PRECISE LONG-TERM PREDICTION OF THE SPACE DEBRIS OBJECT MOTION

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ABSTRACT

For solving some tasks connected with the space debris (SD) problem it is necessary to predict the space object motion with the high accuracy within enough long period of time. Forecasting the dangerous conjunctions of the active space crafts and especially manned vehicles with SD objects; an assessment of the collision risk; re-entry prediction; maintenance of exact space object (SO) catalogues and some other problems are the examples of such tasks.

The high-effective tool (methods and software) for precise computation of different space object orbits is developed in Russian MCC. This tool is based on the modern universal method of numerical integration of the SO motion equations. The given tool is successfully used in MCC for solving the orbit determination and prediction problems. The brief information on a method and software is given. The results of the application of the considered tool to the computation of the different types of the Earth satellite orbits are presented.

1. A NEW ADVANCED METHOD FOR NUMERICAL INTEGRATION OF THE SPACE OBJECTS MOTION EQUATIONS

For the solution of a wide spectrum of the research and applied problems connected with the determination and prediction of SO motion, including the motion of uncontrollable space debris objects, the new universal method of numerical integration of the differential equations of space dynamics has been developed in MCC recently (Kolyuka, 1995).

This still not well known original method can provide the high computing efficiency at integration of the celestial mechanics and space dynamics equations. It belongs to the methods of the high order. This method is constructed on the base of the implicit interpolation-iteration scheme and can be adjusted to the any order of the approximating formula. At the elaboration of this method the authors want it to possess all the most effective advantages of other modern methods as well as to have its own certain advantages.

1.1. The description of a method

The main idea of the method is to find a solution of the differential equations

$$\ddot{\bar{x}} = \bar{f}\left(\bar{x}, \dot{\bar{x}}, t\right) \qquad (1)$$

at each step, having a length H, in the form of the interpolation polynomials:

$$\bar{x}(t) = \sum_{j=-m}^{m} \bar{x}\left(t_{i_j}\right) L_j(\tau),$$

$$\dot{\bar{x}}(t) = \sum_{j=-m}^{m} \dot{\bar{x}}\left(t_{i_j}\right) L_j(\tau),$$

$$L_j(t) = c_j \prod_{\substack{k=-m,\\ k\neq j}}^{m} \left(t - t_{i_k}\right).$$

The nodes of these polynomials is a system of the points:

$$t_{i_j} = t_{i_0} + H\tau_j ,$$

where $\tau_j \in \left[-1,1\right]$, are the symmetric to $\tau_0 = 0$ independent basic step points,
and $-m \le j \le m$, so that total number of points in the given set is $n = 2m + 1$. Clearly, that the value n defines the order of the approximating formula of the method.

These polynomials are the symmetric ones relatively initial point of a current step and present a solution of the equations within some vicinity of this point. Thus, the solution can be obtained with an equal precision at any point of this interval both on the right and on the left from the indicated initial point.

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The system of base points $\{\tau_j\}$ is constructed in a special way. For any n ($n \geq 9$) the set of points $\{\tau_j\}$ is rather close in some sense to the roots of Chebyshev polynomials of a corresponding degree, and at the increasing of the method order all the basic points from the previous set are the components of a new set.

For the current step performance a "history" of solution is used, i.e. a solution's behavior at the previous step. Applying the extrapolation formula this solution is predicted to the current step where it is improved by means of the "corrector" procedure. In the course of realization of the "corrector scheme" the especial pre-computed with high accuracy "matrixes of integration" B1, B2 are used. The elements of these matrixes are:

$$B1_{jk} = \int_0^{\tau_k} L_j(\tau) d\tau ,$$

$$B2_{jk} = \int_0^{\tau_k}\int_0^{u} L_j(u) du d\tau .$$

The method allows checking sharply the precision of the solution at the step.

