THE OPTIMUM PLANNING OF SEARCH FOR A SPACE OBJECT TAKING INTO ACCOUNT THE TEMPORAL STRUCTURAL TRANSFORMATION OF THE SPACE OBJECT CURRENT POSITION UNCERTAINTY DOMAIN

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ABSTRACT

At the First European Conference on Space Debris (Darmstadt, 1993) a presentation was made on development of the theory for optimum planning the search for a space object by rough a priori orbital information with the help of narrow-angle and narrow-beam facilities [1]. The kernel of this theory is the principle of equivalence of the possible search plan elements for different times [2]. Then the theory was developed up to the constructive methods and working search programs for one important case – namely for the assumption of predominant state vector error propagation along the track and neglecting the other errors [3]. As a result, some programs for optimum planning the search for highly elliptical space objects were worked out. These programs were successfully implemented many years ago at the Irkutsk optical station and in 2003 at the electro-optical complex "Okno" in Tajikistan. The report on the test results of these programs was presented at the 4th European Conference on Space Debris in Darmstadt [4]. In this paper the theory and methods are generalized onto the most common case of the state vector error character, the optimum properties of methods being retained.

1. THE COMMON CASE OF THE STATE VECTOR ERROR CHARACTER

However, there often appears the necessity of detecting a space object having the essential state vector errors in different directions in the phase space. The examples of such situations are as the next:

- the presence of very rough a priori orbital information before the launch or after the unsuccessful launch;
- the initial orbit determination by rough measurements or by those spread on the very short arc of orbit;
- availability of a very narrow-angle or narrow-beam sensor to be used for the search (and so even small errors).

In the case of essential state vector errors in different directions (not only along the track) application of the search theory and the equivalence principle is much more complex. The more so, to say strictly, this principle looses its former sense. So, the generalization of the equivalence principle and the further extension of the search theory are needed for the most common case of the state vector error character.

The generalization of the equivalence principle is given in a special poster paper [5]. And here consider how to perfect the search strategy meeting the new complication of the state vector error character.

It will be recalled that under the former conditions (respecting the state vector error only along the track) the optimum planning procedure could be very simplified. It comes to somewhat simple operations in the plane *ut*, where *u* is the argument of latitude and *t* is time (Figs.1,2). Such a comfort was achieved only owing to simplicity of treatment of the equivalence principle under the conditions above.

In the commonest case, as it was shown in [5], the equivalence principle as the main tool of the optimum planning the search acquires more complicated form. And correspondingly, making use of it calls for special skill.

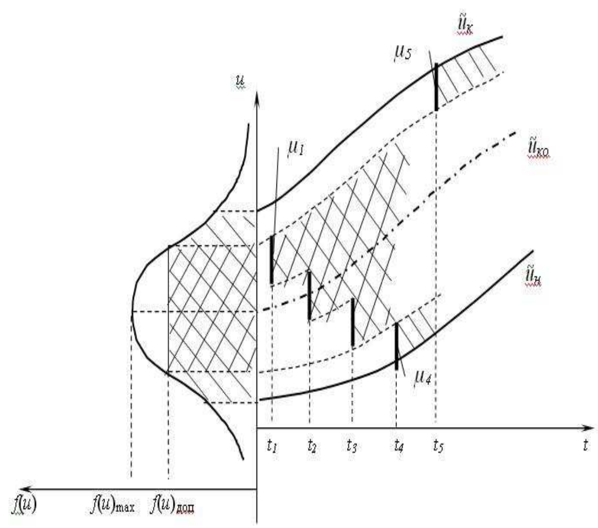

Figure 1. Optimum planning for the classic case 1.

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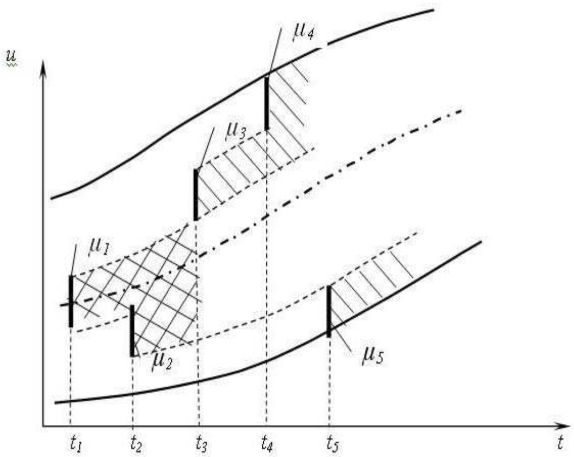

Figure 2. Optimum planning for the classic case 2.

