

# ESTIMATING SOLAR RADIATION PRESSURE FOR GEO DEBRIS

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## ABSTRACT

Earlier investigations and surveys show that over 700 trackable debris objects are drifting or librating in the GEO (geosynchronous Earth orbit) ring. The close monitoring of the motion of these debris objects is becoming increasingly important due to the potential collision risks to operational GEO satellites. A critical element in improving the ephemeris prediction accuracy of these objects is the determination of solar radiation pressure effects on the orbit. A computational procedure employing specially designed PC programs was developed for estimating the solar radiation pressure effect on GEO debris. Results of three sample cases show significant accuracy improvement in long-term ephemeris prediction. The post-fit residuals ( $\sigma_r=3.0$ ,  $\sigma_i=10$ ,  $\sigma_c=1.0$  km) are in good agreement with that processed by an independent tool (TRACE). The noise in the post-fit residuals is believed to be due to the position uncertainty in the two line elements (TLE) data. This efficient method, once implemented, has the potential of achieving real-time monitoring of the GEO debris objects with position accuracy considerably better than 10 km in-track, 3 km radial and 1 km cross-track.

## NOMENCLATURE

GEO = geosynchronous Earth orbit  
a = semi-major axis of GEO disposal orbit  
e = eccentricity of GEO disposal orbit  
 $\Omega$  = RAAN = right ascension of ascending node  
of GEO disposal orbit  
i = inclination of GEO disposal orbit  
 $\omega$  = argument of perigee of GEO disposal orbit  
M = mean anomaly of GEO disposal orbit  
 $C_R$  = index of surface reflection of the spacecraft  
( $0 < C_R < 2$ )  
A/m = area-to-mass ratio of spacecraft (projected area  
normal to Sun's ray)  
g = gravitational acceleration at sea level =  $9.8 \text{ m/s}^2$ .  
SRP = solar radiation pressure

## 1. INTRODUCTION

According to the 2006 ESOC survey [1, 2], there are 1089 cataloged objects in the GEO ring, with 344 operational satellites. The remaining 745 debris objects are drifting or librating in the GEO ring. A prototype

computing system has been designed and tested for real-time monitoring of GEO debris. In order for the prototype system to achieve its design goals, the primary task is to estimate the solar radiation coefficient ( $C_R \cdot A/m$ ), for each of the 745 uncontrolled debris objects. Currently, the only reliable source of orbit data of those inactive and uncontrolled debris objects in space is the so-called two-line elements (TLE) determined and issued by the U.S. Air Force Space Command [3]. However, the software tool, SGP4, which propagates the TLE to the user specified epoch, does not model solar radiation pressure. As a result, the position accuracy of a GEO object with a sizable value of area-to-mass ratio (A/m) may degrade rapidly after a few days.

The purpose of this study is to examine and demonstrate, via numerical simulations, the concept of estimating the solar radiation pressure coefficient ( $C_R \cdot A/m$ ) by fitting a batch (months) of TLE data. In this study, an efficient method to estimate the solar radiation pressure coefficient based on a batch of (months) of TLE data is developed.

## 2. OBJECTIVES AND APPROACH

The objectives of this study are: 1) to design a prototype software system to determine the solar radiation pressure (SRP) coefficient ( $C_R \cdot A/m$ ) of the uncontrolled GEO objects based on several months of TLE data converted to ECI position vectors by SGP4; 2) to demonstrate the improved ephemeris prediction accuracy by using an efficient batch least squares method with a UD (Upper Diagonal) filter; 3) to compare the results from the above system with an independent tool (TRACE).

It is known that the position vector at epoch of a space object determined or computed from a TLE via SGP4 is more accurate than the corresponding velocity components, which are affected by the use of an older Earth gravity model (WGS72). Due to the use of an older gravity model and simplified analytical theory (not considering solar radiation pressure effects), the accuracy of an SGP4-predicted position degrades rather quickly after several days.

