LONG-TERM PERIGEE HEIGHT VARIATIONS OF GEO DISPOSAL ORBITS – A REVISIT

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ABSTRACT

Earlier investigations suggest small initial eccentricity for improved orbit stability of the disposal orbits at super synchronous altitudes. Results of a recent study show that the IADC guideline for disposal of objects in geosynchronous orbits (GEO), which is plus no less than 235 km for perigees of disposal orbits, is appropriate if the initial eccentricity is less than 0.005. According to the ground operations of several GEO missions, the current communications satellites are capable of achieving a final eccentricity of 0.0005 or better for their disposal orbits. This study revisits the problem based on more realistic orbit insertion capability and a high-precision orbit propagation tool, TRACE. Results of analytical investigation and 100-year numerical integration led to two important findings: 1) long-term stability of GEO disposal orbits can be further improved by keeping perigee sun-pointing and initial eccentricity less than 0.0005, 2) the IADC guideline is adequate for maintaining a minimum perigee at 250 km higher than GEO if condition 1) is followed. Long-term perigee variations of inactive objects near GEO altitude were examined in this study.

NOMENCLATURE

GEO = geosynchronous orbit
a = semi-major axis of GEO disposal orbit
e = eccentricity of GEO disposal orbit
$\gamma = n_3^2 R_m/n$
$s = (1-e^2)^{1/2}$
n = mean motion of GEO disposal orbit
n_3 = mean motion of the third body
R_m = mass ratio = 1 for solar perturbation
= 1/82.3 for lunar perturbation
Ω = RAAN = right ascension of ascending node of GEO disposal orbit
$\Delta\Omega = \Omega - \Omega_3$
Ω_3 = Moon's right ascension of ascending node
i = inclination of GEO disposal orbit
i_3 = inclination of the Sun or Moon
ω = argument of perigee of GEO disposal orbit
M = mean anomaly of GEO disposal orbit
$F= S(A/m)P(a_{sun}/r_{sun})2/\mu$
ε = obliquity of the ecliptic
λ_{sun} = ecliptic longitude of the Sun
S = index of surface reflection of the spacecraft $(0 < S < 2)$
A/m = area-to-mass ratio of spacecraft (projected area normal to Sun's ray)
P = solar flux at 1 AU (4.65×10^{-6} Newton/m^2)
a_{sun}, r_{sun} = semi-major axis and radius of the Sun's orbit, respectively
μ = Earth gravitational constant = 398600.4418 km^3/sec^2

1. INTRODUCTION

Earlier investigations [1 and 2], based on 100 to 200 year numerical integrations, reached similar conclusions that GEO disposal orbits were stable by keeping initial eccentricity reasonably small, i.e., less than 0.005. Long-term (10 to 12 year) sinusoidal variations in eccentricity were shown to be caused by lunisolar attractions. Minor sensitivities to initial RAAN, argument of perigee and epoch were also found. The IADC guideline (ΔH=235 + 1000 S A/m km) for determining initial GEO disposal perigee is appropriate if the initial eccentricity is less than 0.005 [2].

Recent studies performed at The Aerospace Corporation on GEO disposal orbit raising procedures revealed some interesting facts. Long-term eccentricity variations can be significantly improved by keeping the initial perigee Sun pointing. An independent study by a French researcher reached the same conclusion [3]. Aerospace results also indicate that the optimum window for disposal orbit insertion with a Sun-pointing perigee depends on the season of the year. During the course of the above studies, it was learned from various GEO mission operations (NASA, DoD and commercial) that a 0.0005 eccentricity can be and has been achieved for their disposal orbits.

The purpose of this analysis is to revisit the long-term perigee stability based on the above findings. Through analytical and numerical investigations, the IADC (Inter-Agency Space Debris Coordination Committee) guideline for computing initial perigee height increase can be more accurately verified with realistic initial parameters. A high-precision numerical integration tool, TRACE [4], is employed to generate 100-year perigee altitude histories. This paper also includes the

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predicted long-term perigee variations of three groups of inactive satellites/objects. These three groups are: disposed GEO satellites with adequate and inadequate perigee increase, and objects abandoned in the GEO libration zones.

