RESEARCH ON THE ARITHMETIC FOR SPACE DEBRIS LONG-TERM ORBIT PROPAGATION IN LEO

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ABSTRACT

Space debris Long-Term orbit propagation is one of the main problems for the space debris environment engineering model. This paper provides arithmetic for the space debris Long-Term orbit propagation in LEO. The arithmetic regards the atmosphere drag as the main perturbation, provides an average method to calculate the effect for the atmosphere drag; then import variable step in order to reduce the amount of calculation; at last validate the correctness of the algorithm by comparing the results between this algorithm and STK software.

1. INTRODUCTION

Since the dawn of the space age, there have been a great number of spacecrafts launched into outer space. However, the side effect of this activity has been space debris. The amount of space debris increases each year and spacecrafts are under threat of collision with the debris [1]. The number of space object which can be tracked is increasing year after year [2], the data can be described with the Two-Line-Element format [3]. It is therefore necessary to build a model for describing the debris environment [4]. At the process of modeling, long-term orbit propagation of space debris is one of the problems for modeling the space debris environment.

This paper provides arithmetic for the long-term orbit propagation of space debris in LEO. The arithmetic is applied to the space debris environment modeling, so we calculate the average position of the space debris. The arithmetic regards the atmosphere drag as the main perturbation, provide an average method to calculate the effect for the atmosphere drag; then provide the orbit propagation strategy for space debris on different orbit; in order to reduce the amount of calculation, a variable step method is applied in the paper; at last validate the correctness of the algorithm by comparing the results between this algorithm and STK software.

2. ARITHMETIC FOR SPACE DEBRIS LONG-TERM ORBIT PROPAGATION IN LEO

Six parameters can determine the position of a space object, but relative to the inertia system, it only needs five parameters to determine the orbit of a space object. If we use the most simple motion model two-body model, the five elements of the orbit are invariable when the object is moving in the space [5]. In fact the orbit elements are varying orderly all the time.

2.1. Principle of Orbit Propagation

The orbit perturbation of a space object contains earth gravity harmonics, atmospheric drag, radiation pressure effects, the third gravitation of sun and moon. The atmospheric drag is not a conservative force, the energy of the space object is reducing unceasingly when the object is affected by the atmosphere drag, so the space object will burn up when the perigee is rather low. The earth gravity harmonics make the semi-major axis and eccentricity varying periodically, and the right ascension of the ascending node (RAAN) and argument of perigee varying symmetrically, this variety will cause some changing of the whole space debris environment. Radiation pressure effects make great influence to the big area-to-mass ratio, the area-to-mass ratio is on the small side as a whole, so this perturbation is ignored in this paper. The third gravitation is rather small in LEO, so this perturbation is not considered in this paper.
We need the whole distribution of space debris when building the space debris environment. In this requirement, the exact position of every debris is not required; we only need the "average poison of every debris" in the time of day.

2.2. Orbit Propagation Strategy in This Paper

Based on the requirement of the space debris environment model, the method of orbit propagation in LEO is as follow:
The space debris with perigee of $130km \le h_p < 1000km$, consider the atmosphere drag and J_2 perturbation of gravity. When the orbit perigee height is below 1000km, the atmospheric drag effect becomes increasingly important. Drag is a nonconservative force and will continuously take energy away from the orbit; it is the main perturbation of space debris in LEO. When we take the modest accuracy, make the following assumptions:
(1) The atmosphere is spherically symmetric;
(2) The atmosphere does not rotate;
(3) Space debris for the constant cross-section.

Only atmospheric drag on the orbital semi-major axis and eccentricity has a clear impact. The influence of atmospheric drag on the semi-major axis and eccentricity can be described as follow:

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$$\frac{da}{dt} = -\frac{a^2 v^3}{\mu B}\rho(h) \quad (1)$$

$$\frac{de}{dt} = -\frac{v}{B}(e+\cos\theta)\rho(h) \quad (2)$$

where, μ is the Earth gravitational constant, $B = \frac{m}{C_D A}$ is the Ballistic coefficient, m is the mass of space debris, A is sectional area of space debris with velocity perpendicular to area, C_D is the drag coefficient, e is the eccentricity, θ is true anomaly. The main factor which leads to the decline of the orbit of space debris is the atmospheric drag. It is shown by (1),(2) that the orbital semi-major axis and eccentricity variety is complicated. Therefore, this article introduced a simplified method in the calculation of the average attenuation. When space debris at each cycle of movement, consider that the orbit is changeless, calculated the change for semi-major axis and eccentricity of each debris, Derived semi-major axis and eccentricity changes disciplinarian with time. Consider a period moving, by the orbital dynamics, we can see

$$\frac{d\theta}{dt} = (1+e\cos\theta)^2\sqrt{\frac{\mu}{p^3}} \quad (3)$$

Can be drawn under the style

$$\frac{da}{d\theta} = -\frac{a^2 v^3 \rho(h)}{\mu B(1+e\cos\theta)^2}\sqrt{\frac{p^3}{\mu}} \quad (4)$$

$$\frac{de}{d\theta} = -\frac{v(e+\cos\theta)\rho(h)}{B(1+e\cos\theta)^2}\sqrt{\frac{p^3}{\mu}} \quad (5)$$

For each set of orbital parameters, Calculated the attenuation coefficient of a, e for each period.

$$D_a = -\frac{a^2}{\mu B}\sqrt{\frac{p^3}{\mu}}\int_0^{2\pi}\frac{v^3\rho(h)}{(1+e\cos\theta)^2}d\theta \quad (6)$$

$$D_e = -\frac{1}{B}\sqrt{\frac{p^3}{\mu}}\int_0^{2\pi}\frac{v(e+\cos\theta)\rho(h)}{(1+e\cos\theta)^2}d\theta \quad (7)$$

Where $r = \frac{p}{1+e\cos\theta}$ is the vector length of position, $p = a(1-e^2)$ is Orbital semi-orthogonal string, $T = 2\pi\sqrt{\frac{a^3}{\mu}}$ is orbit period. After Δt later, Orbital semi-major axis and eccentricity changes are as follows,

$$a = a_0 + \frac{D_a}{T}\Delta t \quad (8)$$

$$e = e_0 + \frac{D_e}{T}\Delta t \quad (9)$$

Where a_0, e_0 are semi-major axis and eccentricity of initial moment and a is the semi-major axis, v is the velocity, ρ is atmospheric density, $h = r - R_\oplus$ is the altitude.

The J_2 perturbation of gravity do not affect the semi-major axis, eccentricity and inclination, it made the RAAN and argument of perigee changing over time evenly when other orbital elements are constant.

$$\frac{d\Omega}{dt} = -\frac{3}{2}\left(\frac{R_\oplus}{p}\right)^2 nJ_2\cos i \quad (10)$$

$$\frac{d\omega}{dt} = -\frac{3}{4}\left(\frac{R_\oplus}{p}\right)^2 nJ_2(1-5\cos^2 i) \quad (11)$$

Where Ω is RAAN, ω is argument of perigee, $n = \sqrt{\mu/a^3}$ is mean motion, $p = a(1-e^2)$ is semi-orthogonal string of the orbit, $J_2 = 1.08264\times10^{-3}$, $R_\oplus = 6378.144km$ is The average radius of the Earth, i is the inclination.

After the discussion above, such space debris orbit propagation formula is as follows:

$$\begin{aligned} a_{i+1} &= a_i + \frac{D_a}{T}\Delta t \\ e_{i+1} &= e_i + \frac{D_e}{T}\Delta t \\ i_{i+1} &= i_i \\ \Omega_{i+1} &= \Omega_i - \frac{3}{2}\left(\frac{R_\oplus}{p}\right)^2 nJ_2\cos i\cdot\Delta t \\ \omega_{i+1} &= \omega_i - \frac{3}{4}\left(\frac{R_\oplus}{p}\right)^2 nJ_2(1-5\cos^2 i)\cdot\Delta t \\ M_{i+1} &= M_0 + n_i\cdot\Delta t \end{aligned} \quad (12)$$

Where $[a_0, e_0, i_0, \Omega_0, \omega_0, M_0]$ are the orbital elements of space debris in initial moment. $[a_i, e_i, i_i, \Omega_i, \omega_i, M_i]$ are the *i*th step orbital elements, Δt is the time difference between the *i*th step and the *i+1*th step, $[a_{i+1}, e_{i+1}, i_{i+1}, \Omega_{i+1}, \omega_{i+1}, M_{i+1}]$ is the *i+1*th step orbital elements.