Based on the above understanding, the concept of this approach is to treat the SGP4 computed positions at epoch

as observations, and to perform differential corrections of the initial state and SRP coefficient with high-precision numerical integration tools. The refined initial state and the SRP coefficient may significantly improve the accuracy of long-term GEO debris ephemeris prediction

### 3. DATA PREPARATION AND VERIFICATION

A data processing diagram is shown below (Fig 1) for the illustration of data preparation for this study.

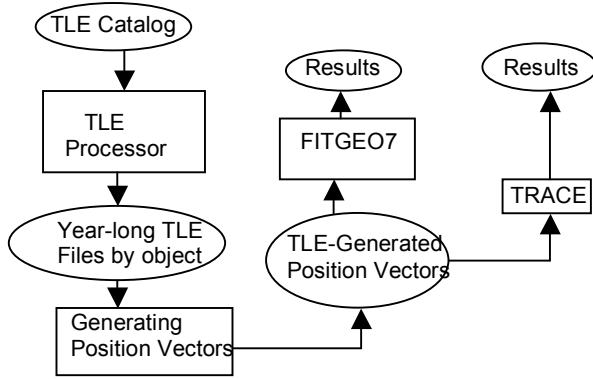


Figure 1 Data flow diagram

A TLE processor developed by The Aerospace Corporation [Ref. 4] sorts out Two-line elements (TLE) for selected GEO debris objects from the TLE catalog and creates a batch TLE file up to a year from the current time. A slightly modified SGP4 converts the TLE batch file into an array of position vectors at or near the epoch of each TLE. These TLE-generated position vectors are treated as observations for determining the SRP coefficient for each debris object.

An orbit determination program called FITGEO7 performs least squares fitting of the array of TLE-generated position vectors. The force models in FITGEO7 include: EGM/WGS84 8x8 Earth gravity, sun-moon gravity and solar radiation pressure (flat plate). A simple batch-least-squares filter using Bierman's UD algorithm [5] efficiently estimates the seven parameters:  $x_0$ ,  $y_0$ ,  $z_0$ ,  $v_{x0}$ ,  $v_{y0}$ ,  $v_{z0}$  and the SRP coefficient ( $C_R \cdot A/m$ ). A-priori covariance, measurement variance, initial value of SRP, epoch and fit span are required. Post-fit residuals (observed – computed) in position errors are rotated to radial, in-track and cross-track and printed out sequentially on the PC screen as well as to an output file. The user can examine the residuals in an interactive mode while the computer is processing and computing the data. The estimated SRP coefficient is determined until the above orbit determination process is converged after several iterations.

An independent tool, TRACE, is used for verifying the results of FITGEO7. The Aerospace Corporation's

trajectory analysis and orbit determination program (TRACE) [6] is used to fit the same array of TLE-generated position vectors. TRACE uses the same perturbing forces: Earth gravity harmonics, lunisolar attractions and solar radiation pressure. The solar radiation pressure coefficient in TRACE is called CPAW and it is defined as:

$$CPAW = 4.65 \times 10^{-6} \cdot (C_R \cdot A/m)/g \quad (1)$$

where  $A/m$  is area-to-mass ratio in  $m^2/kg$ ,  $C_R$  is index of surface reflection and  $g$  is the gravitational acceleration at sea level ( $9.8 m/s^2$ ).

### 4. THREE SAMPLE CASES

Three aged GEO debris objects were arbitrarily selected from the two-line elements (TLE) catalog as testing cases. The mean orbit parameters were:

Object #1  $a = \text{near GEO}$ ,  $e = 0.000469$ ,  $i = 8.95 \text{ deg}$ ,  
 $\Omega = 307.05 \text{ deg}$ ,  $\omega = 144.48 \text{ deg}$

Object #2  $a = \text{GEO}+1190 \text{ km}$ ,  $e = 0.0128$ ,  $i = 15.32 \text{ deg}$ ,  
 $\Omega = 4.54 \text{ deg}$ ,  $\omega = 147.91 \text{ deg}$

Object #3  $a = \text{GEO}+1385 \text{ km}$ ,  $e = 0.00474$ ,  $i = 14.43 \text{ deg}$ ,  
 $\Omega = 4.54 \text{ deg}$ ,  $\omega = 147.91 \text{ deg}$

The ages of the above three debris objects were approximately estimated from the inclination vector space as shown in Figure 2.