2. ANALYTICAL INVESTIGATION

2.1 Eccentricity variations induced by third-body attractions

In order to understand the long-term orbit perturbations and stability of GEO disposal orbits, the doubly averaged equation in eccentricity is derived and analyzed as in Reference 5. The closed-form doubly averaged equation in eccentricity due to third-body perturbations [5] is given below.

$$\begin{aligned} de/dt = -(15/8)e\Sigma\gamma s[&C1 \sin 2(\omega-\Delta\Omega) \\ &+ C2 \sin(2\omega-\Delta\Omega) + C3 \sin 2\omega \\ &+ C4 \sin(2\omega+\Delta\Omega) + C5 \sin 2(\omega+\Delta\Omega)] \end{aligned} \quad (1)$$

where the Σ sign is to sum over the terms for the Sun and Moon. The five coefficients, C1,.., C5, are functions of the inclinations of the GEO and third body with the following forms [5]:

$$\begin{aligned} C1 &= ½ \sin^2 i_3 (\cos i + 1/2 \sin^2 i - 1) \\ C2 &= ½ \sin i \sin 2i_3 (\cos i - 1) \\ C3 &= \sin 2i (3/2 \sin^2 i_3 - 1) \\ C4 &= ½ \sin i \sin 2i_3 (1 + \cos i) \\ C5 &= ½ \sin^2 i_3 (1/2 \sin^2 i - \cos i - 1) \end{aligned} \quad (2)$$

The orbit of the third body is assumed to be circular, and it has been shown that the circular orbit approximation causes no noticeable degradation in long-term propagation accuracy [1]. For very long-term integration (>> 18.6 years), the right ascension of the ascending node of the Moon may be set to be zero and the angle $\Delta\Omega$ can be replaced by Ω. After substituting the GEO disposal inclination (average=7deg) and inclination of the third body (23.5 deg for both the Sun and Moon), the resulting equation assumes the following simplified form.

$$\begin{aligned} de/dt = -(15/8)e\Sigma\gamma s[&-0.0000022 \sin 2(\omega - \Omega) \\ &- 0.000332 \sin(2\omega - \Omega) - 0.1842 \sin 2\omega \\ &+ 0.089 \sin(2\omega + \Omega) - 0.158 \sin 2(\omega + \Omega)] \end{aligned} \quad (3)$$

The rate of argument of perigee, $d\omega/dt$, and the rate of the node, $d\Omega/dt$, of a GEO disposal orbit are, approximately, 0.03 deg/day and –0.008 deg/day. The periods of the last three terms in the above equation vary from 16 to 23 years based on simple conversion from the three combined rates. The above equation (Eq. 3) indicates that the third-body attractions coupled with the secular J_2 effects are responsible for the long-period (10 to 12 year) variation in eccentricity with amplitude of variation proportional to initial eccentricity. These findings have been reported in Reference 1 and will be further confirmed later by numerical integration results.

2.2 Eccentricity variations caused by solar radiation pressure

Following Reference 6, the averaged equations in eccentricity and argument of perigee due to solar radiation pressure can be expressed by the following equations after isolating the dominant terms.

$$de/dt = -(3/2)na^2(1-e^2)^{1/2}F\cos^2(\varepsilon/2) \sin(\lambda_{sun} - \omega - \Omega) \quad (4)$$
$$d\omega/dt = (3/2)na^2(1-e^2)^{1/2}F\cos^2(\varepsilon/2) \cos(\lambda_{sun} - \omega - \Omega)/e$$

The above equations can be better illustrated by defining a new angular variable ϕ such that

$$\phi = \lambda_{sun} - \omega - \Omega \quad (5)$$

Then the two equations become

$$\begin{aligned} de/dt &= -g \sin\phi \\ d\phi/dt &= z - (g/e) \cos\phi \end{aligned} \quad (6)$$

where

$$\begin{aligned} g &= (3/2)na^2(1-e^2)^{1/2}F\cos^2(\varepsilon/2) \\ z &= d\lambda_{sun}/dt - d\Omega/dt \end{aligned}$$