The methodical errors $\delta_n x$ for $\bar{x}$ and $\delta_n \dot{x}$ for $\dot{\bar{x}}$ at any point inside the interval $\left[t_{i_0} - H, t_{i_0} + H\right]$ can be estimated as following:

$$\delta_n x \leq MH^{n+1} \left|\tilde{\omega}(\tau)\right| ,$$

$$\delta_n \dot{x} \leq (n+2) MH^{n+1} \left|\tilde{\omega}(\tau)\right| ,$$

where

$$M = \max_{t \in \left[t_{0_i} - H, t_{0_i} + H\right]} \left|\bar{x}_{n+1}\right| ,$$

$$\tilde{\omega}(\tau) = \prod_{j=-m}^{m} \left(\tau - \tau_j\right) = \tau\left(\tau^2 - \tau_1^2\right)\ldots\left(\tau^2 - \tau_m^2\right).$$

At the same time, it can be shown, that in a selected basic system of points $\left\{\tau_j\right\}$ is always done:

$$\left|\tilde{\omega}(\tau)\right| << 1 .$$

There is an effective way of an optimal length of step H_0 selection. The H_0 value for the given integration step is determined from:

$$\left|\tilde{\Delta}^{(n)} \bar{f}\right| \frac{H_0^n}{H^n} = \varepsilon ,$$

so,

$$H_0 = H \sqrt[n]{\frac{\varepsilon}{\left|\tilde{\Delta}^{(n)} \bar{f}\right|}} ,$$

where ε - the given value of precision;
H - a value of current integration step, for which interpolation polynomials have been built,
$\tilde{\Delta}^{(n)} \bar{f}$ - vector, having as its consisting parts the separated differences of functions that present right parts of the Eq. 1 in the form:

$$\tilde{\Delta}^{(n)} f_\nu = \sum_{j=-m}^{m} \tilde{c}_j f_\nu \left(t_{i_j}\right),$$

and $\tilde{c}_j$ is a set of coefficients, which is determined by a special way.

The method allows enough simple to fulfill a starting step if the initial conditions $\left\{\bar{x}(t_0), \dot{\bar{x}}(t_0)\right\}$ of SO motion are known.

1.2. Features and advantages of a method

The main features and advantages of this method are the following:

- Applicability of the method for solving the differential equations with arbitrary complex right parts.
- The equal high computational effectiveness for all types of the celestial bodies' orbits.
- Simplicity of the method adjustment to any given order (from among possible) and methodical accuracy level.
- Reliable and rather simple control of a step length's value in the course of computation.
- Achievement of any given level of the methodical accuracy and keeping it within a long-term integration of the space objects motion equations.

- Provision of high speed of orbit computation.
- Simplicity of obtaining with required precision of large number of close in time output results (ephemeris).
- Possibility to change the method order in the course of fulfillment of the current step of integration.

2. HIGH-EFFECTIVE TOOL (SOFTWARE) FOR PRECISE PROPAGATION OF THE SPACE OBJECT ORBITS

2.1. The principal points of Software

Software for high-precise propagation of space objects motion based on the method considered above has nowadays two realizations in MCC: in FORTRAN and C++ languages.

These two variants of Software can function on various hardware and system software platforms, in particular, under operational systems UNIX, Windows and MS DOS. For the second variant of Software written in language C++, the convenient graphic user interface in JAVA language is developed.

Software for high-precision propagation of SO motion consists of two mainframes (blocks):

- Block of integration of the differential equations of SO motion, i.e. actually the integrator;
- Block of calculation of the right parts of Eq. 1 realizing chosen model acting on SO forces.

Except this Software includes a number of other additional programs, subroutines, functions and the procedures that are necessary for the maintenance of mentioned blocks working. Here it may be attributed: the programs realizing various models of the Earth atmosphere density and Earth gravitational fields; programs of calculation of the Moon, the Sun and planets ephemeris; programs of the Earth rotation parameters calculations; procedures of transition from one systems of coordinates to the others; the procedures of time transformations; etc.