### 5. RESULTS OF FIT AND PREDICTION

Figures 3, 4 and 5 show the post-fit residuals of the three GEO debris objects, #1, #2 and #3, respectively. Residuals in the in-track direction, as marked by red dots, always have the largest amplitude. The residuals along radial (blue) and cross-track (green) directions have

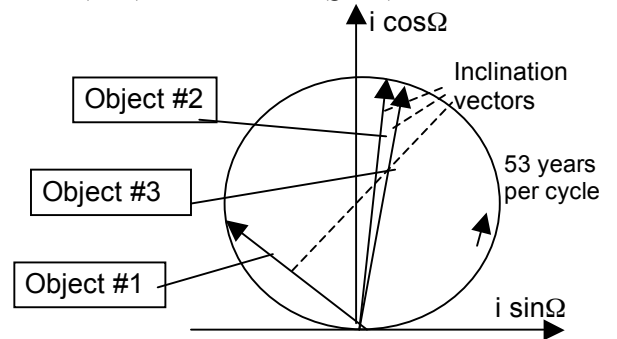


Figure 2 Inclination vectors of the three debris objects

smaller amplitudes. The post-fit residuals from orbit determination solutions using TRACE are shown in the following three figures (Fig 6, 7 and 8). These residuals

are along the X (blue), Y (red) and Z (green) components in the Earth fixed coordinates. The characteristics and amplitudes of the residuals from FITGEO7 are in good agreement with those from TRACE. The one-sigma noise ranges from around 10 km along the in-track direction, 3 km along the radial and 1 km along the cross-track directions. These types of noises are typical for the TLE data (at epoch) at GEO altitude.

The estimated values of SRP coefficient ( $C_R \cdot A/m$ ) of the three objects are: 0.0192 for Object #1, 0.0242 for Object #2 and 0.0282 for Object #3. The unit is  $m^2/kg$ .