Because $e^2 \approx 0$ and $d\lambda_{sun}/dt >> d\Omega/dt$, both g and z can be assumed to be positive constants. The above set of equations implies that ϕ may librate about 0 deg depending on the initial value of eccentricity. For instance, if $\phi = 0$ and $e = g/z$, then e and ϕ will stay constant at their initial values. After eliminating the independent variable t, Eq. 6 can be integrated to

$$e^2 - (2g/z)e \cos\phi = K \quad (7)$$

where $K = e_0^2 - (2g/z)e_0 \cos\phi_0$ is the constant of integration determined from the initial conditions. The above equation (Eq. 7) can be represented by a circle in the (e, ϕ) space as shown in Fig 1.

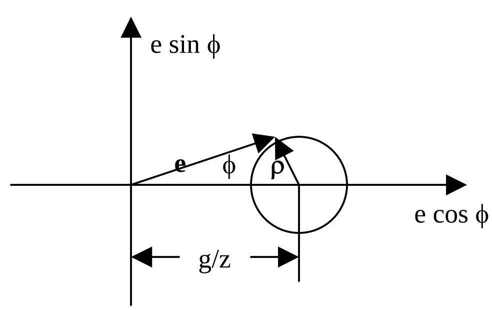

Figure 1. Eccentricity vector in e, • space

The eccentricity vector, **e**, moves along the circle (counter clockwise) at the rate of one rev per year. The radius of the circle, ρ, is a function of initial conditions. The yearly variation of eccentricity depends on the size of ρ. By letting e_0=g/z and ϕ_0=0, ρ is zero and the eccentricity will remain constant and equal to g/z, the eccentricity induced by solar radiation pressure. When ϕ_0=0, by definition (Eq. 5), the eccentricity vector or the perigee of the orbit is Sun-pointing. The induced eccentricity is related to the spacecraft area-to-mass ratio, A/m, by the following equation [7].

$$e_f = g/z = 0.01S(A/m) \tag{8}$$

For typical communications satellites with large solar arrays, the A/m values vary from 0.03 to 0.04 m^2/kg. From the above relation (Eq. 8), e_f = 0.0004 to 0.0005 assuming S=1.25. Therefore, most of the GEO stationkeeping maneuvers with tight longitude control tolerance, ± 0.1 deg, follow the Sun-pointing strategy. This property should be preserved after the end-of-life re-orbit in order to minimize the eccentricity variations of the disposal orbit.

3. NUMERICAL INTEGRATION

Two independent approaches for numerical integration were employed to generate the 100-year orbit histories. They are the semi-analytical mean orbit propagation and the N-body numerical integration. The Aerospace programs representative of these two approaches are, respectively, GEOSYN [6] and TRACE [4]. The perturbing forces included are: Earth gravity harmonics (6x6 EGM 96 in TRACE and 6x6 WGS 84 in GEOSYN), lunisolar attractions and solar radiation pressure. GEOSYN was used in most of the initial analysis due to its high speed in computing 100-year orbit histories. The high-precision N-body numerical integration tool, TRACE, was employed to generate all the long-term orbit results presented in this paper. A few cases of 100-year integration from TRACE were verified by another high-precision semi-analytic propagation tool, MEANPROP [8]. TRACE results are more precise in computing perigee altitudes that include both long and short period oscillations.

The baseline orbital elements of a GEO disposal orbit for 100-year integration are determined from the assumed initial eccentricity of 0.0005 and A/m = 0.035 m^2/kg. The IADC recommended altitude increase is

$$\begin{aligned} \Delta H &= 235 + 1000\ S\ A/m \\ &= 235 + 1000\ 1.3\ (0.035) \\ &= 280.5\ \text{km} \end{aligned} \tag{9}$$

The corresponding semi-major axis is 42467.6 km, e= 0.0005, i = 0.1 deg, Ω = 90 deg, ω = 0 deg, and M = 0 deg. A Sun-pointing geometry is selected by assuming an epoch of 1 July 2018. Figure 2 illustrates the sun-pointing geometry of this example. A midnight-pointing perigee is achieved for the same initial orbital elements by changing the epoch to 1 January 2018.