Developed Software allows carrying out the propagation of motion of both one single celestial body and two or more various SO simultaneously. At the same time in the last case the model of acting forces, the method order and accuracy level can be used individually for each of considered SO.

2.2. Models of Earth satellite motion, realized in Software

2.2.1. Used coordinate systems

As a basic coordinate system (CS) for calculation of Earth artificial satellite (EAS) orbital motion parameters there will be considered the rectangular inertial coordinate system (ICS) which is determined by the position of mean Earth equator and equinox at the standard epoch J2000. Besides, during the realization of integration of the equations of SO motion it is used also rotating rectangular CS, which *XY* plane coincides with a plane of true equator of the Earth, and *OX* axis lays in a plane of Greenwich meridian. Transition from one system of coordinates to another is carried out on known procedures.

2.2.2. Models of acting forces and the equations of SO motion

For the description of the EAS motion the relevant model of the acting forces is accepted. This model generally takes into account:

- Influence of a non-central gravitational field of the Earth,
- Aerodynamic atmosphere drag (for low-Earth artificial satellites),
- Gravitational attraction from the Moon and the Sun,
- Solar pressure upon a SO surface,
- Some other small disturbances.

The corresponding equations of SO motion generally are represented as following:

$$\ddot{r} = -\mu \frac{\bar{r}}{r^3} + MgradU\left(\bar{r}'\right) + \sum_{L,S} \mu_{L/S} \left[\frac{\bar{r}_{L/S} - \bar{r}}{\left|\bar{r}_{L/S} - \bar{r}\right|^3} - \frac{\bar{r}_{L/S}}{r^3{}_{L/S}} \right] + \\ + \bar{F}_{atm} + \bar{F}_{Spr} + \bar{F}_{add} \, ,$$

where

$\bar{r}$ − radius-vector of SO position referred to the inertial CS,

$\bar{r}' = M^T \bar{r}$ – radius-vector of SO referred to the Greenwich CS (M - a matrix of transition between systems of coordinates, T – the symbol of matrix transposition);

μ, $\mu_{L/S}$ − gravitational constants of the Earth and other gravity bodies (L – Moon, S - Sun) correspondingly;

$\bar{r}_L$, $\bar{r}_S$ − geocentric vector of the Moon and the Sun positions,

$U(\bar{r}')$ − non-central part of the Earth gravity potential, which is presented by decomposition in a series of the spherical functions,

$$\bar{F}_{atm} = -\frac{S_b}{2}\rho(\bar{r}', t)\cdot\left|\dot{\bar{r}}'\right|\cdot\dot{\bar{r}}',$$

S_b − ballistic coefficient, ρ − density of an atmosphere,

$\bar{F}_{add}$ − accelerations caused by additional small perturbing forces.

3. RESULTS OF SOFTWARE APPLICATION FOR HIGH-PRECISE PROPAGATION OF VARIOUS ORBITS OF EARTH ARTIFICIAL SATELLITES

For estimation of the Software possibilities in sense of maintenance of required accuracy and speed of calculations at long-term propagation of various orbits of the Earth artificial satellite (EAS) the relevant computation results were obtained. Five different classes of the typical Earth artificial satellites orbits have been considered:

1). SO orbit on which the object was 3 weeks prior to entering the Earth low atmosphere (an orbit of re-entering SO).
2). SO orbit close to the orbit of international space station (ISS).
3). Geosynchronous orbit on which the meteorological SC "Meteor - 3M" is flying now.
4). Geostationary orbit (GSO).
5). High-eccentricity orbit close to an orbit of a communication satellite "Molniya".

The parameters of the chosen orbits are resulted in Tab. 1.

The propagation of the SO motion for considered orbits was carried out at the use of the following models of acting forces (see Tab. 2).