2.3. Variable Step Method in Numerical Integration

During one period moving, believe the orbital elements are unchanged, the semi-major axis and eccentricity are constant, so D_a and D_e are the definite integral of the true anomaly.

2.3.1 Variable Step-complex trapezoid formula

The integration is $\int_a^b f(x)dx = \left[\int f(x)\right]_{x=a}^{x=b}$, integral interval $[a,b]$ can be divided into $2n$ share, y_0, y_1, ……, y_{2n-1}, y_{2n} are points of division, $y_0 = a$ and $y_{2n} = b$, also the y_{2k-1} is midpoint of the interval $[y_{2(k-1)}, y_{2k}]$, $k = 1, 2, \cdots, n$, then the formula can be educed as follow formula

$$M_n = \frac{b-a}{n}\sum_{i=1}^{n} f(y_{2k-1}) \tag{13}$$

Where choose $n = 2^k$ usually.

The other method of integral can be described as the follow formula

$$T_n = (b-a)/2\left[(1/2)f(x_0) + \sum_{j=1}^{n-1} f(x_j) + (1/2)f(x_n)\right] \tag{14}$$

Obtain the average value of the two items, the result is as follow

$$T_{2n} = \frac{1}{2}(T_n + M_n) \tag{15}$$

2.3.2 The Accuracy of Numerical Integration

When calculating the integration, order $n = 2^k$, and $2n = 2^{k+1}$, so the $T[k+1]$ can be decided by the formula

$$T[k+1] = (T[k] + M[k])/2 \quad , \quad k = 0,1, \cdots \tag{16}$$

$M[k]$ can be decided by the former formula, and $T[0]$ is given before the calculation begin. Precision control parameter EPS is pre-given before the calculation, when $|T[k+1] - T[k]| \le EPS$, the calculation finishes.

3. RESULT AND ANALYSE

3.1. The Near Circular Orbit in LEO

The initial orbital elements of the first space debris is $[6878.0km, 0.01, 45.0°, 45.0°, 45.0°, 45.0°]$, the area-to-mass ratio is $0.01m^2/kg$.

Calculate the result applying this method of this article, the atmospheric density model is 1976 standard, the time step is 1 day, the calculation lasts 1 year.

The result orbit elements are $[6846.3km, 0.00695, 45.0°, 220.1°, 350.3°, 49.3°]$.

The Nominal results is calculated by STK, applying HPOP(high-precision orbit propagator) mode, the result consider 3-order model of the Earth gravitational field, 1976 standard atmospheric density model, time step is 20 min, the calculation lasts 1 year. The comparison between the result of this paper and STK is as follow.

Fig1~~5 shows the difference between the result of this paper and STK software for the initial orbit above.

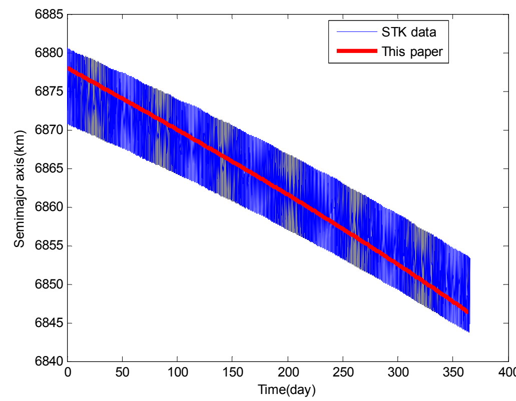

Figure 1. Variation of the semi-major axis (1)

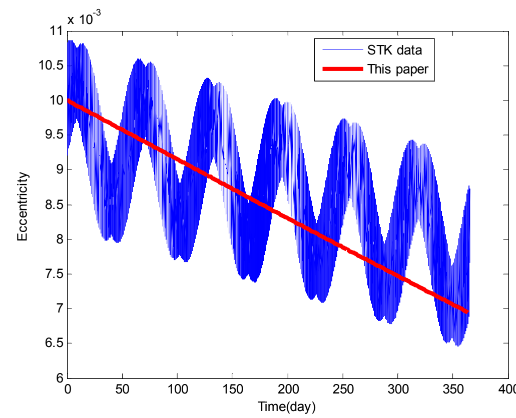

Figure 2. Variation of the eccentricity (1)