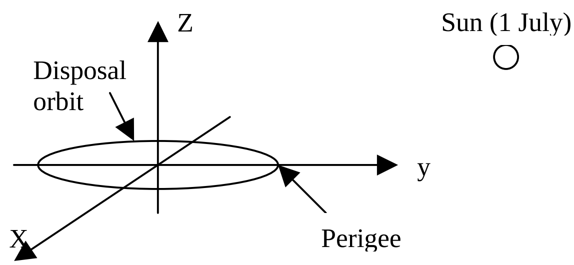

Figure 2. Sun-pointing geometry with 1 July epoch

Figure 3 shows the perigee height history of the 100-year integration using the high-precision TRACE. The perigee height history is well behaved with perigee never below GEO+250 km. However, when the perigee is pointing to the local midnight, a much larger oscillation in perigee height is found in the 100-year integration (Fig. 4). The minimum perigee height above GEO is approaching 200 km. This new finding clearly suggests that the reorbit insertion should be performed with perigee pointing to the Sun. A long-period variation with a period of about 10.5 years, caused by Sun/Moon attractions (Eq. 3) is apparent in both these two cases (Figs 3 and 4) as well as the other 100-year eccentricity histories.

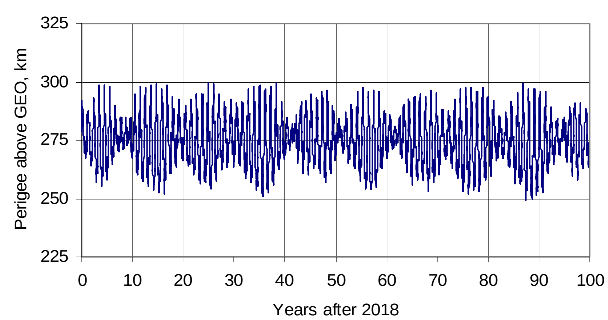

Figure 3. 100-year perigee history (Sun- pointing)

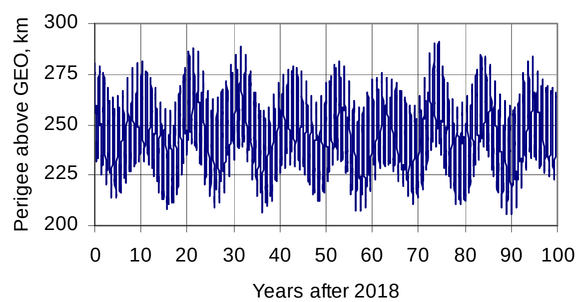

Figure 4. 100-year perigee history (point to midnight)

4. SENSITIVITY TO EPOCH

Several cases of 100-year integration were repeated using TRACE to study the sensitivity of perigee height to epoch. The initial orbit elements are the same as before for each case at July 1 or January 1 of each epoch year. In each 100-year integration, only the minimum perigee height above GEO is recorded and plotted as in Figure 5. The upper curve represents the minimum perigee height for July 1 epoch (Sun-pointing), and the lower curve represents the minimum perigee height for January 1 epoch (perigee pointing to midnight). The epoch is varied from January 1, 2005 to July 1, 2025. Figure 5 clearly shows the sensitivity to epoch.

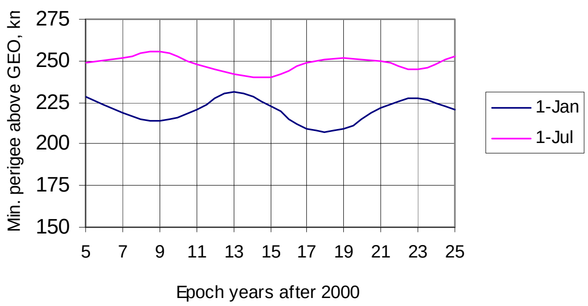

Figure 5. Minimum perigee height vs. epoch

In order to see if there is a seasonal variation, the 100-year integration is repeated with the same initial orbital elements as before and epoch on first day of each month for four selected years (2005, 2009, 2014 and 2018). Figure 6 clearly shows the seasonal variation of the minimum perigee heights. The four curves in Fig 6 reveal another interesting fact that the peak of the seasonal variation changes to valley in about every 4 years, or ¼ of the 18.6-year lunar cycle. The same trend can be seen in Figure 5 where the peaks occur in summer (July 1) for 2009 and 2018.