2. FORMULATION OF THE PROBLEM OF OPTIMUM PLANNING THE SEARCH FOR A SPACE OBJECT, AND ON SOME DIFFICULTIES OF ITS SOLUTION FOR THE COMMON CASE OF THE STATE VECTOR ERROR CHARACTER

So, let the mathematical expectation of the sought for space object state vector error be given in 6-dimentional phase space X_6 at the initial time t_0 (Eq. 1)

$$R_0(t_0) = (X_0, Y_0, Z_0, Vx_0, Vy_0, Vz_0)^T \qquad (1)$$

and the corresponding covariance matrix (Eq. 2)

$$K_0(t_0) = \begin{vmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} & \sigma_{XVx} & \sigma_{XVy} & \sigma_{XVz} \\ 0 & \sigma_Y^2 & \sigma_{YZ} & \sigma_{YVx} & \sigma_{YVy} & \sigma_{YVz} \\ 0 & 0 & \sigma_Z^2 & \sigma_{ZVx} & \sigma_{ZVy} & \sigma_{ZVz} \\ 0 & 0 & 0 & \sigma_{Vx}^2 & \sigma_{VxVy} & \sigma_{VxVz} \\ 0 & 0 & 0 & 0 & \sigma_{Vy}^2 & \sigma_{VyVz} \\ 0 & 0 & 0 & 0 & 0 & \sigma_{Vz}^2 \end{vmatrix}. \qquad (2)$$

Assuming some natural restrictions (not strong enough) this a priori orbital information sets at the moment t_0 a limited compact domain - the sought for space object current position uncertainty domain (CPUD) $D_6(t_0)$. The CPUD is a 6-dimentional ellipsoid with the given probability distribution density function defined on it.
Due to the celestial mechanics laws, when time changes each point $R_6(t_0) \in D_6(t_0)$ is transferred with the help of the homeomorphic mapping F ($F \equiv F_6$) into another point $R_6(t) \in X_6$ with the state vector and covariance matrix parameters corresponding to the time t. If $t = t_1$, then

$$R_6(t_1) = F\,[t_0, R_6(t_0), t_1\,]. \qquad (3)$$

Conformably,

$$D_6(t_1) = \bigcup_{R_6 \in D_6(t_0)} F[t_0, R_6(t_0), t_1] = F\,[t_0, D_6(t_0), t_1\,]. \qquad (4)$$

Let t_1 be the time of beginning the search for the space object. Then according to the equivalence principle [1, 5] checking the point $R_6(t_1)$ at the time t_1 is equivalent to checking the point $R_6(t_2)$ at the time t_2 (any other time moment) in the sense that there is no need to check both points. It is enough to check only one of the two equivalent points. ("To check" means to learn whether the sought for space object is present in the point or absent.)
In other words the mapping $F \equiv F_6$ is one-to-one and to-and-fro continuous operator. So, if a search were carried out in 6-dimentional space, application of the equivalence principle for optimum planning the search would not cause any difficulties. But in real observation practice one has to make a search in 3-dimentional space and for optical sensors in 2-dimentional one – that is in the picture plane (PP) which by the way is moving itself and changing its orientation in space.
When projecting the realized part of the search plan into PP and transforming it in space with change of time the corresponding projection of the mapping F looses the property of homeomorphism [5]. That is why (as shown in [6]) after transition from t_1 to t_2 (the next time moment of observation) the point $A(t_1) = R_2(t_1)$ is transferred not into a point but into some "smear" id est some little domain in PP (Fig. 3).
So, if at the time t_1 in the corresponding PP (PPt_1) some square domain $d_2(t_1)$ (conforming by its form and size to the telescope field of view) was observed, then in PP corresponding to the time t_2 (PPt_2) this domain will be transformed into the distorted square having "washed away" boundaries (Fig. 4).
The character of "washing away" or "smearing" the boundaries is affected by a sum of factors including the state vector evolution and its error propagation, change of the CPUD foreshortening, change of the PP orientation, projecting the 6-dimentional CPUD, its covariance matrix and the mapping F into the PP of the observer. This fact should be taken into account when placing the next element of the search plan to PPt_2.
The difficulties of calculation of F_2-image (in PPt_2) of already realized part of the search plan are accounted by the following causes.