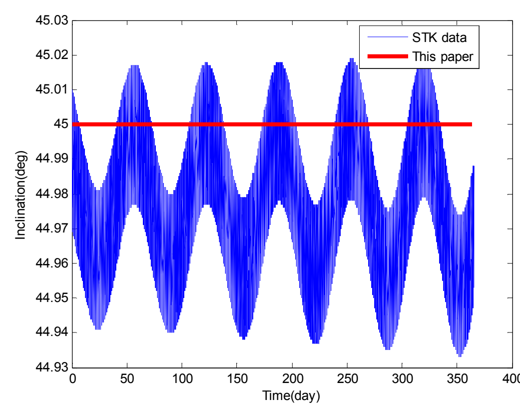

Figure 3. Variation of the inclination (1)

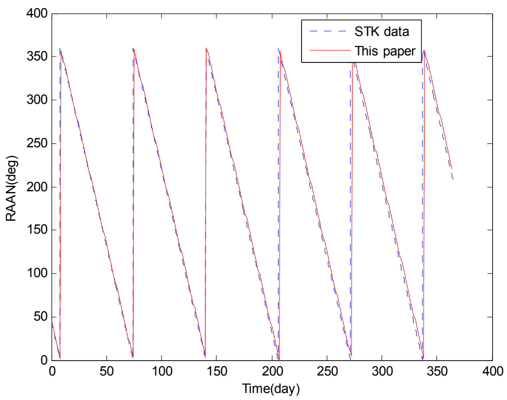

Figure 4. Variation of the RAAN (1)

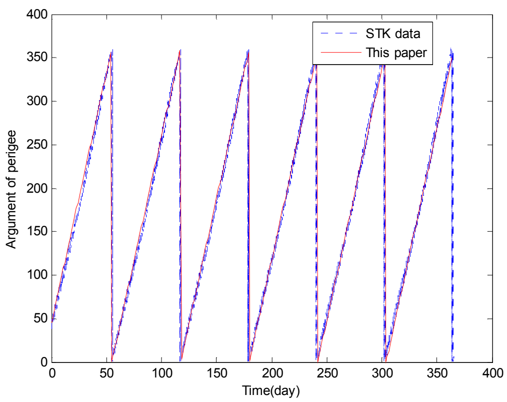

Figure 5 Variation of the argument of perigee (1).

The result shows that the data of this paper is the "average position" of the debris.

3.2. The Large Eccentricity Orbit

The initial orbital elements of the space debris is $[21300.0km, 0.69, 45.0°, 45.0°, 45.0°, 45.0°]$, the area-to-mass ratio is $0.01m^2/kg$ .Calculate the result applying this method of this article, the atmospheric density model is 1976 standard, the time step is 1 day, the calculation lasts 1 year.

The result orbit elements are $[20708.0km, 0.681, 45.0°, 264.1°, 194.5°, 89.1°]$.

The Nominal results is calculated by STK, applying HPOP(high-precision orbit propagator) mode, the result consider 3-order model of the Earth gravitational field, 1976 standard atmospheric density model, time step is 20 min, the calculation lasts 1 year. The comparison between the result of this paper and STK is as follow:

Fig 6~~10 shows the difference between the result of this paper and STK software for the Initial orbit above.

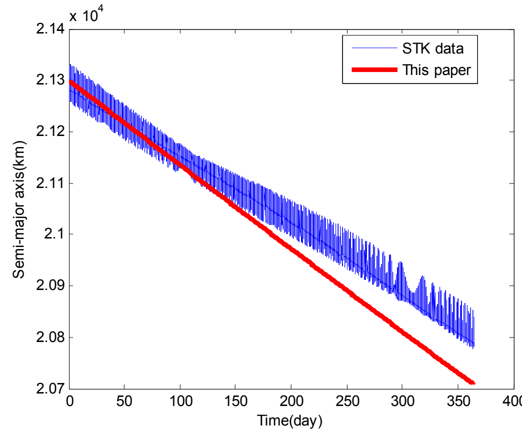

Figure 6. Variation of the Semi-major axis (2)