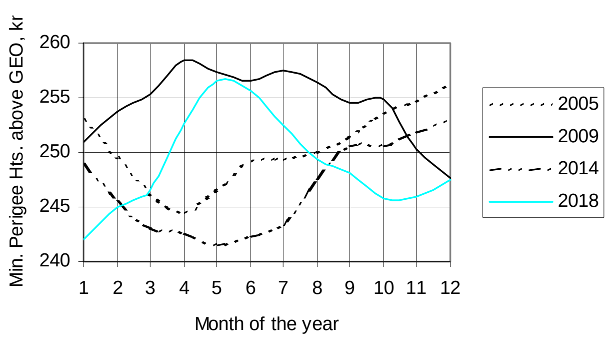

Figure 6 Minimum perigee height vs. month of the year

The above finding reveals that the IADC recommended altitude increase (280.5 km) yields adequate clearance, 250 km or higher, for 100 years if the orbit insertion is performed near the peak season (Figs. 5 and 6) following Sun-pointing geometry.

5. SENSITIVITY TO PERIGEE POINTING

When the perigee is pointing to a direction away from the Sun or midnight, the minimum perigee height from the 100-year integration changes. Figure 7 shows the sensitivity to perigee pointing as a function of the combined angle, $\omega+\Omega$, for two epochs. On July 1, the Sun-pointing geometry occurs when $\omega+\Omega$ is 90 deg. On January 1, the Sun-pointing geometry occurs when $\omega+\Omega$ is 270 deg. The results clearly show that highest value of minimum perigee height occurs when perigee is Sun pointing .

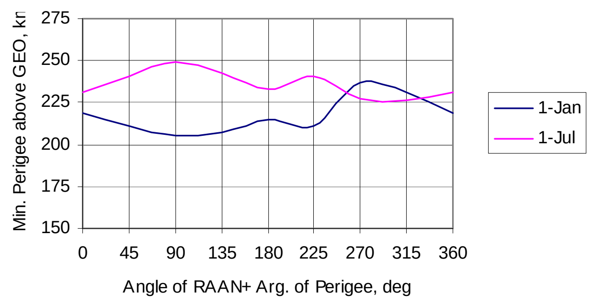

Figure 7. Minimum perigee height vs. (•+Ω)

6. SENSITIVITY TO INITIAL ECCENTRICITY

Based on the findings of earlier studies [1, 2] and the averaged equation (Eq. 3) in eccentricity, the initial eccentricity of a GEO disposal orbit should be small to minimize the long-term variations in eccentricity. Figure 8 shows the minimum perigee height above GEO determined from 100-year integration versus initial eccentricity of disposal orbit. The same set of initial orbital elements is assumed with two epochs, 1 July and 1 January 2018. All the cases satisfy the initial perigee altitude increase, ΔH, determined by the IADC formula (280.5 km).

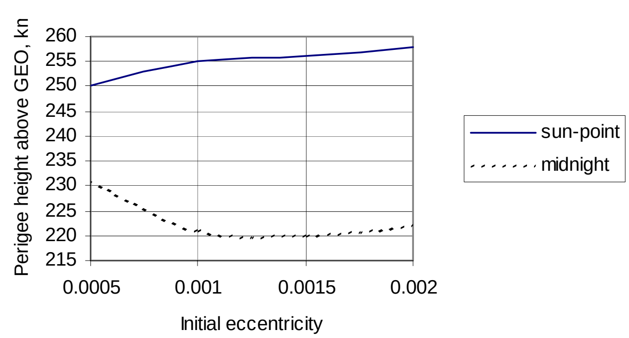

Figure 8. Minimum perigee height above GEO vs. initial eccentricity

The results indicate that smaller eccentricity at 0.0005 tends to keep the minimum perigee higher at 230 km when the perigee is pointing to midnight (dash curve in Fig 8). The results also show that larger initial eccentricity actually increases the minimum perigee height if the Sun-pointing geometry is followed (solid curve in Fig 8). However, larger initial eccentricity with the same initial perigee altitude increase (280.5 km) requires more fuel, which is undesirable for mission operations. Figure 9 gives an estimate of the ΔV requirement as a function of initial eccentricity of a GEO disposal orbit with initial altitude increase of 280.5 km.