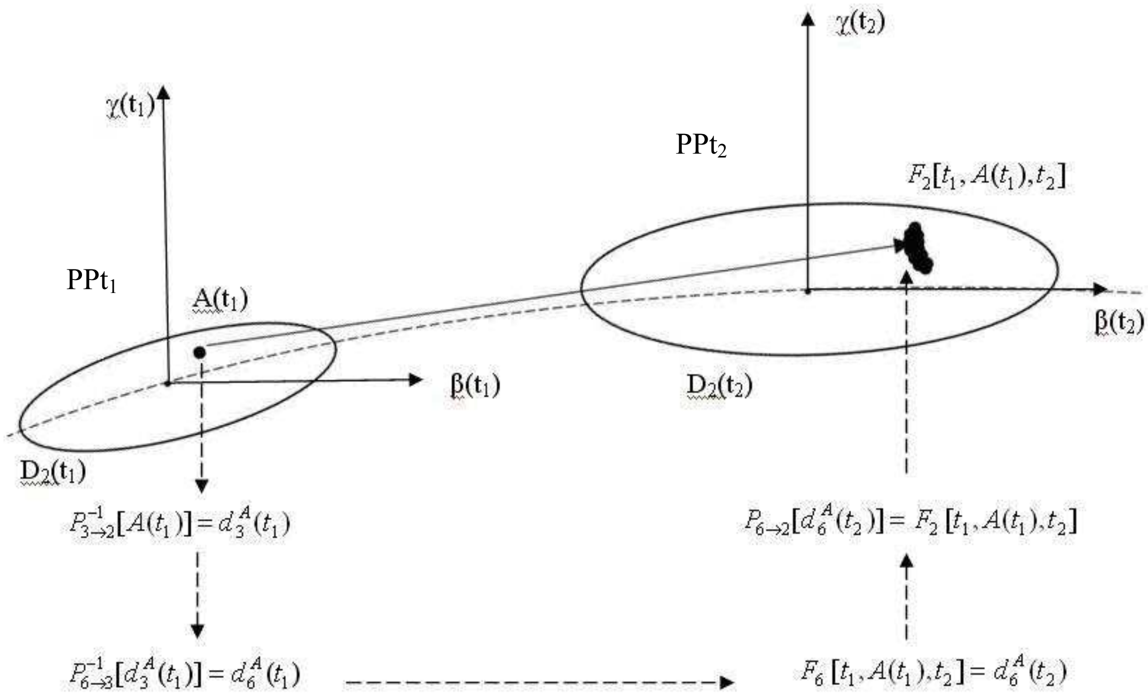

Figure 3. Temporal transformation of a point in the picture plane

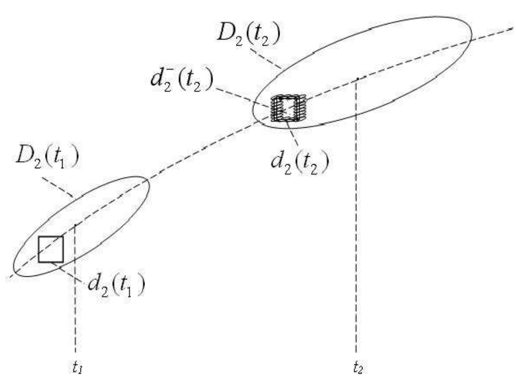

Figure 4. Temporal transformation of the telescope field of view image in PP

Any point $A(t_1)$ in PPt_1 (namely, in the uncertainty ellipse) is a projection of some segment $[A_b(t_1); A_e(t_1)]$ in 3-dimentional space limited by the surface of the 3-dimentional uncertainty ellipsoid. The projection is fulfilled along the sight axis.