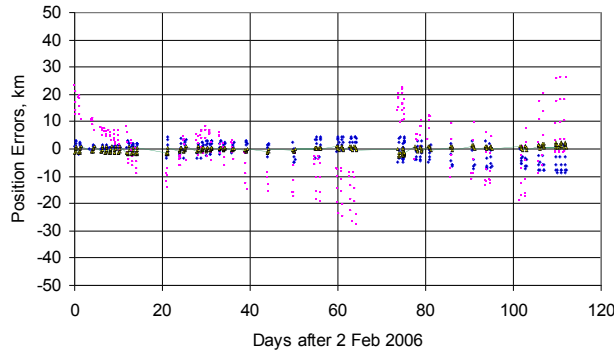


Figure 3 Post-fit residuals of object #1

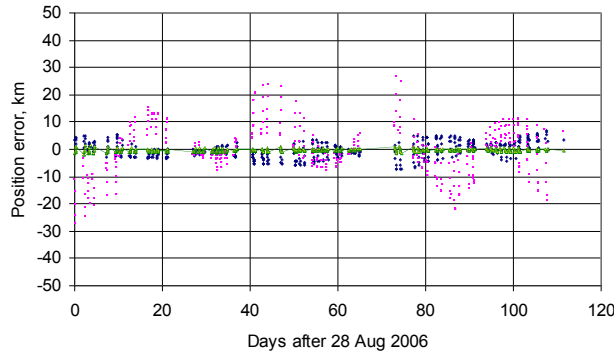


Figure 4 Post-fit residuals of Object #2

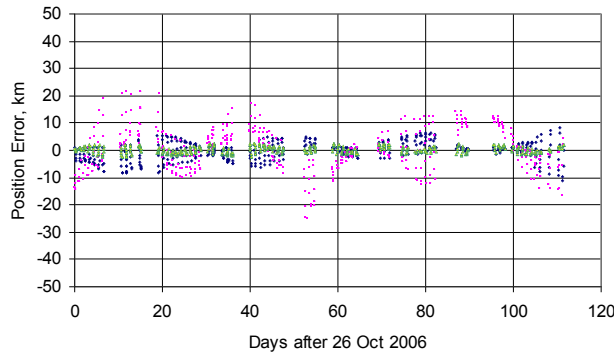


Figure 5 Post-fit residuals of object #3

Assuming a common value of 1.3 for  $C_R$ , the estimated area-to-mass ratios for the three objects are 0.0148,

0.0186 and 0.0217  $m^2/kg$  for Object #1, #2 and #3 respectively. These values are based on 80-day fit and they have small variations at different fit spans. It should be pointed out that the actual purpose of this study is not to study the physical property of the debris objects, but the main goal is to significantly improve the accuracy of long-term position prediction by estimating SRP.

A parallel study of the tumbling effects on estimating SRP is reported in a separate paper by Chao [7]. The effects are found to be small when tumbling rate is different from the orbit rate.

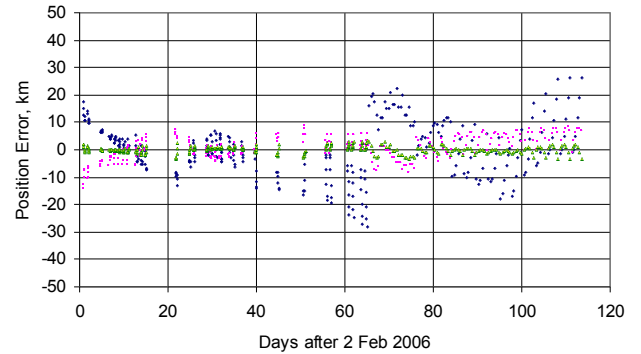


Figure 6 Post-fit residuals of Object #1 using TRACE

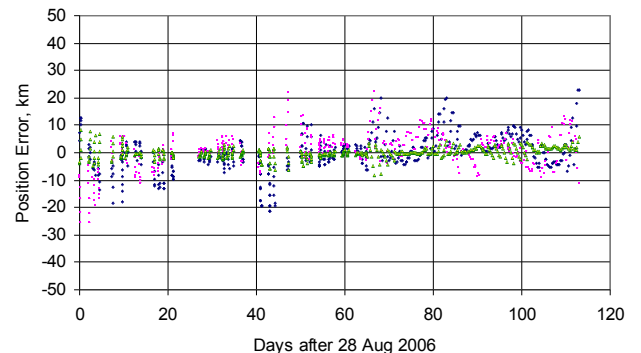


Figure 7 Post-fit residuals of Object #2 using TRACE

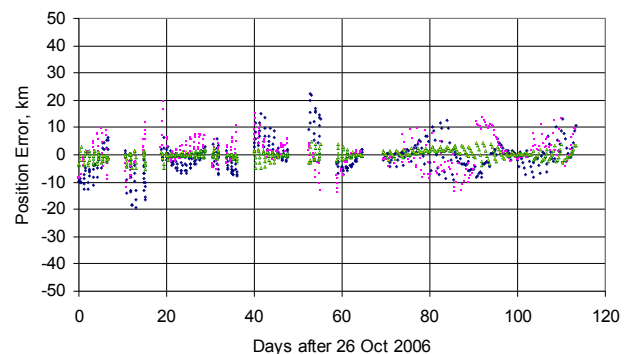


Figure 8 Post-fit residuals of Object #3 using TRACE

The next step is to see how much improvement there is in position prediction accuracy based on the long-arc fit of

the TLE data using high-precision numerical computation. Figure 9 shows the position deviations from the truth (TLE generated positions at epochs) based on numerical integration (FITGEO7) using a single TLE (not the initial state after fit). It clearly shows that the error along the in-track direction (red dots) grows to 400 km after 50 days. The in-track error can grow to as large as 700 km when the same TLE is propagated to 50 days (marked by light blue X in Fig 10) using the simplified analytical propagator, SGP4, which does not model solar radiation pressure.