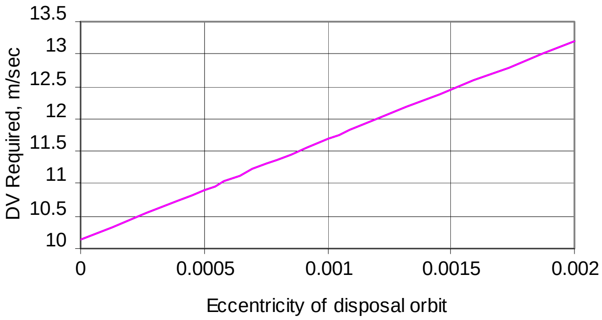

Figure 9. Δ V required for reorbit vs. eccentricity

As seen from the linear relation in Fig 9, an initial eccentricity of 0.002 will cost 2 m/sec or 20% more in ΔV than that from a 0.0005 eccentricity.

7. PREDICTED PERIGEE HISTORIES OF INACTIVE SATELLITES

In Reference 9, Jehn and Hernandez analyzed the active and inactive GEO satellites/objects for the years 1997-2000. They divided the inactive objects into three groups: 1) objects reorbited in compliance with the IADC recommendation, 2) objects moved into orbits with a perigee lower than the IADC recommendation, and 3) objects abandoned in libration orbits. As part of this study, objects of the three groups were propagated for 100 years using TRACE with the 2-line elements dated November 11, 2004. Due to the large volume of 100-year plots, only the minimum, average, and maximum values of perigee height above GEO for each 100-year propagation are plotted. Figure 10 is a plot of min./ave./max. perigee heights for group #1. Nearly all the predicted minimum perigee heights are 200 km or higher than GEO altitude.

The corresponding perigee heights predicted for the second group with inadequate perigee increase are plotted in Fig 11. All the objects, except the second object, have minimum perigee heights less than 200 km above GEO.

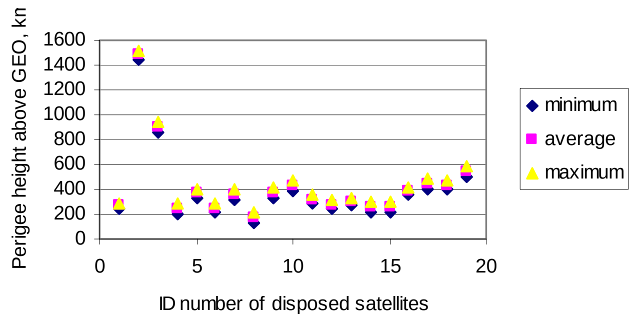

Figure 10. Predicted perigee heights of disposed GEO satellites following IADC recommendation

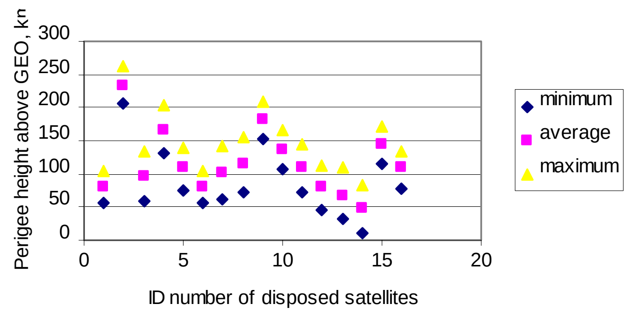

Figure 11. Predicted perigee heights of disposed GEO satellites with inadequate perigee increase

The third group of inactive GEO satellites includes the objects abandoned in the GEO ring and following the longitude libration about the two stable longitudes, 75 deg E and 255 deg E.

