabolic curves depending on the value of k as shown in Figure 7. Figure 8 shows the after-fit residuals of a 150-day fit (Case B1) with tumbling rate equal to orbit rate and simulated data noise. The large sinusoidal signature of the intrack deviations is believed to be induced by the resonance with the orbit mean motion. The cross-track residuals seem to be least effected by the tumbling motion. The estimated values of the constant solar radiation pressure are quite different from the initially assumed values before the differential corrections. This insightful finding leads to a conclusion that none of the three actual GEO debris objects examined in the recent study by Chao and Campbell [1] has a spin rate equal to or near the orbit rate.

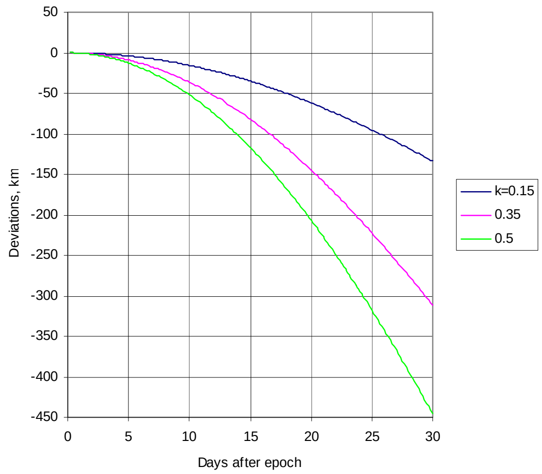

Figure 6. Intrack deviations caused by tumbling at different values of k when tumbling rate is orbit rate (no noise and no fit)

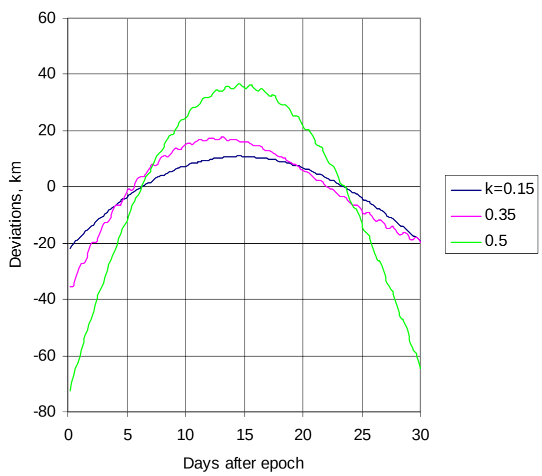

Figure 7. Intrack deviations from truth when tumbling rate is orbit rate (after fit, no noise)

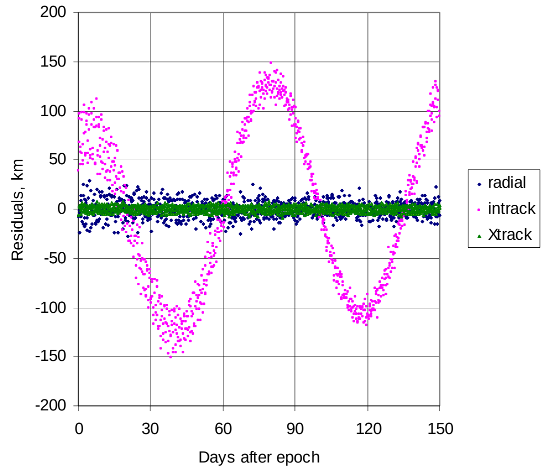

Figure 8. Post-fit residuals of a 150-day fit (Case B1) with tumbling rate = 1/day

5. CONCLUSIONS

Based on the interesting findings of this study, the inherent accuracy in estimating the solar radiation pressure coefficient can be as good as 1% if a long-arc fit span up to 180 days is used and the satellite is not tumbling. The corresponding prediction accuracy can be as good as, in maximum error, 1 km along in-track, 0.3 km along radial and 0.1 km along cross-track up to 30 days. Similar accuracies can be expected when the object is tumbling as long as the rate of attitude change is different from the orbit rate. Results of this study reveal an important phenomenon that the solar radiation pressure significantly affects the orbit motion when the spin rate is equal to the orbit rate. An analytical investigation of the equations of motion to better understand the resonance effect is being conducted.

The results of this study further enhance the findings of the recent study [1] that the long-arc fit of TLE data can significantly improve the long-term orbit prediction accuracy of GEO debris objects. It is hoped that the findings of this study using simulated data and the recent study [1] using TLE data can stimulate interest in the investigation of the uncontrolled attitude motion of a GEO or super-GEO object in order to better account for tumbling in ephemeris prediction. The attitude motion may be determined through independent measurements, such as radar or optical observations.

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7. REFERENCES

1. Chao, C.C., and W. S. Campbell, “Estimating Solar Radiation Pressure for GEO Debris,” Paper to be presented at the 5th European Conference on Space Debris, Darmstadt, Germany, March 30 – April 3, 2009.

2. Hoots, F. R., and R. L. Roehrich, “Models for Propagation of NORAD ELEMENT SETS,” Space Track Report No. 3, Project Space Track, U. S. Air Force Space Command, December 1980.

3. Bierman, G., *Factorization Methods for Discrete Sequential Estimation*, New York: Academic Press, 1977.

4. Liang, A. C., and D. M. Brueck, Private communication, The Aerospace Corporation, November, 2008.

5. Downs, W. D. III. Et al., “TRACE Trajectory Analysis and Orbit Determination Program, Vol. Vol. II: Usage Guide,” The Aerospace Corporation Report SAMSO-TR-71-141, 31 May 1974.