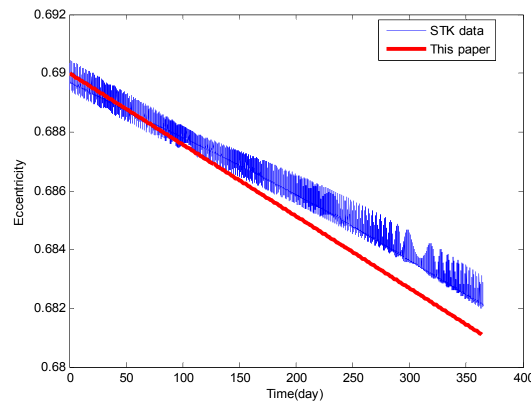

Figure 7. Variation of the eccentricity (2)

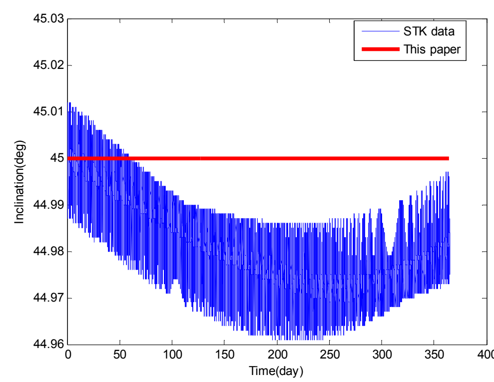

Figure 8. Variation of the Semi-major axis (2)

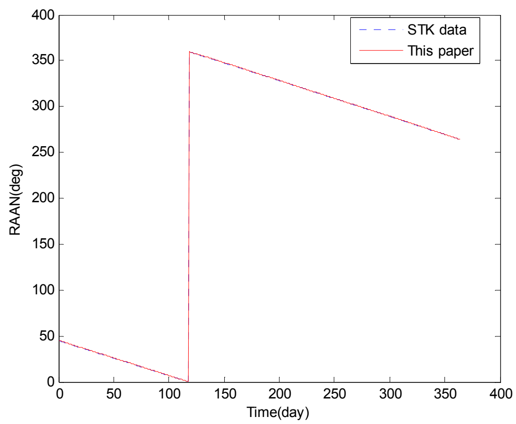

Figure 9. Variation of the eccentricity (2)

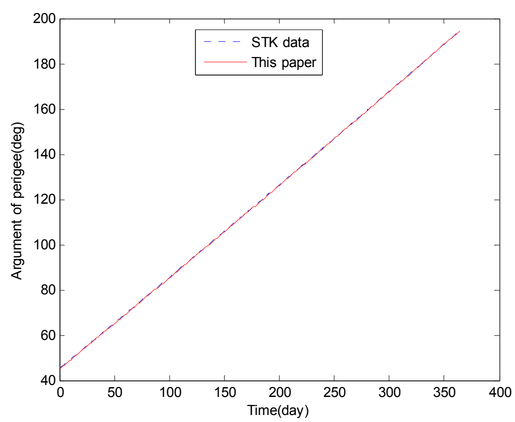

Figure 10. Variation of the argument of perigee (2)

4. CONCLUSION

This paper provides an arithmetic for the long-term orbit propagation of space debris. Based on the space debris motion model, we can educe following conclusions:

1 The atmosphere drag is the most important perturbation in LEO; the area-to-mass ratio is an important parameter of space debris, the greater the area-to-mass ratio is the faster the orbital decay.

2 The propagation of space debris can calculate the "average position" of every debris; this can educe the whole kinematics rule of the space debris environment.

REFERENCES

1. H. Klinkrad. Space Debris: Models and Risk Analysis. Springer, 2006, Published in association with Praxis Publishing Chichester, UK

2. ESA/ ESOC1MASTER 2005 Software User Manual. 2005,ESA,Contract: 18014/03/D/HK(SC)

3. Felix R. Hoots, Ronald L. Roehrich, Models for Propagation of NORAD Element Sets 1988, Spacetrack Report NO.3, NORAD

4. NASA Orbital Debris Program Office , Johnson Space Center , Houston , Texas. The new NASA Orbital Debris Engineering Model ORDEM 2000.

5. Vladimir A. Chobotov, Orbital Mechanics, American Institute of Aeronautics and Astronautics, Inc., Washington, DC, 2003.10 ISBN 1-56347-007-1