Figures 12, 13 and 14 show the longitude histories of three groups of GEO objects in libration about the two stable equilibrium longitudes. The majority of the GEO objects librates about the 75 deg E longitude as shown in Fig 12. Only three objects librate about the 255 deg E longitude (Fig 13). It is interesting to observe that six objects actually move around the two stable points in very long periods, longer than 2000 days (Fig. 14).

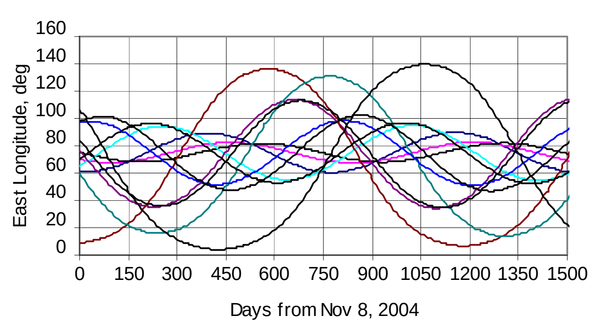

Figure 12. Longitude histories around 75° longitude

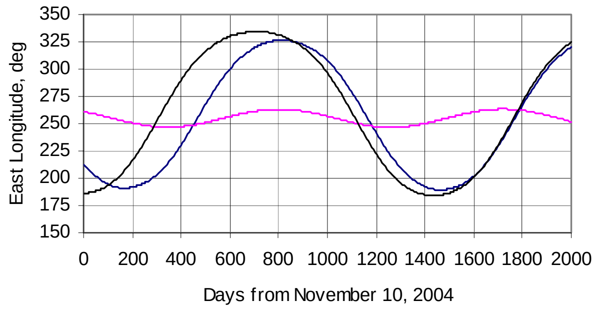

Figure 13. Longitude histories around 255° longitude

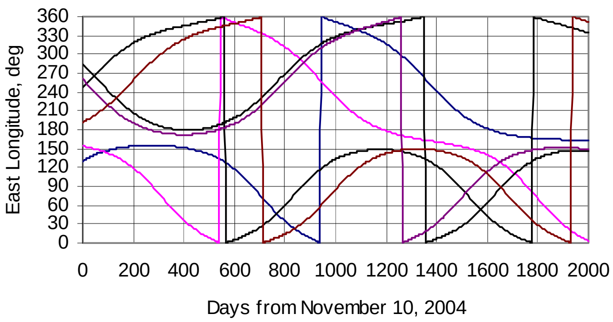

Figure 14. Longitude histories around both stable points

8. ESTIMATED UNCERTAINTY OF PERIGEE HEIGHT PREDICTION

The uncertainties of the long-term predictions of perigee altitudes by TRACE may be estimated from the uncertainties in predicting semi-major axis and eccentricity. The major source of error in predicting semi-major axis and eccentricity of a super-synchronous orbit comes from the uncertainty in modeling the solar radiation pressure that is proportional to the effective spacecraft area-to-mass ratio. Due to the uncertainty in the attitude of an inactive spacecraft, it is difficult to estimate accurately the effective area-to-mass ratio. In the 100-year predictions, a constant effective A/m of 0.03 m^2/kg with a value of 1.3 for S is assumed for all cases. This is believed to be a conservative estimate for solar radiation pressure effects. Actual values may be smaller and thus the induced minimum perigee height would be larger. Numerical tests show that a 20% decrease in A/m will increase the minimum perigee height by about 5 km.

9. CONCLUSIONS

The long-term perigee variations of GEO disposal orbits were analyzed. These 100-year perigee variations were propagated by a high-precision numerical integration tool, TRACE. Based on the results of analytical and numerical investigations, the following conclusions are evident.

1) Long-term eccentricity or perigee stability of GEO disposal orbits can be significantly improved by having the initial perigee pointing to the Sun and performing the reorbit in the most favorable season of the year.

2) With the IADC recommended perigee increase, the minimum perigee height can stay higher than GEO+250 km if the reorbiting is done at the most favorable conditions. The GEO+250 km minimum perigee height not only provides a safe cushion for the 200-km limit but also allows adequate clearance for the 3 deg/day longitude drift required by many of the active GEO satellites.

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