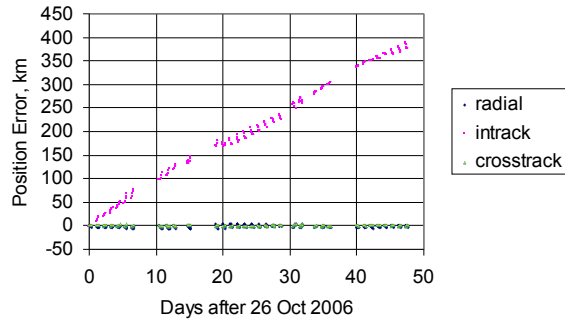


Figure 9 Position errors of prediction based on a single TLE using high-precision tool (FITGEO7)

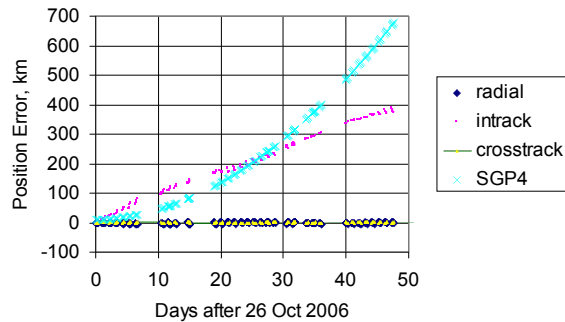


Figure 10 Comparison of position errors of prediction using a single TLE with FITGEO7 and SGP4

Figures 11 to 14 show the significant improvements in long-term prediction accuracy when the predictions are based on an initial state determined using 30- to 180-day fit. The signatures of the position errors in the prediction span of both cases (50 day prediction and the 82-day prediction) are very much similar to that of the post-fit residuals inside the fit span. This comparison clearly shows that the solar radiation pressure effect has been well modeled and there is no secular growth in in-track error. The similarity in magnitude and characteristics of the data noise between the fit span and prediction span also suggests that the true accuracy of the long-term prediction is much less than the 10-km noise of the TLE data. In other words, any prediction errors greater than, say, 3 to 5 km would cause noticeable increase in data

noise in the prediction span. With the much improved prediction accuracy, the potential application of this concept for GEO debris collision analysis looks promising.

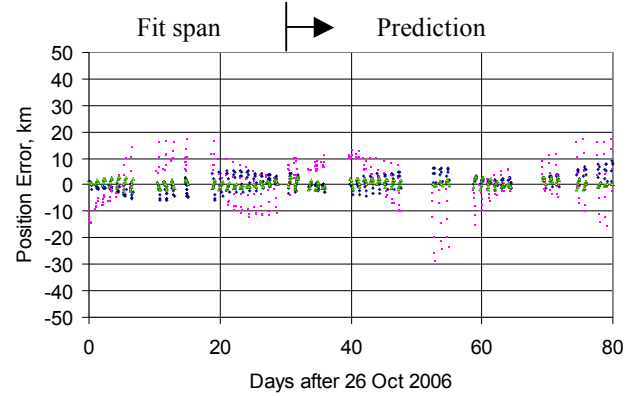


Figure 11 50-day prediction based on a 30-day fit (Object #3)

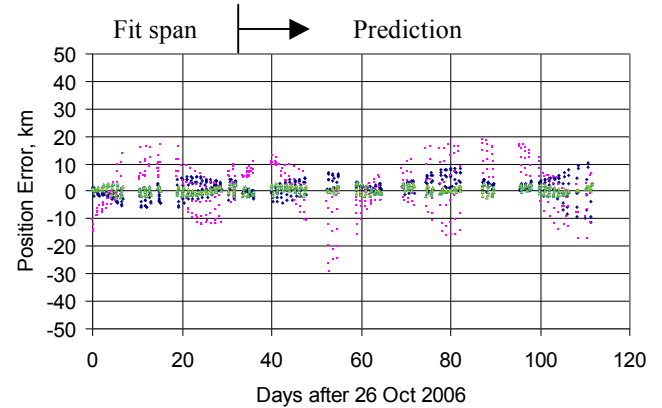


Figure 12 82-day prediction based on a 30-day fit (Object #3)