F_2-image of $A(t_1)$ in PPt_2 is non other than F_2-image of this segment. For constructing F_2-image of $A(t_1)$ in PPt_2 it is necessary to transfer all the points of the segment $[A_b(t_1); A_e(t_1)]$ into PPt_2 with the help of mapping F and projection. This operation could be fulfilled easily (at any rate theoretically) if all points were given as 6-dimentional vectors having the single meanings. But in reality that is not like this.

For example, the vector corresponding to the beginning of the segment (id est to the point $A_b(t_1)$ of the 3-dimentional ellipsoid) in 6-dimentional phase space X_6 looks like Eq. 5

$$R_{Ab}(t_1) = \begin{pmatrix} X_{A_b}(t_1) \\ Y_{A_b}(t_1) \\ Z_{A_b}(t_1) \\ \left\{\dot{X}_{A_b}(t_1)\right\} \\ \left\{\dot{Y}_{A_b}(t_1)\right\} \\ \left\{\dot{Z}_{A_b}(t_1)\right\} \end{pmatrix}, \qquad (5)$$

where the braces mean that the corresponding vector component has not a single value but a set of possible values. For the rest points of the segment the case is similar.

How to solve this problem of non-single-values and to map and project such packet of vectors is shown in [6].

3. THE STRATEGY OF OPTIMUM PLANNING THE SEARCH FOR A SPACE OBJECT

Founding upon this solution a model for temporal structure transformation of CPUD was developed. It allows planning the search for a space object in interactive mode taking into account all this complex process.

Making use of this model allows revealing some useful properties of this process. With due regard for them one has the possibility to reduce and even neutralize many negative consequences of transition from 6-dimentional phase space to real 3- and 2-dimentional embodiment of the search planning space and the search plan itself.

Firstly, upper and lateral boundaries of the field of view in PP are washing away differently during the transition from PPt_1 to PPt_2 , PPt_3 , …, PPt_n , … and so on. The intensity of the washing away process in some direction depends on the content of errors of the state vector and the rates of their changes in different directions. With due regard for the model-computed character of washing away boundaries of already constructed search plan elements, the observer can choose the correct location for the next element position. It is advisable to adjoin it to the boundary being washed away most intensively. This is the first prompting to the observer.

Secondly, washing away of the adjacent boundaries of already constructed search plan elements has no negative influence on the quality of the search plan and consequently on the effectiveness of the sought for space object detection. So, such adjoining boundaries may be ignored and one should concentrate only on the open boundaries. This is the second prompting to the observer.

There are some other helpful properties utilization of which makes the search plan construction more perfect and effective.

Below, the next suboptimum (really very close to optimum) strategy of planning the search for a space object is proposed (Fig. 5).

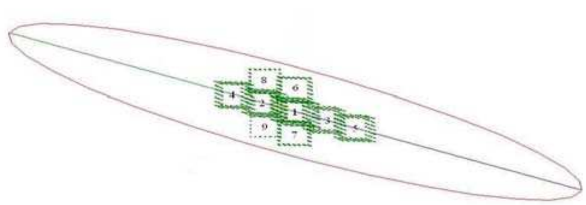

Figure5. Suboptimum strategy of planning the search for a space object

The first plan element corresponding to the time moment of beginning the search t_1 is to be placed at the center of CPUD in PPt_1 (normal to the sight axis) where the probability of detecting the space object is greatest. Then with the time discreteness Δt all the following plan elements are placed at the area of greatest detection probability owing to the probability distribution density function with due regard for the useful CPUD transformation properties, the adjacent smeared boundary of the earlier set element being overlapped. The model calculates the measure of such overlapping.

The value of the time step Δt depends on the exposure time interval for the intelligence signal energy accumulation, the electronic still-scanning time, the time of retargeting the telescope and so on.

If there are some alternatives the next plan element is to be adjacent to the most washed away boundary of one of the preceding elements.

Such a procedure for the search plan synthesis minimizes the mathematical expectation of the detection time with guarantee of the sought for space object detection.

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