As it was already mentioned, the Software allows doing enough simple and convenient adjustment of a method for the required level of accuracy. Such a kind of adjustment can be carried out by changing both the method order *n* and accuracy parameter ε.

Table 1. Parameters of the considered EAS orbits

Type of orbit	Re-entering	ISS	Meteor-3M	GSO	Molniya
Altitude, *H*	300 *km*	400 *km*	1020 *km*	36000 *km*	2850÷37520 *km*
Eccentricity, *e*	0.001	0.002	0.003	0.0	0.653
Inclination, *i*	81.2°	51.6°	99.6°	0.0°	64.5°
Period, *P*	90.5 *min*	92.5 *min*	105.3 *min*	23.9 *hours*	11.95 *hours*

Table 2. Models of acting forces

SO type	Re-entering	ISS	Meteor-3M	GSO	Molniya
Geopotential	8×8	8×8	32×32	32×32	8×8
Atmosphere	+	+	+	–	–
Moon + Sun	–	–	+	+	+
Solar pressure	–	–	+	+	+

For an estimation of Software computing characteristics for each chosen orbit the method order n and accuracy parameter ε was selected so that the required level of accuracy of integration was provided. The estimation of propagation accuracy was carried out by a method of direct and reverse integration of the equations of SO motion. In this case at a direct integration in the typical points allocated on the chosen time interval, the orbital ephemeris were calculated, which are then compared with the results of similar calculations in the same points on a reverse integration after changing (in the last point of a chosen interval) directions of time run. The estimation of the speed of calculation was based on expenses of the computer processor time that required for the performance of a full cycle of calculation of orbits within the chosen interval of the propagation. The resulted characteristics on the computation speed correspond to the expenses for calculations of orbits with the help of the given Software, at its use on Pentium-4 (frequency of 2 GHz) under the operational system Windows-2000.

The Software computing characteristics reflecting its possibilities at numerical integration of the motion equations of examined SO for chosen propagation intervals are given in Tab. 3. Here the assigned levels of accuracy on SO position in an orbit and expenses of processor time of the computer corresponding to these levels for calculations T_c are resulted for each orbit. The level of accuracy is determined as

$$\Delta\rho = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} ,$$

where Δx, Δy, Δz - are the calculation errors in SO coordinates, obtained as a result of integration.

Table 3. Characteristics of numerical integration

Type of orbit	Re-entering			ISS			Meteor-3M			GSO			Molniya		
Propagation interval	10 days (160 orbits)			30 days (470 orbits)			600 days (820 orbits)			120 days (120 orbits)			120 days (240 orbits)		
$\Delta\rho$, m	100	10	1	50	10	1	10	1	0.01	1	0.01	0.001	10	1	0.01
T_c, c	6	9	12	20	23	50	85	122	200	44	60	83	24	28	40

Making comments on the results presented in the table, it is necessary to notice, that time spent on numerical prediction of EAS orbits grows in process of complication of used acting forces models. First of all it concerns the quantity of taken into account harmonics in decomposition of gravitational potential of the Earth. As a whole, the developed Software can provide a higher accuracy of propagation of the orbital motion for SO, not being under the influence of an atmosphere drag. It can be explained, in particular, by that circumstance, that the models of an atmosphere used at the calculations are not smooth enough functions for the precise approximating formula that used in a given method.

4. CONCLUSION

The special Software constructed on the base of a new universal method of the numerical integration is a highly effective tool for the propagation of various space object orbits. Practically for all types of the artificial satellite orbits, the given tool allows to achieve a high accuracy of the SO motion prediction on long-term propagation intervals at rather low expenses of the computation time.

This tool is successfully applied in MCC for solving the different problems connected to the orbital dynamics of space debris.

5. REFERENCES

Kolyuka, Yu.F., Margorin, O.K., The new high-effective method for numerical integration of space dynamics differential equation, *Proceedings of the Spaceflight Dynamics conference*, Toulouse, - France, 1995.