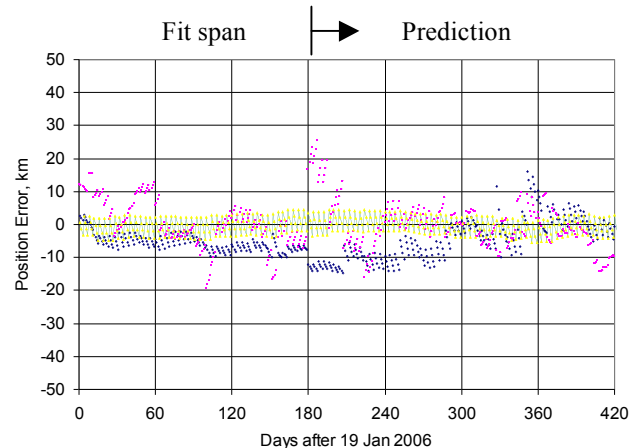


Figure 13 240-day prediction based on 182-day fit

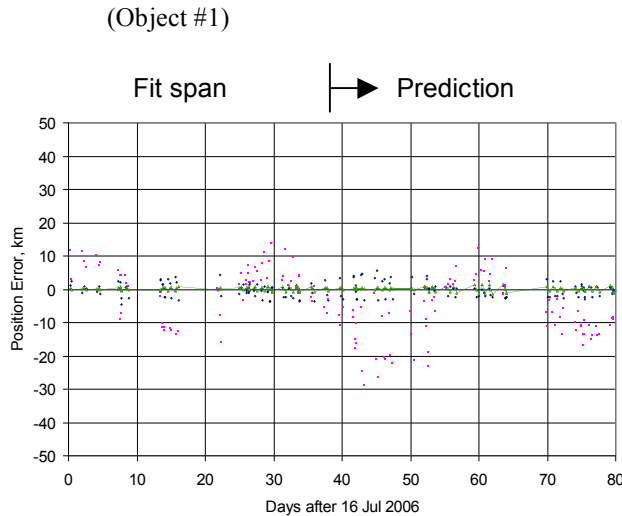


Figure 14 40-day prediction based on 40-day fit  
(Object #2)

## 6. CONCLUSIONS

A computational procedure employing specially designed PC programs was developed for estimating solar radiation pressure effect on GEO debris. Results of three sample cases show significant accuracy improvement in long-term ephemeris prediction. The post-fit residuals ( $\sigma_r=3.0$ ,  $\sigma_i=10$ ,  $\sigma_c=1.0$  km) are in good agreement with those processed by an independent tool (TRACE). The noise in the post-fit residuals is believed to be due to the position uncertainty in the TLE data. This efficient method, once implemented, has the potential of achieving real-time monitoring of the GEO debris objects with position accuracy considerably better than 3 km radial, 10 km in-track, and 1 km cross-track.

## 7. RECOMMENDATIONS FOR APPLICATIONS

The promising improvements in long-term ephemeris prediction call for continued studies as recommended below:

- 1) The 30-day fit may be performed once a month for each of the 700 plus GEO debris objects; the fit span should be optimized for best prediction accuracy for each object.
- 2) The apparent accuracy of this method is limited by the uncertainty of the TLE data, and the true accuracy should be verified by comparing with an active GEO satellite using high-precision ranging or GPS measurements.
- 3) The accuracy might be improved if the TLE produced observations (position vectors) are replaced by the SP (special perturbation) data or

other types of GEO tracking and orbit determination data.

- 4) A sensitivity study using simulated observations should be performed to understand the optimal fit span for estimating solar radiation pressure effects for a slowly tumbling object.

## ACKNOWLEDGEMENTS

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